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Self-assembly and Particle Aggregation in Stratified Fluids

Experimental Discovery and First-principle Mechanisms of Diffusion-induced Flows in Exterior Domains

By Roberto Camassa and Richard M. McLaughlin

The motion of particulates in the ocean as they fall under gravitation constitutes a critical component of the carbon cycle. Phytoplankton can photosynthesize dissolved carbon dioxide gas into dense, solid carbon debris called "marine snow," which facilitates the transport of carbon from the surface waters to the ocean depths. Over the course of eons, the carbon eventually turns into oil. The prevailing view maintains that particulate aggregation occurs through random turbulent oceanic motions that generate collisions between small-scale sticky particulates, which then form aggregates [4].

However, we recently discovered that such particles—when suspended at similar depths in stratified water—can slowly form aggregates in the absence of both external random flows and adhesion [7]. Surprisingly, these particles experience an effective horizontal attractive force that originates from the no-flux boundary condition. This attraction leads to self-assembly, during which particles form molecule-like assemblages that subsequently appear to dynamically solve jigsaw-like puzzles while en route to large-scale particulate discs.

Figure 1a depicts a time-series montage of the formation of a "particulate molecule" that eventually merges with a larger aggregate, and Figure 1b portrays a schematic of the experiment. Figures 1c and 1d present an initial image of the particulate distribution and a visual from 250 minutes later. Our findings offer insights into new mechanisms for large-scale aggregate formation in a wide variety of physical systems, including the ubiquitous stratified ocean waters, potential microplastic organization mechanisms, magnetic dynamos on Mercury and Ganymede, and even origin-of-life questions (such as matter's self-organization in deep ocean vents).

The discovery itself arose through a fortunate series of laboratory mistakes that ultimately led to a "eureka" moment. During a routine outreach demonstration, a setup error revealed this self-assembly phenomenon. Our demo—which seeks to exemplify

See **Particle Aggregation** *on page 3*

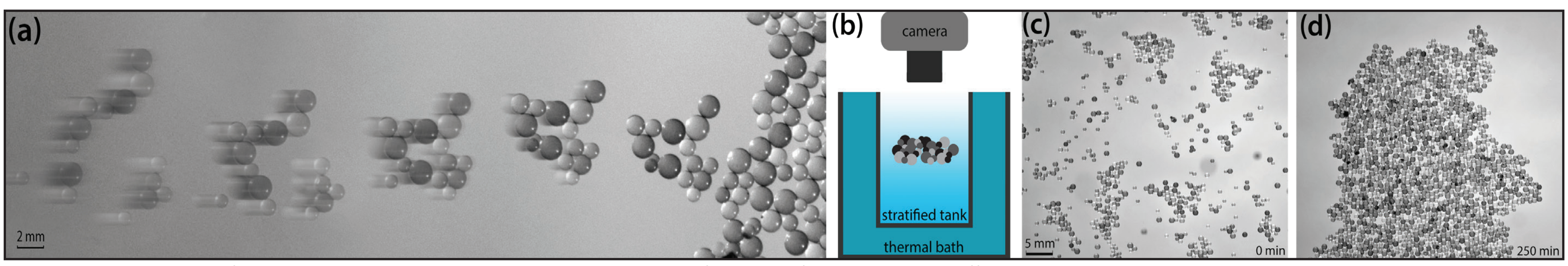

Figure 1. *Experimental self-assembly.* **1a.** *Time-series montage that illustrates the formation of a molecule. Five different output times—with 10 minutes between each instance—appear from left to right (artistic blurring depicts the motion). The spheres' radii and densities are respectively* 0.025-0.05 *cm and* 1.05 *g cc*$^{-1}$*; the top fluid is fresh water of density* 0.997 *g cc*$^{-1}$ *and the bottom is a sodium chloride water solution of density* 1.1 *g cc*$^{-1}$*, with a transition thickness of approximately* 2 *cm.* **1b.** *Schematic of the experimental setup.* **1c.** *Initial cluster.* **1d.** *Final self-assembled cluster. Figure adapted from [7].*

Untangling Topology with California Blackworms

By Matthew R. Francis

Elastic filaments have a tendency to spontaneously tangle, as evidenced by the frequent development of knots in electrical cords, headphones, and garden hoses. However, another class of systems can *detangle* just as rapidly without conscious intervention. Biological polymers inside cells—such as DNA, RNA, proteins, and other complex molecules—primarily reside in a snarled-up state but can spontaneously organize themselves to perform essential processes, particularly cell division.

California blackworms (*Lumbriculus variegatus*) exhibit a macroscopic version of this phenomenon, forming spheroidal tangles that consist of anywhere from five to 50,000 individual worms (see Figure 1). Though completion of this tangling operation can take from several minutes to as long as a half hour, detangling is practically instantaneous. Because blackworm brains only contain a few neurons, the detangling process must be purely biophysical in nature — analogous to the biopolymers inside a cell.

The relatively large size of these worms—which range from about four to 10 centimeters (roughly 1.5 to four inches) in length—makes them a prime study candidate for researchers who are interested in the mathematics of real-world knots, which have physical properties that affect their topological and geometrical structures. On a practical level, these types of systems can help resolve protein folding problems or lead to the creation of new materials that avoid excessive tangling.

"Lots of materials are made up of tangled filaments," Vishal Patil, a mathematician in the Department of Bioengineering at Stanford University, said. "Things like polymer gels and—at larger scales—wool, fabrics, and cellulose are all comprised of tangled filaments."

