Math 231: Final Exam A - Sequence 111111 **Fall 2016**

- Please use the scantron for multiple choice. Since you have test version A, please code the SEQUENCE NUMBER on the scantron as 111111 (all 1's).
- No calculators allowed.
- No partial credit on multiple choice.
- For short answer questions, you must show work for full and partial credit. For short answer questions, all work to be graded needs to go on the test.
- Give exact values instead of decimal approximations unless otherwise specified.
- Sign the honor pledge below after completing the exam.

First and last name . . . .

PID . . . .

Instructor (circle one):

Linda Green Elizabeth McLaughlin Lev Rozansky David Rose

Recitation TA (circle one):

Marc Besson Gonzalo Cazes-Nasitiqui Robert Hunt

Samuel Jeralds Claire Kiers Logan Tathan

Honor Pledge: I have neither given nor received unauthorized help on this exam.

Signature: . . . .

1. (2 pts) True or False: If the velocity of a particle is given by $v(t) = 9 - t^2$, then $\int_0^5 v(t)\,dt$ represents the total distance traveled by the particle between time $t = 0$ and $t = 5$.

   A. True

   B. False

2. (2 pts) True or False: If $\lim_{t\to 0} f(x) = 5$, then $\lim_{t\to 0} \frac{f(x)}{x^2}$ must be ∞.

   A. True

   B. False

3. (4 pts) For what value of a is the following function continuous?

$$f(x) = \begin{cases} \frac{x^2-9}{x-3} & \text{for } x < 3 \\ ax^2 + x - 6 & \text{for } x \geq 3 \end{cases}$$

   A. $a = \frac{1}{3}$

   B. $a = 1$

   C. $a = 3$

   D. $a = 6$

   E. There is no value of a that will make this function continuous.

4. (4 pts) Find $\lim_{x\to 0} \frac{\tan(4x)}{x + \sin(2x)}$

   A. $-\frac{1}{3}$

   B. 0

   C. $\frac{1}{3}$

   D. $\frac{4}{3}$

   E. DNE

5. (4 pts) Find $\lim_{x\to 1^-} \dfrac{4x^2 + x - 5}{|x-1|}$

   A. -9

   B. -3

   C. 0

   D. 9

   E. ∞

6. (4 pts) Find $\lim_{x\to\infty} \dfrac{\sqrt{4x^2 + x - 5}}{x}$

   A. 0

   B. 1

   C. 2

   D. 4

   E. ∞

7. (4 pts) Suppose that $h(x) = \sqrt{f(x)} \cdot \ln(g(x))$. Use the fact that $f(5) = 4$, $f'(5) = 16$, $g(5) = 2$, $g'(5) = 4$ to find $h'(5)$.

   A. $1 + 4\ln 2$

   B. $4 + 4\ln 2$

   C. $4 + \frac{1}{4}\ln 2$

   D. $8 + \ln 2$

   E. $8\ln 2$

8. (4 pts) What is the absolute MINIMUM value of $f(x) = (x-2)^3 + 100$ on the interval $[-2, 4]$?

   A. 0

   B. 12

   C. 36

   D. 48

   E. 100

9. (4 pts) Find the slope of the tangent line to $\frac{x}{y^2} + \frac{x^2}{8} = 3$ at the point $(4, 2)$.

   A. $-\frac{1}{4}$

   B. $-\frac{3}{8}$

   C. $\frac{5}{4}$

   D. $\frac{4}{3}$

   E. $\frac{9}{4}$

10. (4 pts) A function of the form $f(x) = \frac{a}{x^2 + bx}$ has a local extreme point (max or min point) at $(2, 3)$. Find the value of b.

   A. $b = -4$

   B. $b = -2$

   C. $b = 0$

   D. $b = 2$

   E. Cannot be determined from this information.

11. (4 pts) Which of the following limits represents $f'(3)$ if $f(x) = \ln(x + 4)$?

   1) $\lim_{h \to 0} \frac{\ln(3 + h) - \ln(3)}{h}$

   2) $\lim_{h \to 0} \frac{\ln(7 + h) - \ln(7)}{h}$

   3) $\lim_{x \to 3} \frac{\ln(x + 4) - \ln(7)}{x - 3}$

   A. 1, 2

   B. 1, 3

   C. 2, 3

   D. All of them.

   E. None of them.

12. (4 pts) If $g(x) = \int_{\pi/2}^{x} \sqrt{\sin(t) + 5}\, dt$, find $g'(2\pi)$.

A. $\sqrt{5}$

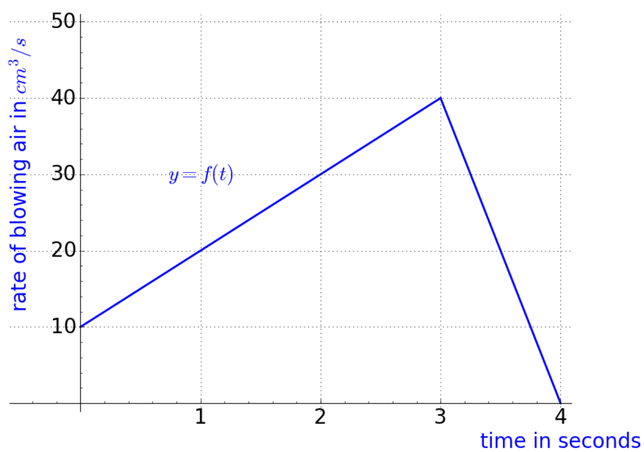
B. $\sqrt{5} - \sqrt{6}$

C. $\frac{1}{2}$

D. $\frac{1}{2\sqrt{5}}$

E. $\frac{1}{2\sqrt{6}}$

13. (7 pts) Josie is blowing up a balloon. The **rate** at which she blows air into the balloon at time t is $f(t)$ cm^3 per second, graphed below. When $t = 0$, the balloon is empty. How many cm^3 of air are in the balloon at time $t = 2$ seconds?



