Math 232 – Fall 2017 – Thomas – Test 2 **Name** Key

#1-6 Multiple Choice (9 points each)

1. Determine the convergence or divergence of the series $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$ using the Integral Test.

$f(x) = \frac{1}{x(\ln x)^2}$, $f(x)$ is positive, continuous, + decreasing from $[2,\infty)$ so we can use integral test

$f'(x) = \frac{-\ln x - 2}{x^2(\ln x)^3}$

$\frac{-\ln x - 2}{x^2(\ln x)^3} < 0$

$-\ln x - 2 < 0$

$-\ln x < 2$

$\ln x > -2$

$x > e^{-2}$

a) Converges since $\int_2^{\infty} \frac{1}{x^3}\, dx$ converges

b) Diverges since $\int_2^{\infty} \frac{1}{x}\, dx$ diverges

c) Converges since $\int_2^{\infty} \frac{1}{x(\ln x)^2}\, dx$ converges

d) Diverges since $\int_2^{\infty} \frac{1}{x(\ln x)^2}\, dx$ diverges

e) Converges since $\int_2^{t} \frac{1}{x(\ln x)^2}\, dx$ converges

$\int_2^{\infty} \frac{1}{x(\ln x)^2}\, dx = \lim_{t\to\infty} \int_2^{t} \frac{1}{x(\ln x)^2}\, dx$

let $u = \ln x$

$du = \frac{1}{x}dx$

$= \lim_{t\to\infty} \int_2 \frac{1}{u^2}\, du = \lim_{t\to\infty} \frac{-1}{u}\Big]_2^t = \lim_{t\to\infty} \frac{-1}{\ln x}\Big]_2^t$

$= \lim_{t\to\infty} -\frac{1}{\ln t} + \frac{1}{\ln 2} = \frac{1}{\ln 2}$ converges

2. Determine the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{n}{n^2-2n+5}$ using the Limit Comparison Test.

$\frac{1}{n}, \frac{n}{n^2-2n+5} > 0$

compare to $\sum_{n=1}^{\infty} \frac{n}{n^2} = \sum_{n=1}^{\infty} \frac{1}{n}$

a) Converges since $\lim_{n\to\infty} \frac{1/n^2}{n/(n^2-2n+5)} = 1$

b) Converges since $\lim_{n\to\infty} \frac{1/n}{n/(n^2-2n+5)} = 1$

c) Converges since $\lim_{n\to\infty} \frac{1/n}{n/(n^2-2n+5)} = 0$

d) Diverges since $\lim_{n\to\infty} \frac{1/n}{n/(n^2-2n+5)} = 1$

e) Diverges since $\lim_{n\to\infty} \frac{1/n^2}{n/(n^2-2n+5)} = 1$

$\lim_{n\to\infty} \frac{\frac{1}{n}}{\frac{n}{n^2-2n+5}} = \lim_{n\to\infty} \frac{1}{n} \cdot \frac{n^2-2n+5}{n}$

$= \lim_{n\to\infty} \frac{n^2-2n+5}{n^2} = \lim_{n\to\infty} \frac{2n-2}{2n}$

$= \frac{2}{2} = 1 > 0$

since $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges + $\lim_{n\to\infty} \frac{\frac{1}{n}}{\frac{n}{n^2-2n+5}} > 0$, $\sum_{n=1}^{\infty} \frac{n}{n^2-2n+5}$ diverges

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3. Using the Remainder Estimate for the Integral Test, find an upper bound for the error using S_3 as an approximation to S. $\sum_{n=1}^{\infty} \frac{1}{n^4}$

$R_n \leq \int_n^{\infty} f(x)\,dx$

a) $-\frac{1}{81}$

b) $-\frac{1}{192}$

c) $\frac{1}{9}$

d) $\frac{1}{81}$

e) $\frac{1}{192}$

$$R_3 \leq \int_3^{\infty} \frac{1}{x^4}\,dx = \lim_{t\to\infty} \int_3^t \frac{1}{x^4}\,dx$$

$$= \lim_{t\to\infty} \left.\frac{-1}{3x^3}\right]_3^t = \lim_{t\to\infty} \left(\underbrace{\frac{-1}{3t^3}}_{\to 0} + \frac{1}{3(3)^3}\right) = \frac{1}{3^4} = \frac{1}{81}$$