Patil's doctoral research explored the mathematics of physical knots, such as those in shoelaces. This work caught the eye of Saad Bhamla, a biomaterials scientist at the Georgia Institute of Technology who was studying blackworms as a potentially intriguing system for the development of analogues for other tangled-filament materials. Patil and Bhamla's subsequent collaboration resulted in a paper that recently published in *Science* [1], wherein Bhamla's group provided experimental measurements of blackworm tangles to reconstruct their full three-dimensional structure. This structure then served as the basis for the mathematical model. "The fact that Saad and Harry [Tuazon], his grad student, discovered this amazing tangling-untangling behavior in these worms suggests something about tangles that can apply to other contexts as well," Patil said.

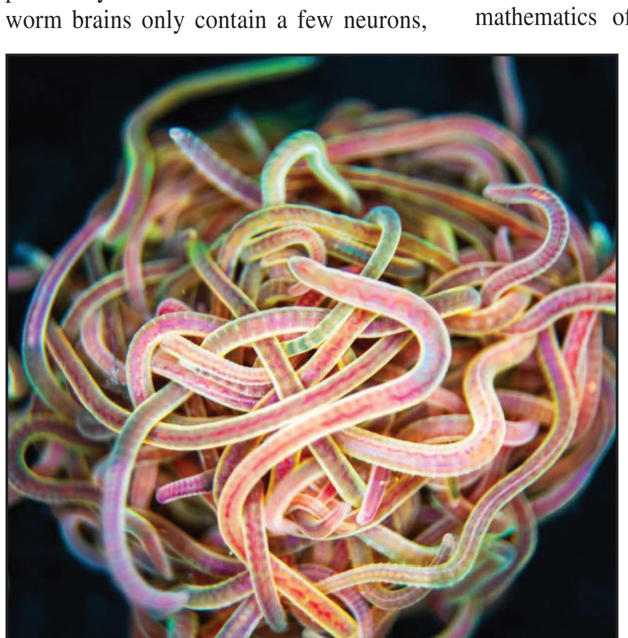

Figure 1. *A tangled mass of approximately 200 California blackworms, roughly 20 millimeters across. Figure courtesy of [1].*

Oh What a Tangled Worm We Weave

Blackworms form tangles to control their temperature, prevent themselves from drying out, and move efficiently as a group. Their intricately constructed clumps can redisperse under threat in a matter of milliseconds. Such behavior indicates that the *way* in which they form a tangle is essential to the rapidity of the untangling process (in contrast to extension cords' seemingly spontaneous ability to hopelessly tie the lawnmower to the bicycles[1]).

To model the system, the researchers first immobilized a worm clump by embedding it in gelatin and scanning it with an ultrasound. Drawing from knot theory, they defined the linking number between worms as

$$Lk_{ij} = \frac{1}{4\pi}\int \Gamma_{ij}\cdot(\partial_s\Gamma_{ij}\times\partial_\sigma\Gamma_{ij})\,ds\,d\sigma,$$

where the matrix

$$\Gamma_{ij} = \frac{\boldsymbol{x}_i(s)-\boldsymbol{x}_j(\sigma)}{|\boldsymbol{x}_i(s)-\boldsymbol{x}_j(\sigma)|}$$

involves the vector difference between curves $\boldsymbol{x}_i$ and $\boldsymbol{x}_j$ that respectively describe worms i and j. To reduce the system's complexity, the modelers defined the *contact link* as $cLk_{ij} = |Lk_{ij}|$ if the worms are in contact and $cLk_{ij} = 0$ if they are not touching. They found that most worms had physical contact with nearly every other individual in the gelatin, thus indicating that strong interactions govern the system (see Figure 2, on page 4).

See **Blackworms** *on page 4*

[1] A point first made by the late Terry Pratchett in his 1992 novel, *Lords and Ladies*.

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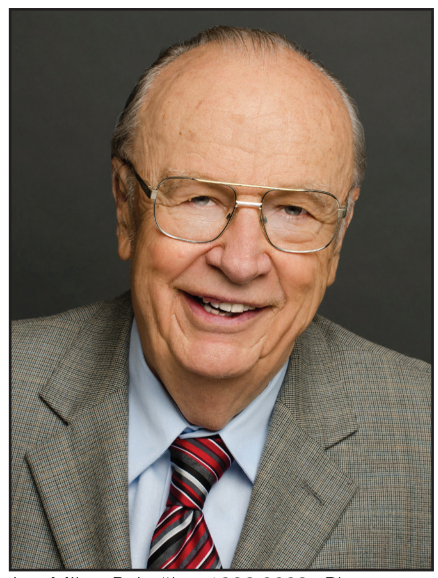
Remembering Ivo Babuška

By Douglas N. Arnold

Ivo Milan Babuška, a Czech-American mathematician and engineer who came of age at the end of the precomputer era, passed away on April 12, 2023. He was 97 years old. As one of the creators of the field of computational science and engineering, Ivo was the preeminent authority on the finite element method (FEM). He was thoroughly dedicated to his research and helped develop FEM into an accurate and reliable methodology for computer simulation. Ivo pursued every project with a remarkable level of curiosity, energy, and discipline and generated a torrent of original, insightful, and high-impact contributions.



Ivo Milan Babuška, 1926-2023. Photo courtesy of the Oden Institute for Computational Engineering and Sciences.

