Math 231: Test 3A
Fall 2016
Instructor: Linda Green

- Calculators are NOT allowed.
- Please code true/false and multiple choice answers on a scantron. These are questions 1 - 11.
- Since you have test version A, please code the "Sequence Number" on the scantron as 111111 (all 1's).
- No partial credit for multiple choice / no work needs to be shown.
- For short answer questions, you MUST SHOW WORK for full and partial credit unless otherwise specified.
- Sign the honor pledge below after completing the exam.

First and last name .....

PID .....

UNC Email .....

Honor Pledge: I have neither given nor received unauthorized help on this exam.

Signature: .....

For the True False questions, assume all functions have first and second derivatives that exist and are continuous on $(-\infty, \infty)$. Remember that True means always true, and False means sometimes or always false.

1. (2 pts) If $f'(5) = 0$ and $f''(5) = 2$, then f has a local max at $x = 5$.
    A. True
    B. False

2. (2 pts) Suppose $f(x) > 0$ and and $f(x)$ is decreasing on $[0, 5]$. A Riemann sum using left endpoints will be an overestimate of $\int_0^5 f(x)\, dx$.
    A. True
    B. False

3. (2 pts) Suppose $F(x)$ is an antiderivative of $f(x)$. Then $\sin(F(x))$ is an antiderivative of $\cos(f(x))$.
    A. True
    B. False

4. (2 pts) If $f(x) < g(x)$ on $[0, 10]$,then $\int_0^{10} f(x) - g(x)\, dx < 0$
    A. True
    B. False

5. (2 pts) If $f'(1) = 0$ and $f'(3) = 0$, then $f(c) = 0$ for some c between 1 and 3.
    A. True
    B. False

6. (5 pts) For a differentiable function $f(x)$, $f(1) = 5$ and $f(2) = 5$ and $f(4) = 6.5$. Which of the following must be true about the derivative $f'(x)$?
    1. $f'(x) = 0.5$ for some x-value
    2. $f'(x) = 1.5$ for some x-value
    3. $f'(x) = 2$ for some x-value
    4. $f'(x) = 5.75$ for some x-value
    5. None of these have to be true.

7. (5 pts) If $\int_2^5 (2f(x) - 6)\,dx = -10$, then what is $\int_2^5 f(x)\,dx$?

   A. -4

   B. -2

   C. 4

   D. 8

   E. 10

8. (5 pts) Suppose that $g(x)$ is a continuous function. Some of the values of $g(x)$ are given in the table below.

x	-1	1	3	5
$g(x)$	-2	3	5	6

   Use a Riemann sum with $\Delta x = 2$ and right endpoints to estimate $\int_{-1}^5 g(x)\,dx$.

   A. 12

   B. 14

   C. 24

   D. 28

   E. 48

9. (5 pts) Express $\int_3^8 (4 - x)\,dx$ as the limit of a Riemann sum using right endpoints.

   A. $\lim_{n\to\infty} \sum_{i=1}^{n} \left(4 - \frac{5i}{n}\right)$

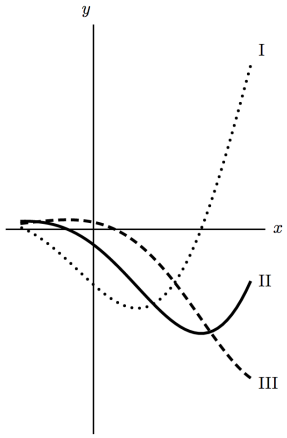
   B. $\lim_{n\to\infty} \sum_{i=1}^{n} \left(7 - \frac{5i}{n}\right)$

   C. $\lim_{n\to\infty} \sum_{i=1}^{n} \left(1 - \frac{5i}{n}\right)\frac{5}{n}$

   D. $\lim_{n\to\infty} \sum_{i=1}^{n} \left(3 - \frac{5i}{n}\right)\frac{5}{n}$

   E. $\lim_{n\to\infty} \sum_{i=1}^{n} \left(4 - \frac{5i}{n}\right)\frac{5}{n}$

10. (5 pts) The figure shows the graphs of $y = f(x)$, $y = f'(x)$, and $y = f''(x)$. Which one is which?



