Math 233 Final Exam Fall 2023

Circle your instructor's name: **Burkhead** **Green** **McLaughlin**

- Clearly print your NAME and PID below.
- Do not hyphenate your PID (scanner cannot read it):

NAME:

PID:

- Closed book, closed notes, no calculators. Use a pencil with eraser.
- **Print your name on each page**
- Show all work on the exam- no extra paper.
- Show photo ID when turning in exam.
- The small number next to each problem is its point worth.
- No credit for the correct solution without supporting work for the free-response questions.
- By putting your name on your paper, you implicitly pledge your adherence to the honor code.
- For all multiple-choice questions, bubble the circle of your answer, as shown. Fully erase changed answers (otherwise the scanner will pick up it up). No partial credit.

○ XXXX

● your answer

○ XXXX

Useful values and formulas on the backside.

Use the blank pages for scratch work (do not remove). This work will not be graded.

θ		$sin\ \theta$	$cos\ \theta$
Rad	**Deg**		
0 / 2π	0	0	1
π/6	30	1/2	$\sqrt{3}/2$
π/4	45	$\sqrt{2}/2$	$\sqrt{2}/2$
π/3	60	$\sqrt{3}/2$	1/2
π/2	90	1	0

$$sin(2\theta) = 2\,sin\,\theta\,cos\,\theta$$

$$cos(2\theta) = cos^2\theta - sin^2\theta = 2\,cos^2\theta - 1 = 1 - 2\,sin^2\theta$$

$$sin^2\theta = \frac{1 - cos(2\theta)}{2}$$

$$cos^2\theta = \frac{1 + cos(2\theta)}{2}$$

$$\rho = \sqrt{x^2 + y^2 + z^2}$$

$$x = \rho \sin\phi \cos\theta$$

$$y = \rho \sin\phi \sin\theta$$

$$z = \rho \cos\phi$$

$$\cos\phi = \frac{z}{\sqrt{x^2 + y^2 + z^2}}$$

$$\tan\phi = \frac{\sqrt{x^2 + y^2}}{z}$$

Scratch work area:

NAME ________________________________

6 **1)** Suppose **a** and **b** are nonzero, orthogonal vectors. Which of the following is true?

○ I. $proj_{\mathbf{a}}\mathbf{b} = proj_{\mathbf{b}}\mathbf{a}$

○ II. $\mathbf{a} \times \mathbf{b} = \mathbf{b} \times \mathbf{a}$

○ I and II

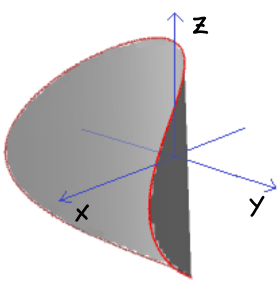
○ neither I nor II

6 **2)** True or False. $\int_0^1 \int_0^1 f(x)f(y)\,dydx = \left[\int_0^1 f(x)dx\right]^2$

○ True

○ False

6 **3)** Choose the correct curve given by $r(t) = \cos t\mathbf{i} + \sin^2 t\mathbf{j} + \sin t\mathbf{k}$.



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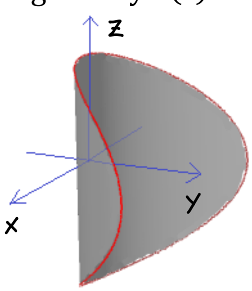

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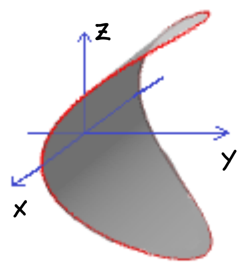

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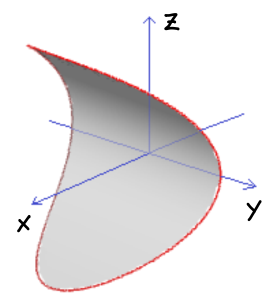

○

7 **4)** Which expression represents the work done by the force field $\mathbf{F} = \langle y^2, e^x \rangle$ moving a particle along the straight-line path from point (1,2) to point (6,5)?

