1. Calculators are allowed.

2. You must show work for full and partial credit, except where otherwise noted.

3. Give exact values instead of decimal approximations, except where otherwise noted.

4. Sign the honor pledge after completing the exam.
   I have neither given nor received unauthorized help on this exam.
   
   [Signature]
1. (20 pts) Consider the three points $A = (0, 2, 3)$, $B = (-2, 1, 4)$, and $C = (1, 5, 5)$.

(a) Find parametric equations for the line through $A$ and $B$.

(b) Find the area of the triangle with vertices $A$, $B$, and $C$.

(c) Find an equation for the plane $P$ through $A$, $B$, and $C$.

(d) Find the angle that the plane $P$ makes with the $x$-$y$ plane. Give your answer in degrees to the nearest tenth.

\[ \overrightarrow{AB} = \langle -2, -1, 1 \rangle \]

Point $A = (0, 2, 3)$

\[
\begin{align*}
x &= 0 - 2t \\
y &= 2 - t \\
z &= 3 + t
\end{align*}
\]

\[
\overrightarrow{AC} = \langle -1, 3, 2 \rangle
\]

\[
\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix}
2 & 3 & k \\
-2 & -1 & 1 \\
1 & 3 & 2
\end{vmatrix} = \begin{vmatrix}
-1 & 1 \\
3 & 2
\end{vmatrix} c - \begin{vmatrix}
-2 & 1 \\
1 & 2
\end{vmatrix} j + \begin{vmatrix}
-2 & 1 \\
1 & 3
\end{vmatrix} k
\]

\[
= -5 \mathbf{i} + 5 \mathbf{j} - 5 \mathbf{k}
\]

Area $= \frac{1}{2} \| \overrightarrow{AB} \times \overrightarrow{AC} \|$ = \frac{1}{2} \sqrt{75} = \frac{5}{2} \sqrt{3}

\[
\mathbf{n} = \overrightarrow{AB} \times \overrightarrow{AC} = -5 \mathbf{i} + 5 \mathbf{j} - 5 \mathbf{k}
\]

Point $A = (0, 2, 3)$

Equation: $-5(x - 0) + 5(y - 2) - 5(z - 3)$

or $-5x + 5y - 5z = -5$

or $-x + y - z = -1$

\[
\mathbf{n} = \langle -1, 1, -1 \rangle \quad \mathbf{n} \text{ for } xy \text{ plane } \lambda = \langle 0, 0, 1 \rangle
\]

\[
\cos \theta = \frac{\langle -1, 1, -1 \rangle \cdot \langle 0, 0, 1 \rangle}{\| \langle -1, 1, -1 \rangle \| \cdot \| \langle 0, 0, 1 \rangle \|}
\]

\[
= \frac{-1}{\sqrt{3}} = -\frac{1}{\sqrt{3}}
\]

\[
\theta = \cos^{-1} \left( -\frac{1}{\sqrt{3}} \right) = 125.3^\circ
\]
2. (10 pts) Find the tangent vector and the unit tangent vector for the curve

\[ \vec{r}(t) = \langle 3t^2, \sin(t) - t \cos(t), \cos(t) + t \sin(t) \rangle \]

at the point \((3\pi^2, \pi, -1)\).

\[ \vec{r}'(t) = \langle 6t, \cos(t) - \cos(t) + ts\sin(t), -\sin(t) + \sin(t) + t\cos(t) \rangle \]

\[ = \langle 6t, 0, t\cos(t) \rangle \]

at \((3\pi^2, \pi, -1)\), \(3t^2 = 3\pi^2\), so \(t = \pi\).

\[ \vec{r}'(\pi) = \langle 6\pi, \pi \sin(\pi), \pi \cos(\pi) \rangle = \langle 6\pi, 0, -\pi \rangle \]

unit tangent vector

\[ \frac{\vec{r}'(t)}{||\vec{r}'(t)||} = \frac{\langle 6\pi, 0, -\pi \rangle}{\sqrt{(6\pi)^2 + 0^2 + (-\pi)^2}} = \frac{\langle 6\pi, 0, -\pi \rangle}{\pi \sqrt{37}} \]

\[ = \left\langle \frac{6}{\sqrt{37}}, 0, -\frac{1}{\sqrt{37}} \right\rangle \]

3. (12 pts) Which of the following represent lines in \(\mathbb{R}^3\)? Circle all correct answers. No work needed.

