Spring 2020 Mentor: Bradley Hicks

**Project:** Character Theory of Finite Groups

**Keywords:** finite groups, finite unipotent groups, character theory, representation theory

**Suggested Prerequisites:** Math 381 or Math 534, Math 578

**Additional Materials:** [Character Theory of Finite Groups](https://www.math.nyu.edu/~kac/tensor3/tletion.pdf)

**Suggested Prerequisites:** Character theory knowledge, basic group theory, and some experience with linear algebra.

This project will focus on the theory of characters of finite groups, working from basic level to advanced concepts. We will start with the basics of character theory, including the definitions of characters and character tables. We will then move on to more advanced topics, such as induced characters and the Frobenius reciprocity theorem.

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Text: Differential Equations and Dynamical Systems by Lawrence Perko

Keywords: dynamical systems, differential equations

Suggested Prerequisites: Perko's book is intended for students with a background in undergraduate differential equations. The course will cover topics such as phase portraits, stability of equilibria, and bifurcations. Students should have a strong foundation in linear algebra and an understanding of basic concepts in analysis.


Keywords: differential equations, boundary value problems

Suggested Prerequisites: The course requires a background in differential equations and some knowledge of linear algebra. Students should be familiar with the basic concepts of differential equations, including first and second order equations, and have a basic understanding of linear algebra.

Text: Introduction to Real Analysis by Robert Brabenec

Keywords: real analysis, limits, derivatives

Suggested Prerequisites: A course in calculus is required. Students should have a strong understanding of limits, derivatives, and integrals as we know them today. The clean and precise treatments of mathematicians saw the need for greater rigor and precision in the subject, leading to the formulation of analysis in the 19th century.

Text: Partial Differential Equations I: Basic Theory by Robert Strichartz

Keywords: partial differential equations, Fourier series

Suggested Prerequisites: The course is primarily for students with a strong background in analysis and some knowledge of differential equations. A course in advanced calculus or real analysis is recommended. The course will cover topics such as Fourier series, distribution theory, and the Fourier transform.

Text: Analysis Project Ideas

Keywords: analysis, dynamical systems, differential equations

Suggested Prerequisites: For the analysis project, students should have completed a course in real analysis and have a strong background in differential equations. The project is not to gain technical fluency with some mathematical concepts, but is instead to help us understand the students' interests and develop their abilities. In some cases, the project may be based on the students' readings.
This text is about...
APP 1
Spring 2020 Mentor: Wesley Hamilton
Text: Dynamical Systems in Neuroscience by Inhaber
Additional Materials: Python or MATLAB
Project: As the book title suggests, students will explore how dynamical systems and ordinary differential equations are used to model neuron firing (in the brain). The project will explore basic properties of dynamical systems, including phase portraits, bifurcations, nullclines, etc.; and then apply these concepts to the biological formulations of neuron firing, such as potassium and sodium channels opening and closing. Students will begin with more accessible models that will be investigated in detail (Hodgkin-Huxley and Fitzhugh-Nagumo), and interested students will have the opportunity to analyze more recent models of neuron firing. Students will learn how to explore these dynamical systems using numerical software such as Python or MATLAB. This project can also be adapted to focus more on the dynamical systems aspect, or focus more on the biological component.
Suggested Prerequisites: Math 383 (basic biology/mastery helps), Math 319, or a pipe.
Keywords: dynamical systems, neuroscience, matlab, python, numerical analysis

APP 2
Spring 2020 Mentor: Wesley Hamilton
Text: Spectral methods II at Tufts
Additional Materials: MATLAB or Python
Project: Students will learn about spectral methods (the Fourier transform), and how they can be used to find accurate approximations to solutions of a wide-range of ordinary and partial differential equations. The discussion of spectral methods (Math 566) would be helpful as well.
Suggested Prerequisites: Math 383 (where eigenvalues and eigenvectors were discussed) and some programming experience (ideally with MATLAB). However, a strong background in MATLAB, along with familiarity with Fourier series, orthogonal polynomials, and programming will also be helpful.
Keywords: spectral methods, numerical analysis, Fourier transforms, matlab, programming

APP 3
Spring 2020 Mentor: Collin Kohli
Text: Nonlinear Dynamics And Chaos by Steven Strogatz
Additional Materials: Dynamical Systems
Project: The subject of this project will be a mathematical understanding of voting. While many democratic countries, states, cities, towns, etc. use voting as a way to construct their government, often voting looks different depending on the laws, rules, or the reason for voting. When we think of voting, we often imagine the scenario of two candidates competing for a seat to represent us, but careful examination of your local ballot will show a variety of voting methods with different rules. Our particular interest will be understanding these situations and more importantly exploring ranked-choice voting, where voters cast ranked ballots ranking candidates and methods for producing a winner from the information. Students will learn about voting methods, explore some of these methods and how to compare, interpret, and justify the outcomes. The goal of the project will be an understanding of Arrow’s Impossibility Theorem, what it actually means, and to give the beginning math major an interesting avenue into thinking like a mathematician about ideas in social science, economics, and politics. The interested student could either perform further exploratory study on some of the voting systems and methods we discuss, or leap into related subjects like weighted voting.
Suggested Prerequisites: This project is best for a student who has completed Math 383 for experience writing proofs, knowledge of sets, functions, and basic combinatorics. The ideal student will have some interest in or knowledge of political policy, economics, or social sciences, as we do not need deep knowledge of this field.
Keywords: voting, social choice, ranked-choice voting, Arrow’s Impossibility Theorem, political science, economics, social science

APP 4
Spring 2020 Mentor: Bradley Bledis
Text: Mathematics of Social Choice by Christoph Borgers
Project: The subject of this project will be an examination of a classic dynamic. This project will simulate the double pendulum with an added restriction on the maximum angle between the two parts. These simulations will be used to explore the chaotic dynamic of the restricted system, and how the dynamics depend on the physical parameters of the double pendulum.
Suggested Prerequisites: Differential equations (Math 383), and even better with the lab. Numerical analysis (Math 386) would be very helpful, along with some familiarity with MATLAB or Python.
Keywords: dynamical systems, numerical analysis, physics, chaos, simulation

APP 5
Spring 2020 Mentor: Wesley Hamilton
Text: Computational (Optimal) Transport by Peyre and Cuturi
Project: The double pendulum is a famous example of a chaotic dynamic. This project will simulate the double pendulum with an added restriction on the maximum angle between the two parts. These simulations will be used to explore the chaotic dynamic of the restricted system, and how the dynamics depend on the physical parameters of the double pendulum.
Suggested Prerequisites: Differential equations (Math 383), and even better with the lab. Numerical analysis (Math 386) would be very helpful, along with some familiarity with MATLAB or Python.
Keywords: dynamical systems, numerical analysis, physics, chaos, simulation

APP 6
Spring 2020 Mentor: Wesley Hamilton
Text: Computing (Optimal) Transport by Peyre and Cuturi
Project: Optimal transport is a very active area of research in both theoretical and applied math (a recent Fields medal was awarded for work on optimal transport). In short, optimal transport is concerned with moving goods, such as sand, from one place to another in the most optimal way. In practice, this means actually moving goods, while in theory, this means finding a "matching" between two probability distributions. In this project we will learn about optimal transport from a very computational standpoint. The focus will be on using computers and algorithms to solve/construct solutions to transportation problems. Along the way we will dip into some probability theory, some algorithm design, and more!
Suggested Prerequisites: Basic probability. Analysis (Math 521) is preferable but not necessary.
Keywords: probability, optimal transport, computational, algorithms, measure theory