○ $\mathbf{F}(6,5) - \mathbf{F}(1,2)$

○ $\int_0^1 ((2+3t)^2 + e^{1+5t})dt$

○ $\int_0^1 ((2+3t)^2 + e^{1+5t})\sqrt{34}dt$

○ $\int_0^1 (5(2+3t)^2 + 3e^{1+5t})dt$

○ $\int_0^1 \left((2+3t)^2(1+5t) + e^{1+5t}(2+3t)\right)dt$

Scratch Work

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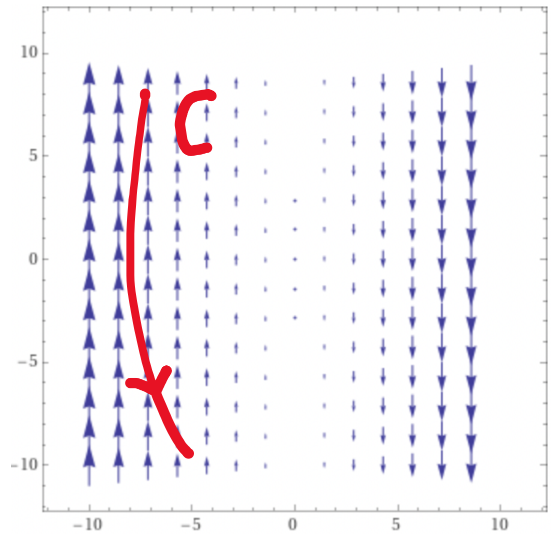
6**5)** Consider the following vector field **F**.

Let C be a curve with indicated orientation. Is the line integral $\int_C \mathbf{F}\cdot\mathbf{T}\,ds$ positive, negative, zero, or not enough information to determine.



○ positive

○ negative

○ zero

○ not enough information to determine

6**6)** Let $g(x,y)$ be a differentiable function. Which of the following statements is true?

○ **I.** $\nabla g(a,b)$ is parallel to the tangent line of $g(x,y)=g(a,b)$ at the point $(x,y)=(a,b)$.

○ **II.** $\nabla g(a,b)$ gives the direction of greatest increase of g at the point $(x,y)=(a,b)$.

○ **I** and **II**

○ neither **I** nor **II**

7**7)** Determine whether $f(x,y)=\begin{cases}\dfrac{1-\cos y}{(x+2)\,y^2} & if\ (x,y)\neq(0,0)\\ \dfrac{1}{2} & if\ (x,y)=(0,0)\end{cases}$ is continuous at (0,0).

○ No, because $f(0,0)$ does not exist.

○ No, because $\lim_{(x,y)\to(0,0)} f(x,y)$ does not exist.

○ No, because $\lim_{(x,y)\to(0,0)} f(x,y)\neq f(0,0)$.

○ Yes, because $f(0,0)$ exists.

○ Yes, because $\lim_{(x,y)\to(0,0)} f(x,y)$ exists.

○ Yes, because $\lim_{(x,y)\to(0,0)} f(x,y)=f(0,0)$.

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7**8)** Which of the following parameterizes the helix $\hat{\mathbf{r}}(t) = \langle 2\cos t, 2\sin t, 4t\rangle, t \geq 0$ by arc length?

○ $\mathbf{r}(\mathrm{s}) = \frac{1}{2\sqrt{5}}\langle 2\cos s, 2\sin s, 4s\rangle, s \geq 0$

○ $\mathbf{r}(\mathrm{s}) = \langle 2\cos\left(\frac{s}{2\sqrt{5}}\right), 2\sin\left(\frac{s}{2\sqrt{5}}\right), \frac{2s}{\sqrt{5}}\rangle, s \geq 0$

○ $\mathbf{r}(\mathrm{s}) = \langle 2\cos(2s\sqrt{5}), 2\sin(2s\sqrt{5}), 8s\sqrt{5}\rangle, s \geq 0$

○ $\mathbf{r}(\mathrm{s}) = \langle 2\cos\left(\frac{1+s}{2\sqrt{5}}\right), 2\sin\left(\frac{1+s}{2\sqrt{5}}\right), \frac{2+2s}{\sqrt{5}}\rangle, s \geq 0$

9) Use the contour plot of $f(x, y)$ to answer each of the following questions.

4 **(a)** Estimate $f_x(0,0)$.

