Math 231: Test 3A
Spring 2016
Instructor: Linda Green

- Calculators are NOT allowed.
- Please code true/false and multiple choice answers on a scantron. These are questions 1 - 12.
- Since you have test version A, please code the "Sequence Number" on the scantron as 111111 (all 1's).
- No partial credit for multiple choice / no work needs to be shown.
- For short answer questions, you MUST SHOW WORK for full and partial credit unless otherwise specified.
- Sign the honor pledge below after completing the exam.

First and last name .....

PID .....

UNC Email .....

Honor Pledge: I have neither given nor received unauthorized help on this exam.

Signature: .....

1. (2 pts) True or False: $\int_3^7 f(x)\,dx = \int_7^3 f(x)\,dx$

   A. True

   B. False

2. (2 pts) True or False: For a function f that is differentiable on $(-\infty, \infty)$, if $f(-4) = -1$ and $f(4) = 9$, then $f'(x) > 1$ for some x value with $|x| < 4$.

   A. True

   B. False

3. (2 pts) True or False: Suppose f is a function whose second derivative f'' exists and is continuous. If $f'(2) = 0$ and $f''(2) < 0$, then f has a local minimum at $x = 2$.

   A. True

   B. False

4. (2 pts) True or False: For a continuous function $f(x)$, if $f'(x) < 0$ for $x < 0$ and $f'(x) > 0$ for $x > 0$, then f has an absolute minimum at $x = 0$.

   A. True

   B. False

5. (2 pts) If $\lim_{x\to\infty} f(x) = 0$ and $\lim_{x\to\infty} g(x) = \infty$, then $\lim_{x\to\infty} f(x) \cdot g(x) = \lim_{x\to\infty} f'(x) \cdot g'(x)$, provided that this second limit exists.

   A. True

   B. False

6. (5 pts) On what interval is $f(x) = 3x^3 - 36x$ both increasing and concave up?

   A. $(-\infty, 2)$

   B. $(-2, 0)$

   C. $(0, 2)$

   D. $(2, \infty)$

7. (5 pts) Express $\int_2^7 x\,dx$ as the limit of a Riemann sum using right endpoints.

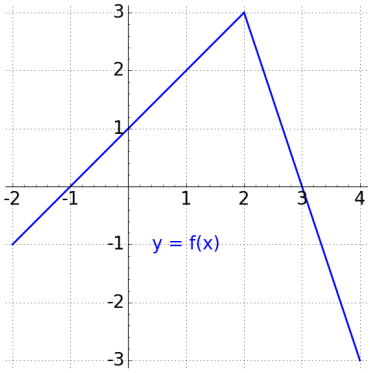
A. $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{5i}{n}$

B. $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{25i}{n^2}$

C. $\lim_{n\to\infty} \sum_{i=1}^{n} 2 + \frac{5i}{n}$

D. $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{10}{n} + \frac{25i}{n^2}$

8. (5 pts) Use the graph of $y = f(x)$ to evaluate $\int_{-2}^{4} f(x)dx$.



A. 2

B. 4

C. 6

D. 8

9. (5 pts) Estimate $\int_0^4 \frac{60}{x+1}\,dx$ using two rectangles and midpoints for sample points.

   A. 45

   B. 64

   C. 90

   D. 112

   E. 160

10. (5 pts) Suppose that we are using Newton's method to estimate $\sqrt{2}$ using the formula $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ and a simple "four function" calculator that does addition, subtraction, multiplication, and division. Which function should we use for $f(x)$?

    A. $f(x) = \sqrt{x}$

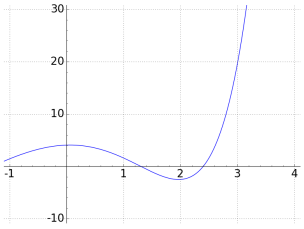
    B. $f(x) = x^2$

    C. $f(x) = \sqrt{x} - \sqrt{2}$

    D. $f(x) = x^2 - 2$

    E. $f(x) = x^2 - 4$

11. (5 pts) Suppose that we wish to use Newton's method to estimate the RIGHT-MOST of the two x-intercepts shown in this graph. Which is the best choice for a starting value x_1?



