A

Final Exam for Math 233 (Fall 2017)

NAME	
UNC EMAIL	PID
INSTRUCTOR	SECTION

Instructions

- This exam consists of **12 exercises**, each worth **10 points**.
- Write **clear solutions** so that you can get partial points for your reasoning!
- Write your **final answer** in the **BOXES** provided.
- Calculators and other materials are **NOT** allowed.
- The duration of the exam is 3 hours.

Honor pledge

I certify that no unauthorized assistance has been received or given in the completion of this work.

SIGNATURE

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Problem 1.

(1) **Find an equation** of the tangent plane to the graph of $f(x, y) = ye^x$ at the point $(0, 2)$. *Express the plane in* $ax + by + cz = d$ *form.*

(2) **Find a linear approximation** of the function $f(x, y) = ye^x$ at the point $(0, 2)$.

(3) **Find an approximation** of the value $2.03e^{0.1}$.

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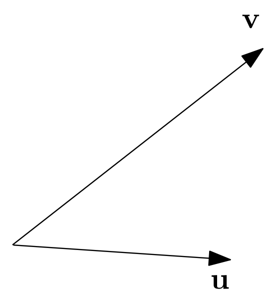
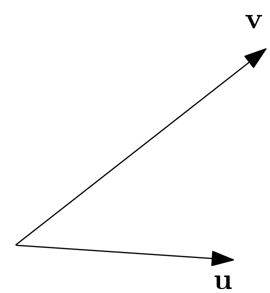
Problem 2.

(a) **Sketch** the vector $2\mathbf{u} + 0.5\mathbf{v}$.



(b) **Sketch** the vector $\text{proj}_{\mathbf{v}}\mathbf{u}$.



(c) Let $\mathbf{u} = \langle 1, 1, 0\rangle$ and $\mathbf{v} = \langle 1, 0, 1\rangle$. **Find the angle** between $\mathbf{u}$ and $\mathbf{v}$.

(d) Let $\mathbf{u} = \langle 1, 1, 0\rangle$ and $\mathbf{v} = \langle 1, 0, 1\rangle$. **Find the area** of the parallelogram of sides $\mathbf{u}$ and $\mathbf{v}$

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Problem 3. Find the **absolute minimum value** of the function

$$f(x, y) = x^2 + 2y^2 - x$$

on the region $x^2 + y^2 \leq 4$.

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Problem 4. Consider the curve in space parametrized by

$$\mathbf{r}(t) = \langle t^2, t^4, t^3 \rangle, \qquad 0 \le t \le 2.$$

(1) Find parametric equations for the tangent line to the curve at the point $(1, 1, 1)$.

(2) Find the work done by the force $\mathbf{F}(x, y, z) = \langle y, 0, z \rangle$ on a particle that moves along the curve described above.

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Problem 5. Let
$$g(s,t) = f\Big(3s + e^{st}\,,\; s^2 + \sin(t^2 + 2t)\Big),$$
where f is an unknown function. Suppose you know that

$$\begin{array}{llll} g(1,0) = 3, & f(1,0) = 5, & f_x(1,0) = 1, & f_y(1,0) = 4, \\ g(4,1) = 0, & f(4,1) = 8, & f_x(4,1) = -1, & f_y(4,1) = 2. \end{array}$$

Find $\dfrac{\partial g}{\partial s}(1,0)$**.**

$\dfrac{\partial g}{\partial s}(1,0) =$

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Problem 6. Consider the vector field

$$\mathbf{F}(x,y) = \left\langle 2xy^2 + 5\,,\, 2x^2y - 3y^2 \right\rangle.$$

(1) Is $\mathbf{F}$ conservative? If the answer is no, please justify. If the answer is yes, find a potential (that is, a function f so that $\nabla f = F$).

(2) **Evaluate** $\int_{C_1} \mathbf{F} \cdot d\mathbf{r}$, where C_1 is given by $\mathbf{r}(t) = t^2\mathbf{i} + \sqrt{t}\mathbf{j}$ between $(0,0)$ and $(1,1)$.

(3) **Evaluate** $\int_{C_2} \mathbf{F} \cdot d\mathbf{r}$, where C_2 is the circle $x^2 + y^2 = 4$ travelled in the counter-clockwise direction starting at $(2,0)$.

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Problem 7. A bug is crawling on a sheet of metal modelled by the xy-plane. The temperature of the sheet is given by the function

$$T(x, y) = 2x^2y + 3xy + 10.$$

(1) If the bug is currently at the point $(3, 1)$, **find the direction** in which the bug should go to **decrease** temperature most rapidly.

(2) **Find the rate of change** of the temperature at the point $(3, 1)$ in the direction of $\langle -1, 1 \rangle$.

(3) Suppose the bug is now at the point $(3, 1)$. **Find an equation** for a curve on which the bug should walk if it wants to stay at exactly the same temperature as it is now.

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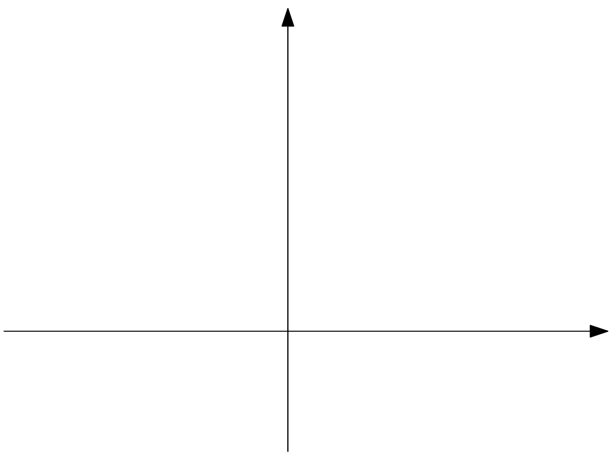
Problem 8. A lamina is shaped as the region in $\mathbb{R}^2$ bounded by the curves $x = 0$, $y = x/2$ and $y = 1$.

Sketch the region that the lamina occupies **(shade it)** on the coordinate plane below and **find the mass** of the lamina if its density is given by

$$\rho(x, y) = x\cos(y^3 - 1) + 2.$$



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Problem 9. Find the **volume** of the solid that lies in the **first octant** and is enclosed by the paraboloid $z = 1 + x^2 + y^2$ and the plane $x + y = 2$.

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Problem 10. Consider the line integral

$$\int_{\partial D} x^2 y \; dx - y^2 x \; dy,$$

where D is the region in the first quadrant enclosed between the coordinate axes and the circle $x^2 + y^2 = 4$, and where ∂D is the boundary curve of the region D traversed in counterclockwise direction.

Use Green's Theorem to **compute** this integral.

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Problem 11. Find the **volume** of the solid that lies within the sphere $x^2 + y^2 + z^2 = 1$, above the xy-plane, and below the cone $z = \sqrt{x^2 + y^2}$.

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Problem 12. Find the **surface area** of the part of the paraboloid $x = y^2 + z^2$ between the planes $x = 0$ and $x = 4$.

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