○ $-5/2$

○ $-2/5$

○ $2/5$

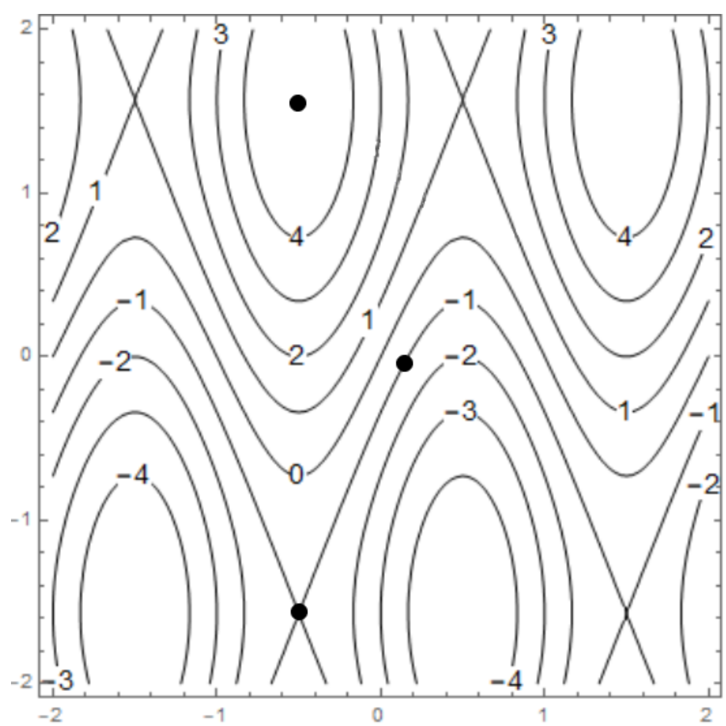
○ $5/2$

4 **(b)** Classify the critical point at $(x, y) = (-0.5, 1.5)$

○ a local maximum

○ a local minimum

○ a saddle point



Scratch Work

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6**10)** Use differentials to approximate the change in the volume of a cylinder, $V = \pi r^2 h$, when r changes from 2 cm to 1.9 cm and h changes from 5 cm to 5.4 cm.

○ Volume increases by about $19.6\pi\ cm^3$

○ Volume decreases by about $16.4\pi\ cm^3$

○ Volume increases by about $3.6\pi\ cm^3$

○ Volume decreases by about $0.4\pi\ cm^3$

○ Volume increases by about $0.04\pi\ cm^3$

7**11)** Convert the following triple integral to spherical coordinates.

$$\int_0^3 \int_0^{\sqrt{9-y^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{18-x^2-y^2}} z\sqrt{x^2+y^2+z^2}\ dz\,dx\,dy$$

○ $\int_0^{\pi/2} \int_0^{\pi/4} \int_0^{\sqrt{18}} \rho^2 \sin\varphi\, d\rho\, d\varphi\, d\theta$

○ $\int_0^{\pi} \int_{\pi/4}^{\pi/2} \int_0^{3} \rho^4 \cos\varphi \sin\varphi\, d\rho\, d\varphi\, d\theta$

○ $\int_0^{\pi/2} \int_0^{\pi/4} \int_0^{\sqrt{18}} \rho^4 \cos\varphi \sin\varphi\, d\rho\, d\varphi\, d\theta$

○ $\int_0^{\pi} \int_0^{\pi/2} \int_0^{3} \rho\cos\varphi\, d\rho\, d\varphi\, d\theta$

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6**12)** Is $\mathbf{F} = 4x^2\mathbf{i} + ye^y\mathbf{j} + z\mathbf{k}$ an irrotational vector field?

○ Yes, because $\nabla \cdot (\nabla \times \mathbf{F}) \neq 0$
○ No, because $\nabla \cdot \mathbf{F} \neq 0$
○ Yes, because $\nabla \times \mathbf{F} = \mathbf{0}$
○ No, because $\nabla \times \mathbf{F} = \mathbf{0}$
○ No, because $\nabla \times \mathbf{F} \neq \mathbf{0}$

7**13)** Find the rate of change of $f(x, y) = x^2 + xe^y$ at the point $(3,0)$ in the direction of $\langle -2, 1\rangle$.

○ -11

○ $\frac{-11}{\sqrt{5}}$

○ $\sqrt{58}$

○ $\frac{-1}{\sqrt{58}}$

7**14)** Find an equation of a plane that contains the point $(2,1,-1)$ and runs in the direction of both the vector $\langle 1,2,3\rangle$ and the $z-$ axis.