Czechoslovakia: The Formative Years

Ivo was born on March 22, 1926 in Prague, Czechoslovakia. When he was 13, Ivo's life was turned upside down by a momentous political upheaval — the first of three times this was to occur. Nazi Germany invaded Czechoslovakia and occupied it until the end of World War II in 1945. The Babuška family lost almost everything and the Nazis closed the universities, sabotaging Ivo's educational plans. Despite the family's hardships, Ivo's father engaged a first-rate mathematics professor to tutor his son in university-level mathematics.

When the war ended and universities reopened, Ivo enrolled in the Czech Technical University to study civil engineering, obtaining his first doctorate in 1951. While he was completing this degree, Eduard Čech—a renowned geometer and director of the Mathematics Institute of the Czechoslovak Academy of Sciences—awarded a fellowship for study at the Institute to Ivo and 11 other promising university graduates. Čech sought to revitalize Czech mathematics and guided Ivo toward the rapidly evolving areas of numerical analysis and applied mathematics. Under Čech's skillful guidance, the Institute soon became home to a close-knit, congenial band of brilliant students and professors who not only studied but also socialized together. Ivo later noted the influence that this extraordinary educational environment had on his own future interactions with students and mentees. In 1955, he earned his second Ph.D. — this time from the Mathematics Institute.

Unfortunately, the restoration of democratic institutions in Czechoslovakia was short-lived. In a second life-altering political upheaval, the Soviet-backed Communist Party of Czechoslovakia staged a coup d'état in 1948. They took complete control of the government and quickly converted the country into a satellite state of Stalin's Soviet Union. It was against this dark background that Ivo finished his formal education and launched his career. Luckily, government officials recognized applied mathematics' necessity to industrial development and allowed the field to progress with minimal interference.

A formative experience in Ivo's education was his role on an interdisciplinary team of civil engineers, materials scientists, and mathematicians who—beginning in 1954—consulted on the six-year construction of the Orlík Dam on the Vltava River. The design process raised many technical challenges, such as determining which construction procedures would likely prevent the formation of harmful cracks in the concrete. Ivo led the mathematical efforts by formulating models based on partial differential equations (PDEs), analyzing them mathematically, and studying them further with numerical computation. Since Czechoslovakia was still in the pre-computer era, a few desk calculator operators carried out the numerical computations under Ivo's direction. The efforts of Ivo and the Orlík research team expedited the construction of the dam, which is still in use today. Ivo's lifelong devotion to interdisciplinary science began at this time.

Upon completing his second Ph.D. in 1955, Ivo was appointed head of the Department of Constructive Methods of Mathematical Analysis within the Mathematical Institute of the Czechoslovak Academy of Sciences — a position he held until he left Czechoslovakia. He published prolifically, and his reputation as a numerical analyst and applied mathematician steadily grew. In 1956, Ivo founded the journal *Aplikace Matematiky* (now known by its English title, *Applications of Mathematics*) and served as its first editor-in-chief. He was granted a Doctor of Science degree in 1960 and received the Czechoslovak State Medal—the nation's highest scientific award—in 1968. In 1964 and 1967, Ivo organized the International Conference on Basic Problems of Numerical Analysis in Czechoslovakia. As a reflection of his growing international reputation, top numerical analysts from around the world—including Sergei Sobolev, Andrey Tikhonov, Gene Golub, Eugene Isaacson, Olof Widlund, Lothar Collatz, and James H. Wilkinson—were in attendance.

The third and most consequential political upheaval for Ivo came in 1968, when the Soviet Union brutally suppressed the Prague Spring reform movement. In August of that year, hundreds of thousands of Soviet and Warsaw Pact troops stormed Czechoslovakia. Fortuitously, Ivo already had plans underway for a one-year visiting professorship at the University of Maryland (UMD) and managed to obtain the necessary documentation to travel by ship to the U.S. with his wife Renata and their young children, Lenka and Vit. The family arrived in Maryland in September 1968.

Maryland: Finding His Life's Work in America

Ivo's visiting appointment at UMD was in the Institute for Fluid Dynamics and Applied Mathematics (IFDAM, later the Institute for Physical Science and Technology). At the end of the year, he accepted a permanent position as a distinguished professor at IFDAM. He remained there for 27 years.

Ivo's research underwent a sharp discontinuity when he emigrated to Maryland. In Czechoslovakia, he had published more than 40 mathematical papers; not one of them even mentioned FEM. After arriving in the U.S., Ivo published roughly 300 more papers. Starting with the very first, virtually every one addressed finite elements; the future *ne plus ultra* of FEM had found his true calling and never looked back. In the next five decades, Ivo continued to produce significant and often seminal results in rapid succession. This work—along with his powerful personality, generous nature, and leadership qualities—made him a driving force in the development of FEM.

In 1970, Ivo and a few colleagues conceived of the need for regular short meetings in the burgeoning area of finite elements. They envisioned an unusual format that did not specify speakers in advance but instead allowed any participant with relevant work to present. To match this informal structure, the group selected a genial name for the conference series: the Finite Element Circus.[1] The first Circus took place at UMD in 1970; future iterations were held at the University of Chicago, Harvard University, and Cornell University. Ivo served as Circus Ringmaster for the first 25 years. 53 years later, the well-attended Circus continues to meet every spring and fall.

