KEY

Math 232 Practice Problems for Exam 1 (some topics may not be on your midterm)

McLaughlin

(1) Set up an integral to find the area bounded by the graphs of $y = 2x^2$ and $y = 3 - 5x$. Do not integrate.

(2) Set up an integral for the length of the arc of the hyperbola $xy = 1$ from the point (1,1) to (2, 1/2).

(3) Find the value of c such that the area of the region bounded by $y = c^2 - x^2$ and the $x - axis$ on $[0, c]$ is equal to the average value of $f(x) = 3x^2$ on [0,2].

a) $\sqrt[3]{6}$

b) $\sqrt[3]{12}$

c) $\sqrt[3]{24}$

d) $\sqrt[3]{8}$

e) $\sqrt[3]{18}$

(4) Evaluate $\int_1^4 \sqrt{x}\ \ln x\, dx$

a) $-\frac{7}{2}\ln 4 - 28$

b) $16 \ln 4 - \frac{28}{3}$

c) $\frac{16}{3}\ln 4 - \frac{28}{9}$

d) $-\frac{7}{2}$

e) $\frac{16}{3}\ln 4 - \frac{7}{2}$

(5) Evaluate $\int cos^{-2}x\ sin^3 x\ dx$.

a) $-\sin x - \csc x + c$

b) $\frac{cos^3 x}{3} - \sec x + c$

c) $\cos x + \frac{sin^4 x}{4} + c$

d) $\cos x + \sec x + c$

e) $\frac{\cos x}{2} - \frac{cos^5 x}{5} + c$

(6) Which definite integral is the area bounded by the graphs of $y = x^2 - 3$ and $y = 15 - x^2$?

a) $\int_{-3}^{3} (12 - 2x^2)\ dx$

b) $\int_{-15}^{15} (12 - 2x^2)\ dx$

c) $\pi \int_{-3}^{3} (18 - x^2)^2\ dx$

d) $\int_{-15}^{15} (18 - 2x^2)\ dx$

e) $\int_{-3}^{3} (18 - 2x^2)\ dx$

(7) Set up the integral to find the volume of the solid found by revolving the region bounded by $y = \sqrt{x}, x = 0 \; and \; x = 1$ about the line $x = 1$. Do not integrate.

(8) Which definite integral is the volume of the solid found by revolving the region bounded by $y = x - 1$, $y = 3 - x$ and the $x - axis$ about the $y - axis$?

a) $\pi \int_1^3 (y^2 - 4y + 10)\, dy$

b) $8\pi \int_0^1 (1 - y)\, dy$

c) $\pi \int_0^1 (1 - 8y^2)\, dy$

d) $\pi \int_1^3 (8 - 8y)\, dy$

e) $8\pi \int_0^1 (y - 1)\, dy$

(9) Find the average value of $y = x \ln x$ on $[1, 5]$.

a) $\frac{25}{8} \ln 5 - \frac{3}{2}$

b) $25 \ln 5 - \frac{3}{2}$

c) $8 \ln 4 - \frac{5}{4}$

d) $\frac{25}{8} \ln 5$

e) $\frac{8}{3} \ln 4 - \frac{5}{4}$

(10) The following integral represents the volume of a solid S. Describe S.

$\int_0^1 e^{2x}\,dx$

a) A solid obtained by rotating the region bounded by $y = e^{2x}, x = 0$ and $x = 1$ about the x – axis.

b) A solid obtained by rotating the region bounded by $y = e^{x}, x = 0$ and $x = 1$ about the x – axis.

c) The base of S is the region bounded by $y = e^{2x},\; x = 0$ and $x = 1$. Cross sections perpendicular to the x – axis are squares.

d) The base of S is the region bounded by $y = e^{x},\; x = 0$ and $x = 1$. Cross sections perpendicular to the x – axis are semicircles.

e) The base of S is the region bounded by $y = e^{x},\; x = 0$ and $x = 1$. Cross sections perpendicular to the x – axis are squares.

(11) Find A.

