Math 231: Test 2A
Fall 2016
Instructor: Linda Green

- Calculators are NOT allowed.
- Please code true/false and multiple choice answers on a scantron. These are questions 1 - 13.
- Since you have test version A, please code the "Sequence Number" on the scantron as 111111 (all 1’s).
- No partial credit for multiple choice / no work needs to be shown.
- For short answer questions, you MUST SHOW WORK for full and partial credit unless otherwise specified.
- Sign the honor pledge below after completing the exam.

First and last name ...........................................................

PID ...............................................................

UNC Email ...........................................................

Honor Pledge: I have neither given nor received unauthorized help on this exam.

Signature: ...........................................................
True means always true. False means sometimes or always false.

1. True or False: \( \frac{d}{dx} x^7 = 7x \cdot x^{7-1} \).
   A. True
   B. False

2. True or False: If \( f \) is differentiable and \( f'(x) > 0 \) for all \( x \) in \( (-\infty, \infty) \), then \( f(x) \) has no local extreme points.
   A. True
   B. False

3. (2 pts) True or False: If \( f(x) \) is a differentiable function on \( (-\infty, \infty) \) and \( f'(5) = 0 \), then \( f \) has a local maximum or local minimum at \( x = 5 \).
   A. True
   B. False

4. (2 pts) True or False: If \( f(x) \) is a polynomial, then \( f(x) \) has an absolute minimum value on the interval \([1, 2]\).
   A. True
   B. False

5. (2 pts) True or False: The function \( f(x) = \frac{1}{x} \) has an absolute maximum value on the interval \([-1, 1]\).
   A. True
   B. False
6. (5 pts) A rectangle is morphing in such a way that its length is INCREASING at a rate of 3 cm per minute and the width is DECREASING at a rate of 2 cm per minute. At what rate will the area of the rectangle be changing when the length and width are both 5 cm?

A. the area will be decreasing at a rate of 15 cm² per minute
B. the area will be decreasing at a rate of 10 cm² per minute
C. the area will be increasing at a rate of 5 cm² per minute
D. the area will be increasing at a rate of 15 cm² per minute
E. the area will be increasing at a rate of 25 cm² per minute

7. (5 pts) Suppose the function \( g(t) = \frac{t^3}{3} - 3t^2 + 8t + 5 \) represents the motion of a particle going up and down at time \( t \) for \( 0 \leq t \leq 10 \). Find the intervals for which the particle is speeding up.

A. (3, 10)
B. (4, 10)
C. (0, 2) \( \cup \) (3, 4)
D. (0, 2) \( \cup \) (4, 10)
E. (2, 3) \( \cup \) (4, 10)

8. (5 pts) For the same function \( g(t) = \frac{t^3}{3} - 4t^2 + 15t + 2 \) that represents the motion of a particle going up and down at time \( t \). Which expression represents the total distance traveled by the particle between \( t = 0 \) and \( t = 10 \)?

A. \( |g'(10) - g'(0)| \)
B. \( |g'(10) - g'(3)| + |g'(3) - g'(0)| \)
C. \( |g(10) - g(0)| \)
D. \( |g(10) - g(4)| + |g(4) - g(2)| + |g(2) - g(0)| \)
E. Cannot be determined from this information.
9. (5 pts) Find the derivative of the function. \( r(z) = \ln(\arctan(Mz)) \)

A. \( \frac{M}{Mz(1 + M^2z^2)} \)

B. \( \frac{M}{\arctan(Mz)} \)

C. \( \frac{M}{(1 + z^2) \arctan(Mz)} \)

D. \( \frac{M}{(1 + M^2z^2) \arctan(Mz)} \)

E. \( \frac{M + z}{(1 + M^2z^2) \arctan(Mz)} \)

10. (5 pts) Find the derivative of the function. \( p(x) = 2^{\sec(5x)} \)

A. \( 5 \ln 2 \cdot 2^{\sec(5x)} \)

B. \( 5 \ln 2 \cdot 2^{\sec(5x) \tan(5x)} \)

C. \( 5 \ln 2 \cdot 2^{\sec(5x)} \sec(5x) \tan(5x) \)

D. \( 5 \ln 2 \cdot 2^{\sec(5x)} \sec(x) \tan(x) \)

E. \( 5 \ln 2 \cdot 2^{\sec(5x) \tan(5x)}[\sec(5x) \tan^2(5x) + \sec^3(5x)] \)
A function \( g(x) \) is defined on the interval \([-4, 4]\). The DERIVATIVE \( g'(x) \) is graphed below.

11. (5 pts) Using the graph of the DERIVATIVE \( g'(x) \) above, find the interval(s) on which the function \( g(x) \) is increasing.
   
   A. \((-1, 2)\)
   B. \((-2, -1) \cup (-0.5, 2)\)
   C. \((-3.4, 0) \cup (1.2, 3)\)
   D. \((-4, -2) \cup (0.5, 2)\)
   E. Cannot be determined from this information.

12. (5 pts) Using the graph of the DERIVATIVE \( g'(x) \) above, find all critical points of the function \( g(x) \) on the interval \((-4, 4)\).
   
   A. \( x = 2 \)
   B. \( x = -2, 0.5 \)
   C. \( x = -2, 0.5, 2 \)
   D. \( x = -3.4, 0, 1.2, 3 \)
   E. Cannot be determined from this information.

13. (5 pts) Using the graph of the DERIVATIVE \( g'(x) \) above, find the interval(s) on which the function \( g(x) \) is negative.
   
   A. \((-1, 2)\)
   B. \((-2, -1) \cup (-0.5, 2)\)
   C. \((-3.4, 0) \cup (1.2, 3)\)
   D. \((-4, -2) \cup (0.5, 2)\)
   E. Cannot be determined from this information.
14. The diameter of a circle is measured to be 80 meters with a possible error of 0.3 meters. Use differentials to approximate the error and the relative error in the calculated area of the circle.

\[ \text{error} = \boxed{\text{error}} \quad \text{relative error} = \boxed{\text{relative error}} \]
15. Find the derivative of $f(x) = (4 + 7x)^3$. 
16. Find the slope of the tangent line to the curve
\[
\sin(x - y) = y^2 + xy
\]
at the point \((\pi, 0)\).
17. Find the absolute maximum and minimum values of \( f(x) = xe^{-2x} \) on the interval \([0, 5]\). As needed, you can use the following facts:

- \( f'(x) = e^{-2x} - 2xe^{-2x} \)
- \( f''(x) = 4xe^{-2x} - 4e^{-2x} \)
- \( e \approx 3 \)

Write your final answer as an exact value, not an approximation or decimal.
18. Choose ONE of the following problems to work. You only need to complete one.

(a) A fish is reeled in at a rate of 2 feet per second from a point 10 feet above the water. At what rate is the angle between the line and the water changing when there is a total of 20 feet of line out?

(b) A flood lamp is installed on the ground 200 feet from a vertical wall. A six foot tall man is walking towards the wall at the rate of 10 feet per second. How fast is the tip of his shadow moving down the wall when he is 50 feet from the wall?