Contributions to FEM Theory and Practice[2]

In the 1970s, Ivo's work centered on the development of a mathematical theory of FEM. One of his first American papers presented a formal framework for the rigorous numerical analysis of FEM. The paper contained one of Ivo's most important contributions: his introduction of the *inf-sup condition* and *discrete inf-sup condition* into numerical PDEs and his proof of their precise relation to the stability and convergence of Galerkin methods. In other publications, Ivo identified what became some of the major extensions of basic FEM, including *Lagrange multipliers*, *penalties* to enforce conformity or boundary conditions, and *treatment of singularities* in the data. Each of these topics later attracted scores of researchers — evidence of Ivo's prescience. In 1972, Ivo delivered an extraordinary series of 10-hour lectures during an international symposium on the mathematical foundations of FEM in Baltimore, Md. A written version of more than 350 pages was published in the meeting's proceedings and became the bible for the mathematical theory of finite elements. Later that decade, he introduced the

See **Ivo Babuška** *on page 5*

[1] https://sites.google.com/view/fecircus/home

[2] Due to space limitations, this section only touches upon some of Ivo's most enduring contributions. His many collaborators are not mentioned by name.

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Particle Aggregation

Continued from page 1

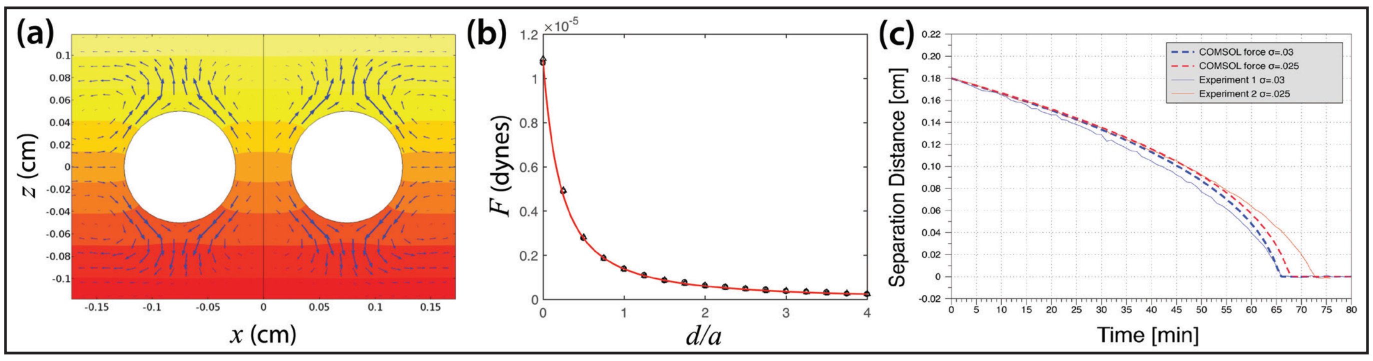

Figure 3. *COMSOL force calculations for three-dimensional simulations of two identical spheres of radius a that are separated by an amount d in linear stratification at $Pe=5.35$.* **3a.** *Steady-state density and flow field for two spheres at $d=0.045$ cm in a vertical plane that slices the north and south poles.* **3b.** *Steady attractive force on one sphere, which we compute by numerically integrating the projected stress tensor. The physical parameters are $a=0.045$ cm, $\kappa=1.5\times10^{-5}$ cm²/s, $\sigma=0.03$ g/cm⁴, $g=981$ cm/s², and $\mu=0.0113$ poise.* **3c.** *Comparison of Stokesian dynamics and two-sphere experiments for two linear background density fields: $\sigma=0.03$ g/cm⁴ and $\sigma=0.025$ g/cm⁴. Figure courtesy of [7].*

stratification's ability to trap particulates within the transition layer between top and bottom densities for a short time period—drops hundreds of thousands of glass spheres (with radii of 50 microns) into a stratified tank. The particles form a trapped cloud within the transition layer, and each particle is coated in the buoyant upper fluid; the collective cloud suspends for a transient period until the particles diffusively shed their lightweight upper fluid coatings. The system then begins to "rain" through a Rayleigh-Taylor-like multiphase instability, and all suspended particulates eventually reach the tank bottom. This phenomenon is closely related to our previous discovery that a dense sphere can actually bounce off of an internal density transition layer due to related mechanisms [1], which led to experimental, computational, and analytical studies of single body behavior [5, 6].

During one demonstration of this raining phenomenon, we made the bottom layer too salty and the particulate was buoyant with respect to the lower fluid. As such, no rain-out phase was evident after 10 minutes of observation. We checked the densities with our densitometers and confirmed the error. But rather than immediately clean up the mess, we left the system overnight; our procrastination was incredibly fortunate because the extremely slow dynamics had time to play out. The next morning, the diffuse particulate system had self-assembled into a large-scale aggregate disc. We knew that we had stumbled onto an intriguing observation and consequently launched an intense experimental, theoretical, and computational campaign over the next several years.

We suspected that the underlying mechanism pertained to a so-called *diffusion-induced flow*. Independent of one another, Owen Phillips and Carl Wunsch analytically discovered such flows for the simple case of a tilted infinite plane in a linearly stratified fluid [8, 9]. Their works derived mathematically exact, steady solutions of the incompressible Navier-Stokes equations coupled to a solute field that is governed by an advection-diffusion equation. The initially horizontal isolines of salt concentration do not orthogonally intersect the tilted plane, as required by the no-flux boundary condition. This situation causes a turning of the isolines, which then creates a buoyancy mismatch that drives a flow up the inclined plane along a narrow boundary layer near the surface.

To isolate this phenomenon's potential role in the context of our research, we first performed numerous benchmark experiments with single stationary spherical and spheroidal objects. We then pursued dynamic two-body experiments to document the finite-time collapse and observed diffusion-induced flows in substantially more complex geometries. Interestingly, previous work had demonstrated that a neutrally buoyant wedge that is suspended within the fluid can actually derive propulsion through such diffusion-induced flows, with the wedge migrating horizontally through the fluid [2]. Although this type of individual propulsion is not possible for a single symmetric body like a spheroid, the associated diffusion-induced flows can generate strong collective dynamics with multiple bodies.