$$R_3 \leq \frac{1}{81}$$

4. Determine if the infinite series is convergent or divergent. If convergent, find the sum. $\sum_{n=1}^{\infty} \frac{3}{\sqrt[3]{n}}$

a) Converges to 0

b) Converges to 1

c) Converges to 3

d) Converges to $\frac{1}{\sqrt{3}}$

e) Diverges

$$\sum_{n=1}^{\infty} \frac{3}{n^{1/3}} = 3\sum_{n=1}^{\infty} \frac{1}{n^{1/3}}$$

p-series diverges since $p < 1$

5. Determine if the ~~sequence~~ series is convergent or divergent. If convergent, find the sum.

$\sum_{n=0}^{\infty}\left(\frac{2}{e+1}\right)^n$

geometric series

$$\sum_{n=0}^{\infty}\left(\frac{2}{e+1}\right)\left(\frac{2}{e+1}\right)^{n-1} = a_0 + \sum_{n=1}^{\infty}\left(\frac{2}{e+1}\right)\left(\frac{2}{e+1}\right)^{n-1}$$

a) Converges to 0

b) Converges to 1

$$\left|\frac{2}{e+1}\right| < 1 \text{ so converges } + \sum_{n=1}^{\infty} = \frac{a}{1-r}$$

c) Converges to $\frac{e+1}{e-1}$

$$a_0 = \left(\frac{2}{e+1}\right)\left(\frac{2}{e+1}\right)^{-1} = 1$$

d) Converges to $\frac{2}{e-1}$

$$\sum_{n=1}^{\infty}\left(\frac{2}{e+1}\right)\left(\frac{2}{e+1}\right)^{n-1} = \frac{\frac{2}{e+1}}{1-\frac{2}{e+1}} = \frac{\frac{2}{e+1}}{\frac{e+1-2}{e+1}} = \frac{2}{e+1}\cdot\frac{e+1}{e-1} = \frac{2}{e-1}$$

e) Diverges

$$\sum_{n=0}^{\infty}\left(\frac{2}{e+1}\right)\left(\frac{2}{e+1}\right)^{n-1} = a_0 + \sum_{n=1}^{\infty}\left(\frac{2}{e+1}\right)\left(\frac{2}{e+1}\right)^{n-1} = 1 + \frac{2}{e-1}$$

$$= \frac{e-1+2}{e-1} = \frac{e+1}{e-1}$$

6. Determine if the ~~infinite series~~ sequence is convergent or divergent. $\left\{\frac{1}{8}, \frac{2}{27}, \frac{3}{64}, \frac{4}{125}, \dots\right\}$

a) Converges to 0

$$a_n = \frac{n}{(n+1)^3}$$

b) Converges to 1

c) Converges to $\frac{1}{3}$

$$\lim_{n\to\infty}\frac{n}{(n+1)^3} = \lim_{n\to\infty}\frac{1}{3(n+1)^2} = 0$$

d) Converges to $\frac{1}{6}$

e) Diverges

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#7-9 True/False (3 points each)

7. ~~An~~ If An infinite series $\sum_{n=1}^{\infty} a_n$ is convergent ~~if~~ then $\lim_{n\to\infty} a_n = 0$. TRUE FALSE

8. An infinite sequence $\{a_n\}$ is convergent *only if* $\lim_{n\to\infty} a_n = 0$. TRUE FALSE

counter example: $a_n = 2(1)^n$ which converges + $\lim_{n\to\infty} a_n = 2$

9. The integral $\int_{-2}^{2} \frac{1}{(x-1)^2}\,dx = \left.\frac{-1}{x-1}\right]_{-2}^{2} = -\frac{4}{3}$. TRUE FALSE

discontinuity at $x=1$

$\int_{-2}^{2} \frac{1}{(x-1)^2}dx = \lim_{t\to1^-}\int_{-2}^{t}\frac{1}{(x-1)^2}dx + \lim_{t\to1^+}\int_{t}^{2}\frac{1}{(x-1)^2}dx = \lim_{t\to1^-}\left.\frac{-1}{x-1}\right]_{-2}^{t} + \lim_{t\to1^+}\left.\frac{-1}{x-1}\right]_{t}^{2}$

$= \lim_{t\to1^-}\left[\frac{-1}{t-1} + \frac{1}{-3}\right] + \lim_{t\to1^+}\left[\frac{-1}{1} + \frac{1}{t-1}\right] = \infty - \frac{1}{3} - 1 + \infty = \infty$ diverges

$\cancel{\lim} \frac{-1}{x-1}$

$\cancel{\lim} \frac{1}{x-1}$

#10-12 Free Response/Partial Credit (#10 worth 13 points, #11-12 worth 12 points each)

10. Determine the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{\cos^2 3n}{1+(1.5)^n}$

Geometric, Integral, Telescoping, p-series does not work

comparison Test:

① $0 \le \frac{\cos^2(3n)}{1+1.5^n} \le \frac{1}{1+1.5^n} \le \frac{1}{1.5^n} = \frac{1}{1.5}\left(\frac{1}{1.5}\right)^{n-1}$

$\frac{\cos^2(3n)}{1+1.5^n} \le \frac{1}{1.5}\left(\frac{1}{1.5}\right)^{n-1}$

② $\sum_{n=1}^{\infty}\frac{1}{1.5}\left(\frac{1}{1.5}\right)^{n-1}$ converges b/c geometric series where $|r| = \left|\frac{1}{1.5}\right| < 1$

③ Since $\sum_{n=1}^{\infty}\frac{1}{1.5}\left(\frac{1}{1.5}\right)^{n-1}$ converges + $\frac{\cos^2(3n)}{1+1.5^n} \le \frac{1}{1.5}\left(\frac{1}{1.5}\right)^{n-1}$,

By comparison test $\sum_{n=1}^{\infty}\frac{\cos^2(3n)}{1+1.5^n}$ converges

11. Determine the convergence or divergence of the series using a telescoping series for $\sum_{n=2}^{\infty} \frac{1}{(n-1)(n+2)}$. If convergent, find the sum.

$$\sum_{n=2}^{\infty} \frac{1}{(n-1)(n+2)} = \sum_{n=2}^{\infty} \frac{A}{n-1} + \frac{B}{n+2} = \sum_{n=2}^{\infty} \frac{\frac{1}{3}}{n-1} + \frac{-\frac{1}{3}}{n+2}$$

$\boxed{\text{Converges to } \frac{11}{18}}$

$1 = A(n+2) + B(n-1) = An + 2A + Bn - B$

$0 = A + B$

$1 = 2A - B$

$1 = 3A$

$\frac{1}{3} = A \quad B = -\frac{1}{3}$

$$\lim_{n\to\infty} S_n = \frac{\frac{1}{3}}{1} + \cancel{\frac{-\frac{1}{3}}{4}} + \frac{\frac{1}{3}}{2} + \cancel{\frac{-\frac{1}{3}}{5}} + \frac{\frac{1}{3}}{3} + \cancel{\frac{-\frac{1}{3}}{6}} + \dots + \cancel{\frac{\frac{1}{3}}{n-3}} + \frac{-\frac{1}{3}}{n} \to 0 + \cancel{\frac{\frac{1}{3}}{n-2}} + \frac{-\frac{1}{3}}{n+1} \to 0$$
$$+ \cancel{\frac{\frac{1}{3}}{n-1}} + \frac{-\frac{1}{3}}{n+2} \to 0 = \frac{1}{3} + \frac{1}{6} + \frac{1}{9} = \frac{6}{18} + \frac{3}{18} + \frac{2}{18} = \boxed{\frac{11}{18}}$$

$\frac{11}{18}$

12. Determine the convergence or divergence of $\int_1^{\infty} \frac{2+e^{-x}}{x}\,dx$ using the Comparison Theorem

close to $\frac{1}{x}$

① $\frac{1}{x} < \frac{2}{x} < \frac{2+e^{-x}}{x}$

② $\int_1^{\infty} \frac{1}{x}\,dx$ diverges b/c p-series $p \le 1$

③ since $\int_1^{\infty} \frac{1}{x}\,dx$ diverges and $\frac{1}{x} < \frac{2+e^{-x}}{x}$,

By comparison Theorem $\int_1^{\infty} \frac{2+e^{-x}}{x}\,dx$ diverges