$\int x \tan x \sec^2 x\,dx = \frac{x\tan^2 x}{2} - A$

a) $A = 2\int \sec^2 x\,dx$

b) $A = \frac{1}{2}\int \tan^2 x\,dx$

c) $A = \frac{1}{2}\int \csc^3 x\,dx$

d) $A = 2\int x \cos x\,dx$

e) $A = 2\int \tan^2 x\,dx$

(12) Evaluate $\int_{\pi/6}^{\pi/2} 5\cos^3 x\,dx$

a) 5/8

b) 8/3

c) 5/24

d) 25/24

e) 25/8

(13) Evaluating the integral $\int \frac{2x-3}{x^3+3x}\,dx$ using partial fraction decomposition would result in the sum of integrals $\int \frac{A}{x+E}\,dx + \int \frac{Bx+C}{x^2+D}\,dx$ where constants A, B, C, D, E sum to:

a) 4

b) 7

c) 5

d) 2

e) 3

(14) Evaluate $\int tan^2(3x)\,dx$

(15) Evaluate $\int cos^3(x)\,sin^6(x)\,dx$

(16) Consider solving the integral $\int \sqrt{4-x^2}\,dx$. After making the appropriate trigonometric substitution, which integral must be solved to complete the solution?

a) $16 \int cos^2\theta \;\; d\theta$

b) $4 \int \sin\theta \cos\theta \;\; d\theta$

c) $4 \int cos^2\theta \;\; d\theta$

d) $2 \int tan^2\theta \;\; d\theta$

e) $16 \int \tan\theta \sin\theta \;\; d\theta$

True or False for problems #17, 18

(17) There exist constants A and B such that $\frac{x(x^2+4)}{x^2-4} = \frac{A}{x+2} + \frac{B}{x-2}$.

(18) Integrating the parabola $y = x^2 - 4$ from $x = -2$ to $x = 2$ is equivalent to finding the area bounded by $y = x^2$ and $y = 4$.

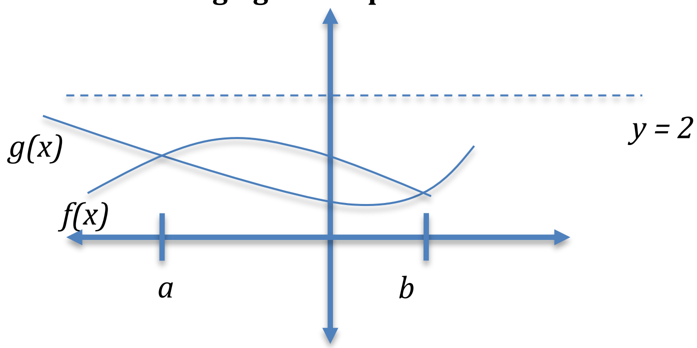
(19) Evaluate $\int x\, cos^2(x)\, dx$

(20) Evaluate $\int x\sqrt{16+x^2}\,dx$.

(21) Using the Partial Fraction Decomposition method, express the integral $\int \frac{x+2}{(x-3)^2}\,dx$ as a sum of integrals $\int \frac{A}{(x-3)}\,dx + \int \frac{B}{(x-3)^2}\,dx$. Find A and B. Show all work for credit. Do not integrate.

A=__________ B=__________

Use the following figure for problems #22 and #23.



(22) The volume of the solid generated by revolving the area bounded by $f(x)$ and $g(x)$ about the line $y = 2$ is equal to:

a) $\lim_{n\to\infty} \pi \sum_{i=1}^{n} \left[\left(2 - g(x_i^*)\right)^2 - \left(2 - f(x_i^*)\right)^2\right] \Delta x$

b) $\lim_{n\to\infty} \pi \sum_{i=1}^{n} \left[\left(g(x_i^*)\right)^2 - \left(f(x_i^*)\right)^2\right] \Delta x$

c) $\lim_{n\to\infty} \pi \sum_{i=1}^{n} \left[\left(2 + g(x_i^*)\right)^2 - \left(2 + f(x_i^*)\right)^2\right] \Delta x$

d) $\lim_{n\to\infty} \pi \sum_{i=1}^{n} \left[\left(g(x_i^*)\right)^2 - \left(2 - f(x_i^*)\right)^2\right] \Delta x$

e) $\lim_{n\to\infty} \pi \sum_{i=1}^{n} \left[\left(2 - g(x_i^*)\right)^2 - \left(f(x_i^*)\right)^2\right] \Delta x$

(23) The base of a solid S is the region bounded by $f(x)$ and $g(x)$. Cross-sections perpendicular to the $x - axis$ are squares. Which of the following is the volume of S.