The Phillips/Wunsch solution reveals some interesting scales for diffusion-induced flows. The characteristic velocity and length scales for the Phillips solution are $U=\kappa\left(\frac{g\sigma}{\kappa\mu}\right)^{1/4}$ and $L=\left(\frac{\kappa\mu}{g\sigma}\right)^{1/4}$, where g is the gravitational acceleration, μ is the dynamic fluid viscosity, κ is the salt diffusivity, and σ is the slope of the background density field (all assumed to be constant). The length scale L defines the boundary layer's thickness above the tilted flat plane, and the velocity scale U defines its strength. We use this velocity scale to nondimensionalize the equations of motion for the velocity field $\boldsymbol{u}$, pressure P, density ρ (which varies only during the evolution of the salt concentration), position $\boldsymbol{x}$, and time t (for a single sphere of radius a) via $\tilde{\boldsymbol{x}}=\frac{\boldsymbol{x}}{a}$, $\tilde{\boldsymbol{u}}=\frac{\boldsymbol{u}}{U}$, $\tilde{\rho}=\frac{\rho}{\sigma a}$, $\tilde{P}=\frac{Pa}{\mu U}$, and $\tilde{t}=\frac{t\kappa}{a^2}$. The resulting nondimensional partial differential equation (PDE) system (dropping tildes and primes) for the incompressible fluid velocity and concentration field is respectively

$$Re\,\rho\left[\frac{1}{Pe}\frac{\partial\boldsymbol{u}}{\partial t}+\boldsymbol{u}\cdot\nabla\boldsymbol{u}\right]= -\nabla P-Pe^3\rho\hat{z}+\Delta\boldsymbol{u} \tag{1}$$

$$\frac{\partial\rho}{\partial t}+Pe\,\boldsymbol{u}\cdot\nabla\rho=\Delta\rho. \tag{2}$$

Here, the Reynolds number is $Re=\frac{\sigma a^2 U}{\mu}$ and the Péclet number is $Pe=\frac{a}{L}$. For all of our experiments, $Re<0.001$, so we employed the so-called Stokes approximation to set the left side of (1) to zero while retaining nonlinear effects through nonzero Pe in the advection-diffusion of the salt concentration. As such, the system's only remaining parameter is the Péclet number. We assume that the boundary conditions are no-slip for the velocity and no-flux for the tracer on any physical boundary.

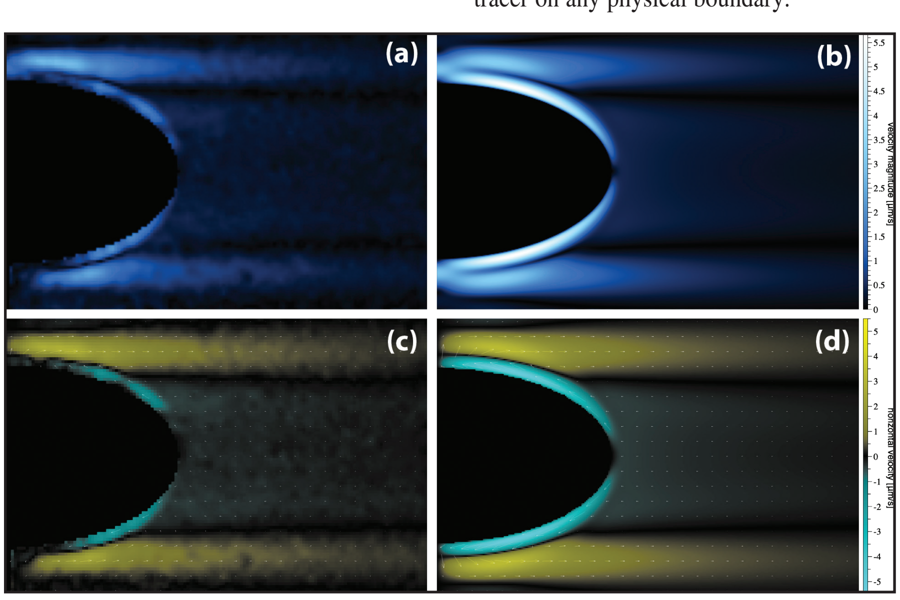

Figure 2. *Comparison of the experiment and simulation for the flow field in a vertical plane that slices the spheroid through its north and south poles. Here, $Pe=52$, $\mu=0.016$ poise, $\kappa=1.5\times10^{-5}$ cm² s⁻¹, the spheroid vertical radius $=0.5$ cm, the width $=2$ cm, and the density gradient $\sigma=0.002$ g cm⁻⁴.* **2a.** *Flow speed from particle imaging velocimetry.* **2b.** *Flow speed from COMSOL.* **2c.** *Horizontal flow velocity from particle imaging velocimetry.* **2d.** *Horizontal flow velocity from COMSOL. Figure courtesy of [7].*

Given our experimental observations of diffusion-induced flows that self-assemble into large-scale compact discs, we first examined the case of a stationary single oblate spheroid in a linearly stratified fluid. We used particle tracking velocimetry to measure the exterior flows to the compact domain, and the finite element package COMSOL[1] generated solutions of the coupled PDEs. Figure 2 provides a strong quantified comparison of the experimental and computational flows. The plots reveal an attractive flow into the body's equator, which is spun up and down and ejected at the north pole as a stronger horizontal flow that fans radially outward. This equatorial flow drives the self-assembly in Figure 1 (on page 1), which causes nearby particles to coalesce into a larger disc.