A. $f(x)$: I, $f'(x)$: III, $f''(x)$: II

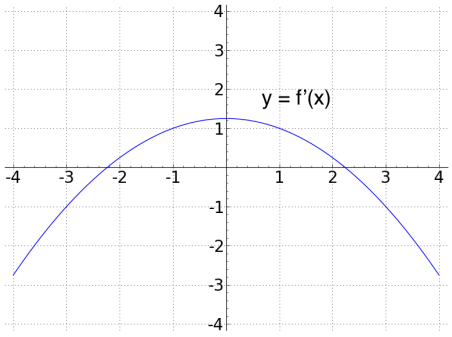
B. $f(x)$: II, $f'(x)$: I, $f''(x)$: III

C. $f(x)$: II, $f'(x)$: III, $f''(x)$: I

D. $f(x)$: III, $f'(x)$: I, $f''(x)$: II

E. $f(x)$: III, $f'(x)$: II, $f''(x)$: I

11. (5 pts) Given the graph of the DERIVATIVE $f'(x)$ defined on the interval $[-4, 4]$ and drawn below, find the interval(s) on which the ORIGINAL FUNCTION $f(x)$ is concave up.



A. $(-4, 0)$

B. $(0, 4)$

C. $(-2.2, 2.2)$

D. $(-4, -2.2) \cup (2.2, 4)$

E. $f(x)$ is never concave up

12. (16 pts) Suppose $f(x)$ is a differentiable function defined for all x. The *derivative* and *second derivative* of $f(x)$ are given by:

$$f'(x) = 3(x+4)(x+2)^{1/3}$$

$$f''(x) = \frac{4x+10}{(x+2)^{2/3}}$$

Answer the following questions, and write "None" if the requested feature does not occur.

(a) What are the critical number(s) for $f(x)$?

$x =$ [ ]

(b) What are the critical number(s) for $f'(x)$?

$x =$ [ ]

(c) At what x-values does $f(x)$ have local max(es)? local min(s)?

MAX at $x =$ [ ] MIN at $x =$ [ ]

(d) At what x-values does $f(x)$ have inflection point(s)?

$x =$ [ ]

(e) On what interval(s) is $f(x)$ decreasing?

Answer: [ ]

(f) On what interval(s) is $f(x)$ concave down?

Answer: [ ]

(g) On what interval(s) is $f(x)$ both decreasing and concave down?

Answer: [ ]

13. (16 pts) Suppose $f''(x) = \dfrac{3}{\sqrt{x}} + 5\cos(x)$, $f(0) = 0$, and $f(\pi) = 10$. Find $f(x)$.

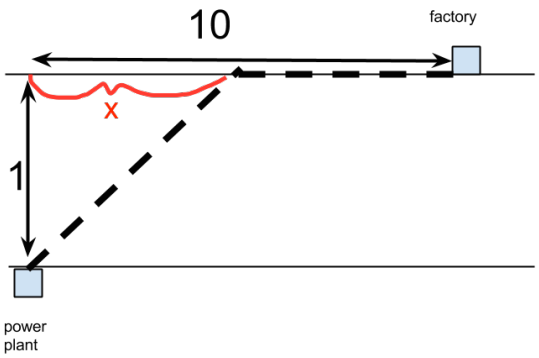
f(x) =

14. (14 pts) Evaluate $\lim_{x\to\infty}\left(1+\frac{2}{x}\right)^{4x}$.

Answer: $\square$

15. (14 pts) Pick ONE of the two questions to answer. For credit, you must use calculus in your solution.

(a) On one side of a river 1 mile wide is an electric power station; on the other side, 10 miles upstream, is a factory. It costs \$300 per mile to run cable over land and \$500 per mile under water. What value of x in the diagram will give the cheapest way to run cable from the station to the factory? Hint: find distances in terms of x, then convert distances to costs.



(b) You wish to make a cylinder with a base and sides but no top. The cylinder must have surface area of 100 cm^2. Give the radius of the cylinder that will maximize volume.

Answer: