Math 231: Test 3A
Spring 2016
Instructor: Linda Green

• Calculators are NOT allowed.
• Please code true/false and multiple choice answers on a scantron. These are questions 1 - 12.
• Since you have test version A, please code the “Sequence Number” on the scantron as 111111 (all 1’s).
• No partial credit for multiple choice / no work needs to be shown.
• For short answer questions, you MUST SHOW WORK for full and partial credit unless otherwise specified.
• Sign the honor pledge below after completing the exam.

First and last name .................................................................

PID .................................................................

UNC Email .................................................................

Honor Pledge: I have neither given nor received unauthorized help on this exam.

Signature: .................................................................
1. (2 pts) True or False: \( \int_{3}^{7} f(x) \, dx = \int_{7}^{3} f(x) \, dx \)
   
   A. True
   B. False

2. (2 pts) True or False: For a function \( f \) that is differentiable on \((-\infty, \infty)\), if \( f(-4) = -1 \) and \( f(4) = 9 \), then \( f'(x) > 1 \) for some \( x \) value with \( |x| < 4 \).
   
   A. True
   B. False

3. (2 pts) True or False: Suppose \( f \) is a function whose second derivative \( f'' \) exists and is continuous. If \( f'(2) = 0 \) and \( f''(2) < 0 \), then \( f \) has a local minimum at \( x = 2 \).
   
   A. True
   B. False

4. (2 pts) True or False: For a continuous function \( f(x) \), if \( f'(x) < 0 \) for \( x < 0 \) and \( f'(x) > 0 \) for \( x > 0 \), then \( f \) has an absolute minimum at \( x = 0 \).
   
   A. True
   B. False

5. (2 pts) If \( \lim_{x \to \infty} f(x) = 0 \) and \( \lim_{x \to \infty} g(x) = \infty \), then \( \lim_{x \to \infty} f(x) \cdot g(x) = \lim_{x \to \infty} f'(x) \cdot g'(x) \), provided that this second limit exists.
   
   A. True
   B. False

6. (5 pts) On what interval is \( f(x) = 3x^3 - 36x \) both increasing and concave up?
   
   A. \((-\infty, 2)\)
   B. \((-2, 0)\)
   C. \((0, 2)\)
   D. \((2, \infty)\)
7. (5 pts) Express \( \int_{2}^{7} x \, dx \) as the limit of a Riemann sum using right endpoints.

A. \( \lim_{n \to \infty} \sum_{i=1}^{n} \frac{5i}{n} \)

B. \( \lim_{n \to \infty} \sum_{i=1}^{n} \frac{25i}{n^2} \)

C. \( \lim_{n \to \infty} \sum_{i=1}^{n} 2 + \frac{5i}{n} \)

D. \( \lim_{n \to \infty} \sum_{i=1}^{n} \frac{10}{n} + \frac{25i}{n^2} \)

8. (5 pts) Use the graph of \( y = f(x) \) to evaluate \( \int_{-2}^{4} f(x) \, dx \).

A. 2
B. 4
C. 6
D. 8
9. (5 pts) Estimate \( \int_{0}^{4} \frac{60}{x + 1} \, dx \) using two rectangles and midpoints for sample points.

A. 45  
B. 64  
C. 90  
D. 112  
E. 160

10. (5 pts) Suppose that we are using Newton’s method to estimate \( \sqrt{2} \) using the formula \( x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \) and a simple “four function” calculator that does addition, subtraction, multiplication, and division. Which function should we use for \( f(x) \)?

A. \( f(x) = \sqrt{x} \)  
B. \( f(x) = x^2 \)  
C. \( f(x) = \sqrt{x} - \sqrt{2} \)  
D. \( f(x) = x^2 - 2 \)  
E. \( f(x) = x^2 - 4 \)

11. (5 pts) Suppose that we wish to use Newton’s method to estimate the RIGHT-MOST of the two x-intercepts shown in this graph. Which is the best choice for a starting value \( x_1 \)?

A. 0  
B. 1  
C. 2  
D. 3
12. (5 points) The function \( f(x) = \sin(x) - \cos(x) + x \) has inflection points at what x-values on the interval \([0, 2\pi]\)?

A. \( \frac{\pi}{4} \) and \( \frac{3\pi}{4} \)
B. \( \frac{\pi}{4} \) and \( \frac{5\pi}{4} \)
C. \( \frac{5\pi}{4} \) and \( \frac{7\pi}{4} \)
D. \( \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \) and \( \frac{7\pi}{4} \)
E. No inflection points.

13. (7 pts) Find the general antiderivative of \( f(x) = \frac{3\sqrt{x} + 1}{x} \).

Answer: 

\[ \int \frac{3\sqrt{x} + 1}{x} \, dx = \text{Answer} \]
14. (12 pts) Evaluate \( \lim_{x \to 0^+} (e^{2x} + 4x)^{1/x}. \)

Answer: 6
15. (12 pts) Sketch a graph of a function $f(x)$ with the following properties.

- $f(0) = 0.$
- $\lim_{x \to \infty} f(x) = 2$
- $\lim_{x \to -\infty} f(x) = 3$
- $f'(x) < 0$ for $x < 2$ and $f'(x) > 0$ for $x > 2$
- $f''(x) < 0$ for $x < 0$ and for $x > 3$ and $f''(x) > 0$ for $0 < x < 3$
- $f$ has an absolute minimum value of $-2$
16. (12 pts) Suppose $f''(x) = 4x + \cos(x)$, $f'(0) = 2$, and $f(0) = 5$. Find $f(x)$. 

Answer: 8
17. (12 pts) Pick ONE of the two questions to answer.

(A) Find the $x$ and $y$ coordinates of the point(s) on the graph of $y = 3 \sqrt{x}$, closest to the point $(5, 0)$.

(B) You need to make a rectangular enclosure with an area of 6000 $m^2$ that is divided into 3 sections by walls parallel to one of its sides. The external walls cost $3$ per meter in length and the interior walls cost $2$ per meter in length. What are the dimensions that minimize the cost of the walls?

Answer: 

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