(a) All \((x, y, z)\) such that \(x = -2t, y = 3t + 1,\) and \(z = 4t + 6\) for \(t \in \mathbb{R}\). \[ \text{line} \]

(b) \(t, t^2, t^3\) for \(t \in \mathbb{R}\)

\[ \text{not a line (a curve)} \]

(c) All \((x, y, z)\) such that \(5x + 4y + 3z = 2\) and \(x + 4y - 7z = 17\)

\[ \text{line} \]

(d) All \((x, y, z)\) such that \(5y + 4 = 8z - 7\)

\[ \text{not a line (a plane)} \]

(e) All \((x, y, z)\) such that \(5(x - 3) - 6(y + 2) + 3z = 0\)

\[ \text{not a line (a plane)} \]

(f) \(\vec{r}(t) = t \hat{i} + t^2 \hat{j} + 2t \hat{k}\) for \(t \in \mathbb{R}\).

\[ \text{piece of a line - actually a ray} \]
4. Consider the line \( x + 5 = \frac{y}{2} = \frac{z}{3} - 1 \) and the plane \(-x + 2y - z = 7\).

(a) (5 pts) Verify that the line and the plane do not intersect.

(b) (10 pts) Find the (shortest) distance between the line and the plane.

Hint: First, find any point \( Q \) on the line and any point \( P \) on the plane. Next, find the vector \( PQ \). The distance is then \( \frac{|PQ|}{|n|} \).

\[ \begin{align*}
(1) & \quad x + 5 = \frac{y}{2} = \frac{z}{3} - 1 \\
\text{Solve} & \\
(2) & \quad x + 5 = \frac{z}{3} - 1 \\
(3) & \quad -x + 2y - z = 7 \\
\end{align*} \]

Substitute into (3):

\[\begin{align*}
- x + 2(2x + 10) - (3x + 18) &= 7 \\
- x + 4x - 3x + 20 - 18 &= 7 \\
2 &= 7 \quad \text{no solution}
\end{align*}\]

b) Point \( P \) on plane \((0, 0, -7)\)

Point \( Q \) on line \((-5, 0, 3)\)

\[\text{Point } Q \text{ on line: } (-5, 0, 3) \checkmark \]

\[\begin{align*}
\overrightarrow{PQ} &= \langle -5, 0, 10 \rangle \\
\overrightarrow{n} &= \langle -1, 2, -1 \rangle \\
\text{compute } \overrightarrow{PQ} \cdot \overrightarrow{n} &= \frac{5 - 0 - 10}{\sqrt{6}} \checkmark \\
\text{distance} &= \left| \frac{-5}{\sqrt{6}} \right| = \frac{5}{\sqrt{6}} \checkmark
\end{align*}\]

\[\text{or use formula: for distance between point and plane}\]

\[D = \frac{|-1(-5) + 2(0) - 1(3) - 7|}{\sqrt{1^2 + 2^2 + 1^2}} = \frac{5}{\sqrt{6}} \checkmark \]

6 pts
5. (12 pts) Match the equations with the graphs. No work needed.

(a) \( x^2 - y^2 + z^2 = 0 \) \( \rightarrow 6 \)
(b) \( -x^2 + y + z^2 = 1 \) \( \rightarrow 4 \)
(c) \( x - y + z = 1 \) \( \rightarrow 1 \)
(d) \( -x^2 + y^2 + z^2 = 1 \) \( \rightarrow 5 \)
(e) \( -y^2 + z = 1 \) \( \rightarrow 2 \)
(f) \( -x^2 - y^2 + z = 1 \) \( \rightarrow 3 \)
6. (10 pts) True or false. True means always true. False means sometimes or always false. No work needed.

(a) F \quad b \cdot (\vec{a} \times \vec{b}) = 0
(b) T \quad \vec{a} \times \vec{a} = 0
(c) T \quad The cross product of two unit vectors is a unit vector.
(d) F \quad The dot product of two unit vectors is a unit vector.
(e) T \quad ||\vec{a} \times \vec{b}|| \leq ||\vec{a}|| ||\vec{b}||

\text{answer is false here;}
\text{for unit vectors } \vec{v} \text{ and } \vec{w},
\text{if } \theta \neq 90^\circ, \text{then}
||\vec{v} \times \vec{w}|| < 1 \text{ so not unit vector}