a) $\lim_{n\to\infty} \pi \sum_{i=1}^{n} [f(x_i^*)]^2 \Delta x$

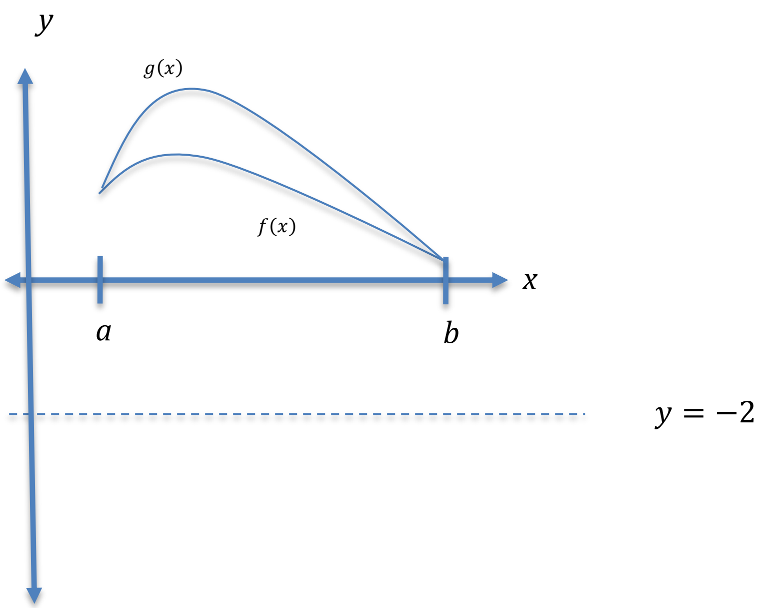
b) $\lim_{n\to\infty} \sum_{i=1}^{n} [f(x_i^*) - g(x_i^*)]\Delta x$

c) $\lim_{n\to\infty} 2\pi \sum_{i=1}^{n} [f(x_i^*)^2 - g(x_i^*)^2] \Delta x$

d) $\lim_{n\to\infty} \sum_{i=1}^{n} [f(x_i^*) - g(x_i^*)]^2 \Delta x$

e) $\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\, g(x_i^*) \Delta x$

Use the following figure for problems #24 and #25.



(24) The area bounded by the graphs of $y = f(x)$ *and* $y = g(x)$ is equal to:

a) $\lim_{n\to\infty} \sum_{i=1}^{n} \pi\big(g(x_i^*) - f(x_i^*)\big)^2 \Delta x$

b) $\lim_{n\to\infty} \sum_{i=1}^{n} (f(x_i^*) - g(x_i^*))\Delta x$

c) $\lim_{n\to\infty} \sum_{i=1}^{n} \big(g(x_i^*) - f(x_i^*)\big) \Delta x$

d) $\lim_{n\to\infty} \sum_{i=1}^{n} \pi(g(x_i^*)^2 - f(x_i^*)^2) \Delta x$

e) $\lim_{n\to\infty} \sum_{i=1}^{n} \pi\big(f(x_i^*) - g(x_i^*)\big)^2 \Delta x$

(25) The volume of the solid generated by revolving the area bounded by the graphs of $y = f(x)$ *and* $y = g(x)$ about the line $y = -2$ is equal to:

a) $\lim_{n\to\infty} \pi \sum_{i=1}^{n} (g(x_i^*) - f(x_i^*) + 2)^2 \Delta x$

b) $\lim_{n\to\infty} \pi \sum_{i=1}^{n} [(g(x_i^*) - 2)^2 - (f(x_i^*) - 2)^2] \Delta x$

c) $\lim_{n\to\infty} \sum_{i=1}^{n} (g(x_i^*) - f(x_i^*) - 4) \Delta x$

d) $\lim_{n\to\infty} \pi \sum_{i=1}^{n} [(g(x_i^*) + 2)^2 - (f(x_i^*) + 2)^2] \Delta x$

e) $\lim_{n\to\infty} \pi \sum_{i=1}^{n} [(f(x_i^*) + 2)^2 - (g(x_i^*) + 2)^2] \Delta x$

(26) A tank in the shape of a right circular cone is full of water. If the height of the tank is 12 feet and the radius of the top is 3 feet, which integral is the work done in pumping the water to a height 10 feet above the top of the tank. (Water weighs 62.4 pounds per cubic foot.) Let $\delta = 62.4$

a) $\frac{\delta\pi}{16}\int_0^{22}(22x^2 - x^3)\,dx$

b) $\frac{\delta\pi}{16}\int_0^{12}(12x^2 - x^3)\,dx$

c) $\delta\pi\int_0^{22}(x^3 - 12)\,dx$

d) $10\delta\pi\int_0^{12}(12x^2 - x^3)\,dx$

e) $\frac{\delta\pi}{16}\int_0^{12}(22x^2 - x^3)\,dx$