    A. 0

    B. 1

    C. 2

    D. 3

12. (5 points) The function $f(x) = \sin(x) - \cos(x) + x$ has inflection points at what x-values on the interval $[0, 2\pi]$?

A. $\frac{\pi}{4}$ and $\frac{3\pi}{4}$

B. $\frac{\pi}{4}$ and $\frac{5\pi}{4}$

C. $\frac{5\pi}{4}$ and $\frac{7\pi}{4}$

D. $\frac{\pi}{4}$, $\frac{3\pi}{4}$, $\frac{5\pi}{4}$, and $\frac{7\pi}{4}$

E. No inflection points.

13. (7 pts) Find the general antiderivative of $f(x) = \frac{3\sqrt{x} + 1}{x}$.

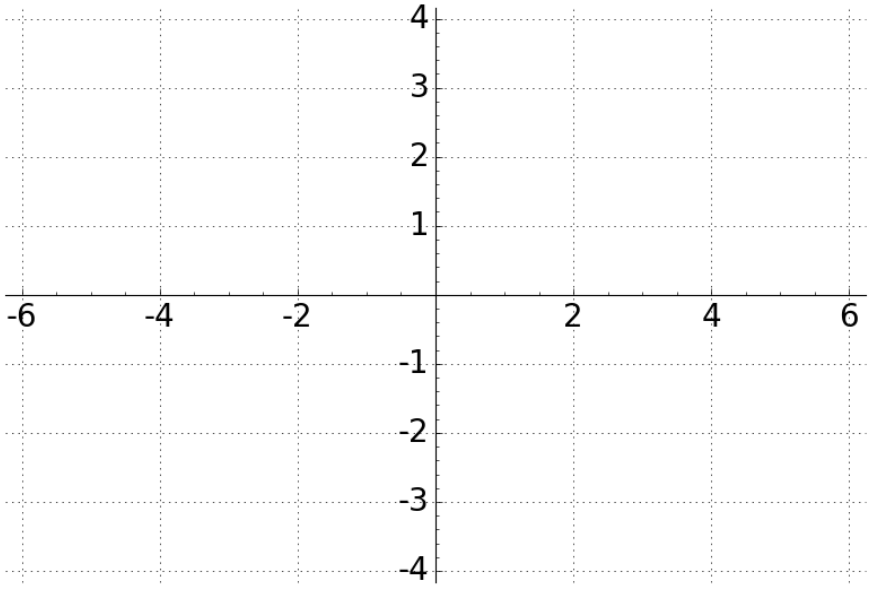
Answer:

14. (12 pts) Evaluate $\lim_{x \to 0^+} (e^{2x} + 4x)^{1/x}$.

Answer: $\boxed{}$

15. (12 pts) Sketch a graph of a function $f(x)$ with the following properties.

- $f(0) = 0$.
- $\lim_{x\to\infty} f(x) = 2$
- $\lim_{x\to-\infty} f(x) = 3$
- $f'(x) < 0$ for $x < 2$ and $f'(x) > 0$ for $x > 2$
- $f''(x) < 0$ for $x < 0$ and for $x > 3$ and $f''(x) > 0$ for $0 < x < 3$
- f has an absolute minimum value of -2



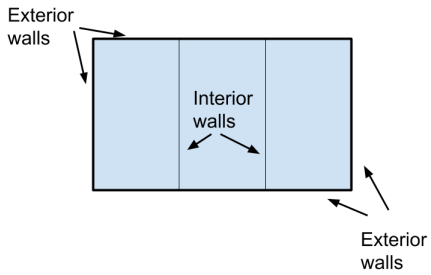
16. (12 pts) Suppose $f''(x) = 4x + \cos(x)$, $f'(0) = 2$, and $f(0) = 5$. Find $f(x)$.

Answer:

17. (12 pts) Pick ONE of the two questions to answer.

(A) Find the x and y coordinates of the point(s) on the graph of $y = 3\sqrt{x}$, closest to the point $(5, 0)$.

(B) You need to make a retangular enclosure with an area of 6000 m^2 that is divided into 3 sections by walls parallel to one of its sides. The external walls cost \$3 per meter in length and the interior walls cost \$2 per meter in length. What are the dimensions that minimize the cost of the walls?



Answer: