Math 232 – Fall 2017 – Thomas – Test 3   Name ___________________________________

I, ________________________________ , have neither given nor received unauthorized aid on this test.

- *Show all work! Work may include explanations in phrases/sentences. Use proper mathematical notation and make complete mathematical statements.*
- *If you need more room to write, get blank paper from me. Do not use your own paper.*
- *Only Scientific Calculators are allowed.* ***NO*** *Graphing Calculators. Test is designed to be completed without a calculator.*
- ***Exact solutions only.***
- *Multiple choice and True/False will be graded correct or incorrect, free response will be graded based on partial credit (NO WORK NO CREDIT)*

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#1-5 Multiple Choice (11points each)

1. Find a power series representation for $f(x) = \frac{1}{9+x^2}$ and determine its interval of convergence.

   a) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{9^n}$ with interval of convergence $(-3,3)$

   b) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{9^{n+1}}$ with interval of convergence $(-3,3)$

   c) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{9^n}$ with interval of convergence $(-1,1)$

   d) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{9^{n+1}}$ with interval of convergence $(-1,1)$

   e) $\sum_{n=0}^{\infty} \frac{x^{2n}}{9^{n+1}}$ with interval of convergence $(-3,3)$

2. Find the Maclaurin series for $f(x) = x^5 \sin x^2$.

   a) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+7}}{(4n+7)(2n+1)!}$

   b) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+2}}{(2n+1)!}$

   c) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+7}}{(2n+1)!}$

   d) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+6}}{(2n+1)!}$

   e) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+6}}{(2n+6)(2n+1)!}$

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3. Using series to find the $\lim_{x\to 0} \frac{x^3-3x+3tan^{-1}(x)}{x^5}$ will simplify to finding which of the following limits?

   a) $\lim_{x\to 0}\left(\frac{3x^2}{5} - \frac{x^4}{3} + \frac{x^6}{2} - \cdots\right)$

   b) $\lim_{x\to 0}\left(\frac{3}{5} - 3x + \frac{x^3}{3} - \frac{x^7}{7} + \cdots\right)$

   c) $\lim_{x\to 0}\left(\frac{3x^2}{7} - \frac{x^4}{3} + \frac{x^6}{2} - \cdots\right)$

   d) $\lim_{x\to 0}\left(\frac{3}{5} - x^2 + x^4 - \cdots\right)$

   e) $\lim_{x\to 0}\left(\frac{3}{5} - \frac{3x^2}{7} + \frac{x^4}{3} - \cdots\right)$

4. Find the first 3 nonzero terms in the Taylor series for $\mathrm{f(x)} = \cos \mathrm{x}$ centered at $\mathrm{a} = \pi$.

   a) $(x-\pi) - \frac{(x-\pi)^3}{3!} + \frac{(x-\pi)^5}{5!}$

   b) $(\pi x) - \frac{(\pi x)^3}{3!} + \frac{(\pi x)^5}{5!}$

   c) $(x-\pi) + \frac{(x-\pi)^2}{2!} - \frac{(x-\pi)^4}{4!}$

   d) $-1 + \frac{(\pi x)^2}{2!} - \frac{(\pi x)^4}{4!}$

   e) $-1 + \frac{(x-\pi)^2}{2!} - \frac{(x-\pi)^4}{4!}$

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5. Find the sum of the series $1 + 2 + \frac{4}{2!} + \frac{8}{3!} + \frac{16}{4!} + \cdots$

   a) e^2

   b) -1

   c) $\ln 3$

   d) 2^k

   e) $\sin 2$

#6-7 True/False (5 points each)

6. The Radius of Convergence for $\sum_{n=0}^{\infty} \left(\frac{x}{2}\right)^n$ is $R = \frac{1}{2}$.   **TRUE**   **FALSE**

7. If a series $\sum_{n=0}^{\infty} a_n$ is convergent, then the series $\sum_{n=0}^{\infty} a_n$ is also absolutely convergent.

   TRUE   **FALSE**

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#8-10 Free Response/Partial Credit(#8-9 worth 13 points each, #10 worth 9 points)

8. Write the function as a power series $f(x) = \ln(x^2 + 2)$.

9. Use a power series to approximate the definite integral $\int_0^{\frac{1}{10}} e^{-x^2}\ dx$ with $|error| \leq \frac{1}{300}$.

(Helpful information: $\left(\frac{1}{10}\right)^2 = \frac{1}{100}$ $\left(\frac{1}{10}\right)^3 = \frac{1}{1000}$ $\left(\frac{1}{10}\right)^4 = \frac{1}{10000}$, etc)

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10. Determine the convergence or divergence of $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt[3]{n}}$. Must prove all statements.