Next, we considered the behavior of the two-body problem. We first studied some three-dimensional COMSOL simulations to compute the diffusion-induced flow that is induced by two fixed spheres in steady state [7]. Figure 3a illustrates the velocity distribution in the symmetry plane that cuts vertically through the north and south poles of both spheres. We computed the viscous stress integral around one of the spheres to determine the horizontal force that the diffusion-induced flow exerts on the sphere for a range of fixed separation distances (see Figure 3b). We fit the data to a power law force $\frac{F}{F_0}=\left(\frac{d}{a}+0.3249\right)^{-1.466}$ with a prefactor $F_0=2.0865\times10^{-6}$ dynes, where d is the separation distance and a is the particle radii. The size of prefactor F_0 is reasonable; balancing Stokes drag with this force yields particle velocities on the order of 2.5 microns per second, which is roughly the average speed that we observed experimentally.

We then modified a well-tested Stokesian dynamics numerical algorithm [7]—based on the theory for multibody low Reynolds number dynamics [3]—by adding this computationally measured force. Figure 3c depicts the simulation's output for an experiment with two attracting bodies that collapses in finite time for two slightly different stratifications. Note the reasonable quantitative agreement. For the N body problem, use of the same modified Stokesian dynamics algorithm finds that self-assembly and particle aggregation strongly resemble the experimental observations in Figure 1 (on page 1) [7]. The low Péclet limit has mathematically exact flows and density perturbations for spheres with both insulating and absorbing boundary conditions. These solutions confirm that a complete flow reversal occurs for sufficiently diffusive spheres, revealing the possibility of much more complex self-assemblies.

Despite our discovery, we have barely scratched the surface of this rich class of problems. In the immediate future, we hope to assess the role of particle geometry, add non-insulating boundary conditions with different diffusivities between the solid and fluid, improve first-principles derivation of the effective force field, and study the details of the finite-time collapse that creates the self-assemblies. We are actively exploring the ramifications of many of these problems and anticipate that exciting new research in this field will keep us busy for many years to come.

[1] https://www.comsol.com

Acknowledgments: We thank Daniel Harris, Robert Hunt, and Zeliha Kilic, our co-authors on the original work that appeared in *Nature Communications* [7]. Additionally, we received funding through the National Science Foundation (grant DMS-1910824) and the Office of Naval Research (grants ONR N00014-18-1-2490 and ONR N00014-23-1-2478).

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Roberto Camassa is the Kenan Distinguished Professor of Mathematics at the University of North Carolina (UNC) at Chapel Hill. He received the Kruskal Prize Lecture from the SIAM Activity Group on Nonlinear Waves and Coherent Structures in 2022. Richard M. McLaughlin is a professor of mathematics at UNC Chapel Hill. He has chaired the Department of Mathematics for more than 10 years. Together, McLaughlin and Camassa run the Joint Fluids Laboratory at UNC, which hosts a 120-foot modular wave tank with state-of-the-art-scientific instrumentation and wave-making capabilities, as well as smaller devices that probe intriguing fluid phenomena.

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Gilbert Strang Reflects on His Rich Academic Career and Lifelong Friendship with Linear Algebra and SIAM

By Lina Sorg

Longtime SIAM member Gilbert (Gil) Strang recently retired from his position as a professor of mathematics at the Massachusetts Institute of Technology (MIT), delivering his final lecture[1] to a standing ovation on May 15, 2023. Strang—who spent a collective 66 years at MIT as a student, instructor, and faculty member—is especially well-known within the applied mathematics community for his popular undergraduate linear algebra course; since 2001, he has publicly hosted videos of each lecture[2] on MIT's OpenCourseWare platform.[3] His lectures have been viewed more than 20 million times and are renowned among mathematicians and non-mathematicians alike for their engaging and clear delivery.

In addition to his robust career in academia, Strang has written 20 books — including six editions of the famed *Introduction to Linear Algebra*.[4] Since 1986, he has self-published all of these texts through Wellesley-Cambridge Press.[5] Strang has also remained an active member of SIAM over the years. He served as Vice President for Education from 1991 to 1996, was SIAM President in 1999 and 2000, and chaired the SIAM Committee on Science Policy[6] from 2001 to 2002. Strang maintains a strong connection to SIAM's Publications Department and has served as an editor for the *SIAM Journal on Numerical Analysis*,[7] *SIAM Journal on Matrix Analysis and Applications*,[8] and *SIAM Review*;[9] much of his published research has appeared in these and other SIAM journals, as well as *SIAM News*. In 2003, Strang received the SIAM Prize for Distinguished Service to the Profession.[10]

Despite his recent retirement, the 88-year-old Strang shows no signs of slowing down. During a recent conversation with *SIAM News*, he discussed his lifelong career at MIT, passion for the subject of linear algebra, dedication to both teaching and writing, continued relationship with SIAM, and future plans for his next stage of life.

***SIAM News*: You were at MIT for 66 years as both a student and faculty member. What kept you there for so long?**

Gil Strang: I attended MIT as an undergraduate, was a Rhodes Scholar at the University of Oxford for two years, and then went to the University of California, Los Angeles for my Ph.D. After I graduated, MIT offered me a job as an instructor. I thought I might work in industry, but when the job offer came I naturally said "yes." That was more than 60 years ago.