A. 10

B. 20

C. 30

D. 40

E. Cannot be determined from this information.

14. (4 pts) Which integral is equal to $\int_0^1 x^2 \sqrt{x^2+3}\, dx$? Hint: use u-substitution.

A. $\int_0^1 (u-3)\sqrt{u}\, du$

B. $\int_3^4 (u-3)\sqrt{u}\, du$

C. $\frac{1}{2}\int_0^1 \sqrt{u-3}\sqrt{u}\, du$

D. $\frac{1}{2}\int_3^4 \sqrt{u-3}\sqrt{u}\, du$

E. $\int_3^4 u^2\sqrt{u}\, du$

15. (4 pts) Find y' if $y = x^{\arctan(x)}$. (You can assume $x > 0$.)

A. $y' = \arctan(x)x^{\arctan(x)-1}$

B. $y' = \frac{\arctan(x)}{x} + \frac{\ln(x)}{1+x^2}$

C. $y' = x^{\arctan(x)}\ln(x)$

D. $y' = x^{\arctan(x)}\left(\arctan(x) + \frac{x}{1+x^2}\right)$

E. $y' = x^{\arctan(x)}\left(\frac{\arctan(x)}{x} + \frac{\ln(x)}{1+x^2}\right)$

16. (7 pts) A large piece of ice in the shape of a perfect cube is melting. Its volume is decreasing at a rate of 60 cm^3 per minute. Find the absolute value of the rate at which its surface area is changing when its side length is 10 cm.

Answer: ______________ cm^2/min

17. (8 pts) Consider the function $f(x) = 1 + \sqrt{x}$.

(a) Find the linear approximation for $f(x)$ at $a = 1$.

$L(x) =$ [ ]

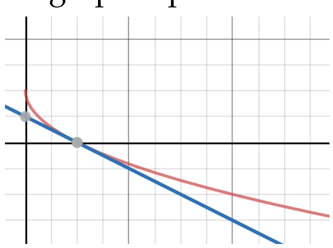
(b) In order to use this linear approximation to approximate the number $1 + \sqrt{0.7}$, what value of x would you plug into your answer from (a)?

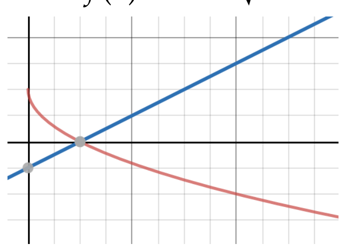
$x =$ [ ]

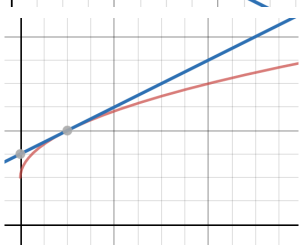
(c) Approximate $1 + \sqrt{0.7}$

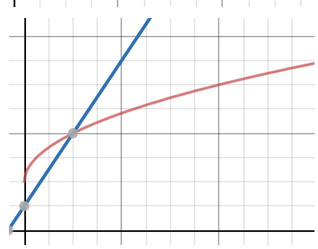
Answer: [ ]

(d) Which graph represents the function $f(x) = 1 + \sqrt{x}$ and its linearization?

A. 

B. 

C. 

D. 

(e) Is your estimate in part (c) an overestimate or an underestimate of the actual value of of $1 + \sqrt{0.7}$?

A. Overestimate

B. Underestimate

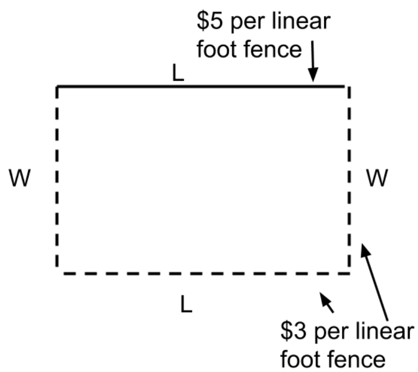
18. (8 pts) A particle is moving with the given data. Find a formula for the position of the particle. Note: $a(t)$ is acceleration, $v(t)$ is velocity, and $s(t)$ is position.

$$a(t) = \sin(t) + 3\cos(t), s(0) = 0, v(0) = 2$$

$s(t) =$ [ ]

19. (7 pts) The town of Chapel Hill plans to build a park, that will be fenced in with two types of fencing as shown. The fencing costs \$3 per linear foot for the cheaper fence (dotted lines) and \$5 per linear foot for the more expensive fence (solid line). The town can spend no more than \$4800 on fencing.

- Use calculus to find the dimensions of a park with the largest possible area.
- Use the first or second derivative test to check that your answer gives a maximum.



$L =$ ☐ $W =$ ☐

20. (7 pts) Compute $\displaystyle\int \frac{5^{\sqrt{x}}}{\sqrt{x}}\,dx$

Answer:

21. (7 pts) Sketch the graph of a function defined on $(-\infty, \infty)$ with exactly one discontinuity that satisfies:

- $f(2) = 3$
- $\lim_{x\to-\infty} f(x) = 1$ and $\lim_{x\to\infty} f(x) = 0$
- $\lim_{x\to 0} f(x) = -\infty$
- $f'(x) < 0$ for $x < 0$ and $x > 2$ and $f'(x) > 0$ for $0 < x < 2$
- $f''(x) < 0$ for $x < 0$ and $0 < x < 3$ and $f''(x) > 0$ for $x > 2$