

**Math 231: Test 1A**  
**Spring 2016**  
**Instructor: Linda Green**

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- Calculators are NOT allowed.
- Please code true/false and multiple choice answers on a scantron. These are questions 1 - 13.
- Since you have test version A, please code the Section field on the scantron as 111111 (all 1's).
- No partial credit for multiple choice / no work needs to be shown.
- For short answer questions, you must show work for full and partial credit if specified.
- Sign the honor pledge below after completing the exam.

First and last name .....

PID .....

UNC Email .....

Honor Pledge: I have neither given nor received unauthorized help on this exam.

Signature: .....

1. (2 pts) True or False: If  $f$  and  $g$  are continuous at  $x = a$  with  $g(a) \neq 0$ , then  $h(x) = \frac{f(x) + \sin(x)}{g(x)}$  is continuous at  $x = a$ .
- A. True  
B. False
2. (2 pts) True or False: For  $f(x) = \frac{1}{x}$ , since  $f(-2) = -\frac{1}{2}$  and  $f(2) = \frac{1}{2}$ , there must be a number  $c$  between  $-2$  and  $2$  such that  $f(c) = 0$ .
- A. True  
B. False
3. (2 pts) True or False: If  $f(x)$  is continuous at  $x = 2$  then  $f(x)$  is differentiable at  $x = 2$ .
- A. True  
B. False
4. (2 pts) True or False: If  $f(c)$  exists and  $f$  is continuous, then  $\lim_{x \rightarrow c} f(x)$  exists.
- A. True  
B. False
5. (2 pts) True or False: If  $\lim_{x \rightarrow c} f(x) = 0$  and  $\lim_{x \rightarrow c} g(x) = 0$  then  $\lim_{x \rightarrow c} \frac{f(x)}{g(x)}$  is either  $0$ ,  $\infty$ ,  $-\infty$ , or DNE.
- A. True  
B. False
6. (5 pts) Find  $\lim_{x \rightarrow 4} \frac{x + 7}{|x - 4|}$
- A. 1  
B.  $\frac{7}{4}$   
C.  $-\infty$   
D.  $\infty$   
E. DNE

7. (5 pts) Find  $\lim_{x \rightarrow 5} \frac{\sqrt{5} - \sqrt{x}}{(x - 5)}$

A. 0

B.  $\frac{1}{2}$

C.  $-\frac{\sqrt{5}}{10}$

D.  $2\sqrt{5}$

E. DNE

8. (5 pts) Find  $\lim_{x \rightarrow 0} \frac{x}{\tan(5x)}$

A. 0

B.  $\frac{1}{5}$

C. 1

D. 5

E. DNE

9. (5 pts) Find  $\lim_{t \rightarrow \infty} \frac{5}{t^2 e^t}$

A. 0

B. 5

C.  $-\infty$

D.  $\infty$

E. DNE

10. (5 pts) Find  $\lim_{x \rightarrow \infty} (\sin(x) + 5)$

A. 0

B. 5

C. 6

D.  $\infty$

E. DNE

11. (5 pts) If  $f'(-4) = 3$  and  $f(-4) = 10$ , find the equation of the tangent line to  $y = f(x)$  at  $x = -4$ .
- A.  $y = 3x - 4$
  - B.  $y = 3x + \frac{10}{3}$
  - C.  $y = 3x + 10$
  - D.  $y = 3x + 22$
  - E. There is not enough information to determine the equation of the tangent line.
12. (5 pts) The function  $p(t) = t^3 + 2$  represents the depth of a submarine in meters at time  $t$  minutes, as the submarine descends directly down. Find the submarine's average velocity between  $t = 1$  and  $t = 3$ .
- A. 12 meters per minute
  - B. 13 meters per minute
  - C. 15 meters per minute
  - D. 27 meters per minute
  - E. 29 meters per minute
13. (5 pts) As in the last problem, the function  $p(t) = t^3 + 2$  represents the depth of a submarine in meters at time  $t$  minutes, as the submarine descends directly down. Find the submarine's velocity at exactly  $t = 3$ .
- A. 12 meters per minute
  - B. 13 meters per minute
  - C. 15 meters per minute
  - D. 27 meters per minute
  - E. 29 meters per minute

14. (8 pts) Each of the following limits represents the derivative of some function  $f(x)$  at some number  $a$ . State a formula for  $f$  and the value of  $a$ . You DO NOT need to find the limits or show work.

(a)  $\lim_{x \rightarrow -1} \frac{e^x - \frac{1}{e}}{x + 1}$

$f(x) =$

$a =$

(b)  $\lim_{h \rightarrow 0} \frac{(5 + h)^2 - 25}{h}$

$f(x) =$

$a =$

15. (10 pts)  $y = x^3 \cos(x)$ . Find  $\frac{d^2y}{dx^2}$  at  $x = \pi$ . Show work.

16. (6 pts) Find the vertical and horizontal asymptotes (if any) of  $f(x) = \frac{6x - 12}{x^2 - 4}$ .

Horizontal Asymptote(s):

Vertical Asymptote(s):

17. (6 pts) Is it possible to find a value for  $c$  that makes the function  $g(x)$  below continuous? If so, find it. If not, indicate why not. Show work to justify your answer.

$$g(x) = \begin{cases} 3, & \text{if } x \leq 1 \\ cx^2, & \text{if } 1 < x < 2 \\ 10x - 5, & \text{if } 2 \leq x \end{cases}$$

18. (10 pts)  $y = \frac{xe^x}{3x^5 - \tan(x)}$ . Find  $y'$ . Show work. DO NOT simplify.

19. (4 pts) If  $\lim_{x \rightarrow 2} \frac{A(x+B)}{x^2 - 6x + 8} = 5$ , find  $A$  and  $B$ .

$A =$

$B =$

20. (6 pts) The graph of  $y = f(x)$  is drawn below. On the blank axes, sketch a graph of  $y = f'(x)$ .