The Department of Mathematics at MIT is great. The students are good, the faculty is fun to work with, and we [my wife and I] like the Boston area — everything fit. It's nice when that happens.

SN: What do you enjoy most about teaching and working with students?

GS: I enjoy figuring out how to present something clearly. My thoughts often go to the basic linear algebra course: how to start it and how to make it work. Everything in that linear algebra course is known, but presenting the ideas clearly is the challenge. So that was really an adventure. The students respond because they understand it, they know that it's useful, and they're ready to learn. They are good students to teach and much of my writing resulted from that course, which connected me with departments all over the country. It has been such a pleasure.

In May 2023, Gilbert (Gil) Strang retired from his position as a professor of mathematics at the Massachusetts Institute of Technology, where he spent a collective 66 years as both an undergraduate student and then a long-term faculty member. Photo courtesy of Gil Strang.

SN: You were one of the first faculty members to upload your lectures to MIT's OpenCourseWare platform, a freely accessible online collection of MIT course content. How did that come about?

GS: It was a case of good luck and perfect timing. I had been thinking about making a recording of my classes for a while, and I actually recorded my linear algebra class a few months before I even heard of OpenCourseWare. When [then-MIT President Charles Vest] wanted courses, I already had material. The whole idea of MIT making courses available for public viewing was happening at the same time that I was videotaping my class. Most of the courses at MIT are not videotaped (it would be too expensive), but the larger classes do have a video component. My recordings were ready to go, so the timing was a fortunate accident. I get a lot of really nice messages from viewers.

SN: What are the benefits of making this type of content freely accessible to a broader audience?

GS: Linear algebra is a beautiful subject, and by good luck it has also become necessary and super important. A lot of people now have a better idea of how linear algebra is used, why it's significant, and how it develops. Linear algebra has always been a topic in pure math, but it needed applications as well. One such application is called least squares—everybody's doing that nowadays—and there are many other applications, of course. It's just the right subject at the right time.

It's fun to think about where linear algebra is now; it has changed so much from when I took it as a sophomore. It is much more interesting and useful, and linear algebra classes are growing at all universities.

SN: Tell us about some of the current application areas of linear algebra.

GS: Well, certainly deep learning. Deep learning is an amazing system that just gradually evolved. It wasn't a success in the beginning; it required new ideas and a lot of it was experimental. For each new set of data, you need to construct a matrix that weights the data. Constructing that matrix is the biggest consumer of computer time. In deep learning, you have to judge the importance of the data in order to weight it. It's a mixture of optimizing matrix weights and designing the architecture of the whole system. Given the many features in the data,

See **Gilbert Strang** *on page 7*

1 https://www.youtube.com/watch?v=lUUte2o2Sn8
2 https://ocw.mit.edu/courses/18-06-linear-algebra-spring-2010/video_galleries/video-lectures
3 https://ocw.mit.edu
4 https://math.mit.edu/~gs/linearalgebra/ila6/indexila6.html
5 http://www.wellesleycambridge.com
6 https://www.siam.org/about-siam/committees/committee-on-science-policy-csp
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8 https://www.siam.org/publications/journals/siam-journal-on-matrix-analysis-and-applications-simax
9 https://www.siam.org/publications/journals/siam-review-sirev
10 https://www.siam.org/prizes-recognition/major-prizes-lectures/detail/siam-prize-for-distinguished-service-to-the-profession

Blackworms

Continued from page 1

"We want to understand how you can control the tangled topological state just by looking at the motion of individual filaments," Patil said. "We built a minimal mathematical model to explain how filament motion can give rise to these different topological states, which means that [the model] is hopefully generalizable to a lot of other filament systems."

Strictly speaking, mathematical knot theory only describes the topology and geometry of string-like shapes that form closed loops. However, scientists who work with open filaments like shoelaces, worms, or DNA can still utilize many concepts from knot theory, including essential topological features and geometrical measures. In the blackworm scenario, dramatic changes in the linking number describe a topological phase transition from the untangled to tangled state and back again.

As the Worm Turns

To investigate the system's dynamics, the experimental team placed the worms in a shallow fluid and tracked their trajectories as they formed and unformed clumps. Giving one worm a very gentle electric shock caused the entire clump to unravel, revealing the way in which the "message" to untangle spread amongst the group. "[The worm] starts to do a figure eight motion where it forms a clockwise loop, then an anticlockwise loop, and then a clockwise loop," Patil said. This wavelike motion passes down the worm's body and alerts the other worms in the tangle that they should start wiggling free as well.

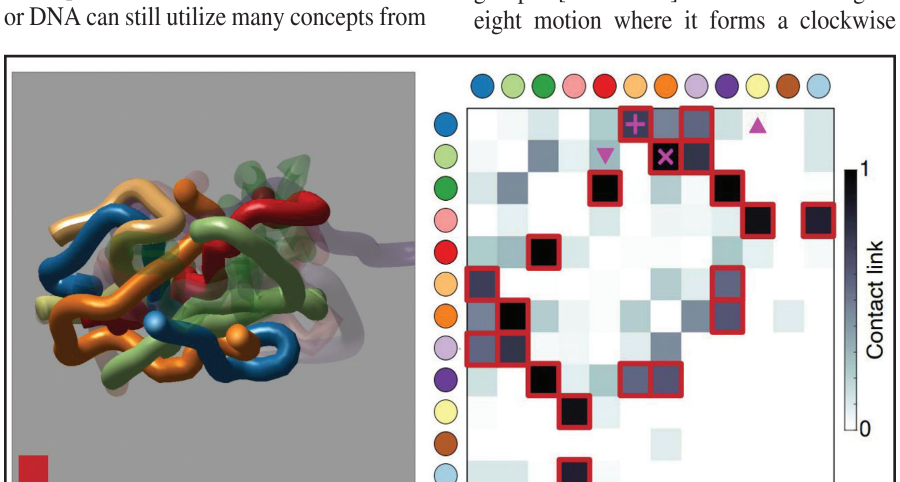

Figure 2. *A mapping of the contact between worms that was created via ultrasound imaging, and a corresponding representation of the contact link matrix cL. The colors of the row and column labels correspond to the worms in the tangle, and the bounded squares indicate strong correlations. Figure courtesy of [1].*