○ $2x - y - 3 = 0$
○ $x + 2y + 3z - 1 = 0$
○ $2x - y - 3z - 9 = 0$
○ $x - y + 1 = 0$
○ $3x + 2y + 2z - 6 = 0$

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12**15)** Determine a set of parametric equations for the tangent line to the curve given by $r(t) = \langle 5t - 6,\ -e^{3-t},\ \frac{9}{t^2} \rangle$ at the point (9, -1,1). Show all work.

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12**16)** Let $\mathbf{F} = \langle tan^{-1}x, \cos y + 2xy \rangle$. Using Green's Theorem, evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is the boundary of the triangle with vertices (0,0), (4,0) and (0,4) oriented in the counterclockwise direction. Show all work.

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12 **17)** Consider the oriented curve C: $y = \cos(\pi x) + 2$, from $(0,3)$ to $(1,1)$.
Use the Fundamental Theorem of Line Integrals to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ for $\mathbf{F} = \langle 2x + 2xy^2, 2y + 2yx^2 \rangle$. Show all work.

Verify that **F** is conservative.

Find a potential function ϕ for **F**.

Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$.

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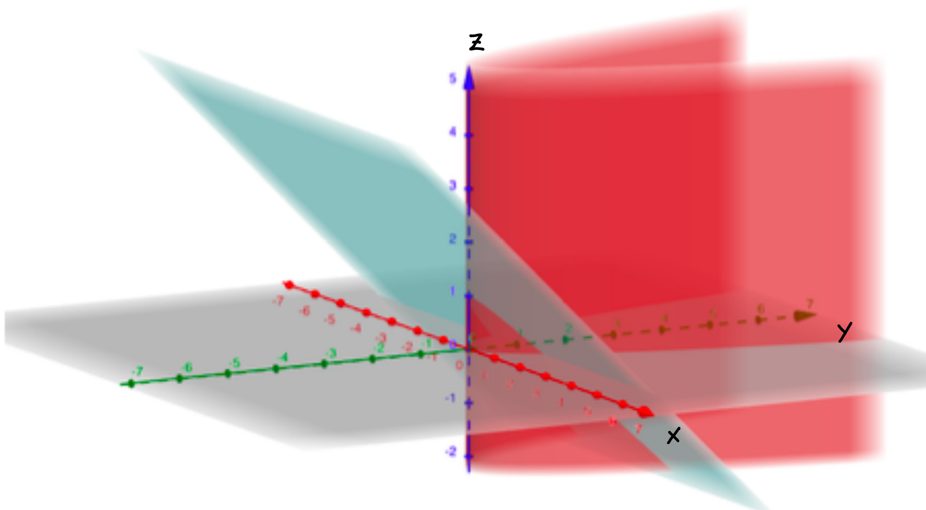
12 **18)** Set up a triple integral that gives the volume of the solid region bounded by the plane $z = 1 - y$, the paraboloid $y = x^2$, and the xy – plane using the two orders of integration given below. **Do not evaluate.**



a) $dz\,dy\,dx$

b) $dx\,dy\,dz$

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12**19)** Evaluate the integral by changing the order of integration. Show all work.

$$\int_0^{\pi/4} \int_{2x}^{\pi/2} \frac{\sin(y)}{y}\, dy dx$$

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12**20)** Find an equation of the tangent plane to $x^2 y \sin z = 1$ at the point $\left(-2, 1, \frac{5\pi}{6}\right)$. Show all work.

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12**21)** Evaluate the integral $\iint_E \frac{1}{\sqrt{16-x^2-y^2}}\, dA$, where E is the region $\{(x, y) \mid 1 \le x^2 + y^2 \le 4, x \ge 0, y \ge 0\}$. Show all work.

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12**22)** Find and classify (local maximum, local minimum, or saddle point) the critical points of the function $f(x, y) = x^2 + 2y^2 - x^2y$, if they exist. Show all work.

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12**23)** Find $\iint_S \mathbf{F}\cdot\mathbf{n}\,\mathrm{d}S$, the upward flux across S of the vector field $\mathbf{F} = \langle 0, z, y\rangle$ where S is the plane $z = 8 - x - 2y$, in the first octant. Show all work.