Patil realized that he could mathematically simplify the motion by reducing it to two dimensions and treating out-of-plane motion projectively. He also tracked the trajectory of just the head of each worm, which allowed him to simulate the system via a set of simple coupled stochastic differential equations. Focusing on the direction in which the worm turns—represented by $\theta(t) = \arg \dot{\boldsymbol{x}}(t)$, where $\boldsymbol{x}$ is the two-dimensional position of the worm head—the stochastic differential equations are as follows:

$$\dot{\boldsymbol{x}} = v\boldsymbol{n}_\theta + \boldsymbol{\xi}_T$$

$$\dot{\theta} = \sigma(t;\lambda)\alpha + \xi_R,$$

with noise terms $\{\boldsymbol{\xi}_T, \xi_R\}$. One can estimate the time-averaged velocities $v = \langle|\dot{\boldsymbol{x}}(t)|\rangle$ and $\alpha = \langle|\dot{\theta}|\rangle$ from experimental data, and $\sigma(t;\lambda)$ switches between $+1$ and -1 at rate λ. The dimensionless chirality parameter $\gamma = \alpha/2\pi\lambda$ is inversely proportional to the number of direction changes; small γ indicates that many direction changes occur and the blackworm cluster subsequently detangles.

"When you switch direction rapidly, you untangle," Patil said. "And when you turn in the same direction for longer, you tangle." He cautioned that the simplified model is not a full mathematical description of blackworm untangling, though its straightforward explanation might be applicable to other systems.

"What's quite interesting is that our ability to [observe] the structure of tangled objects remains kind of limited," Patil said. "If we have a tangled ball of wires, it's very hard to extract just a single wire. Worms obviously do a lot of stuff when they're tangling and untangling in a complex biological system, [and] this is a simple model of one motion. There's a very subtle difference in the transition between these two [states]. But if you can explain it mathematically, then you know *why* it works."

In particular, this research helps to elucidate the reason why many non-biological systems can tangle more easily than they untangle, based on the types of motion that are involved in the snarl. A cluster of electrical cords does not spontaneously produce the wavelike motion that blackworms exhibit, but the molecules inside cells—which undergo the kinds of chemical stimuli that govern biological processes—very well might.

References

[1] Patil, V.P., Tuazon, H., Kaufman, E., Chakrabortty, T., Qin, D., Dunkel, J., & Bhamla, M.S. (2023). Ultrafast reversible self-assembly of living tangled matter. *Science*, *380*(6643), 392-398.

Matthew R. Francis is a physicist, science writer, public speaker, educator, and frequent wearer of jaunty hats. His website is BowlerHatScience.org.

Ivo Babuška

Continued from page 2

famous *angle conditions*, wrote a seminal sequence of papers on *numerical homogenization*, and developed groundbreaking algorithms and theory for *a posteriori error estimation* and *mesh adaptivity*.

The 1980s were the years of the *p-version of FEM* wherein Ivo and his colleagues changed the asymptotical framework to increase polynomial degree instead of decreasing element size. Later in the decade, they developed the hybrid *hp-version of FEM*. Around the same time, Ivo also introduced *mesh-dependent norms*—a novel tool for the analysis of certain kinds of finite elements—and *generalized finite elements*, a new class of FEM.

Ivo entered the 1990s by coauthoring an authoritative survey article on *FEM for eigenvalue problems*. He then explored questions of *robustness* for different FEMs and played a big part in explicating *locking* behavior (the failure of standard numerical methods in certain parameter domains) and designing new methods to overcome it. Ivo conducted similar path-breaking work on *pollution*, another robustness issue that prevents accurate numerical solutions to the Helmholtz wave equation in the range of *large wave numbers*. Finally, some of his most cited work in the late 1990s details his invention of the *partition of unity finite element method* and study of other forms of *meshless methods*.

By this time, Ivo's work was widely respected and appreciated; he received three honorary doctorates (two more would come later), the Gold Medal of Charles University, the Birkhoff Prize in Applied Mathematics from the American Mathematical Society (AMS) and SIAM, and the John von Neumann Medal from the U.S Association for Computational Mathematics.

Attendees of the 1967 International Conference on Basic Problems of Numerical Analysis, which took place at Liblice Castle in Czechoslovakia, gather for a group photo. Ivo Babuška organized both the 1964 and 1967 iterations of this meeting, which attracted some of the top numerical analysts from around the world. Numbered from left to right: (1) Ivo Babuška, (2) Lothar Collatz, (3) Andrey Tikhonov, (4) Bert Hubbard, (5), Hans Jörg Stetter, (6) James H. Wilkinson, and (7) Frank W.J. Olver. Photo courtesy of Douglas Arnold.

Texas: The Grand Old Man of FEM

In 1995, Ivo retired from UMD as an emeritus distinguished professor. However, the word "retire" is rather misleading. Ivo's longtime friend and collaborator, J. Tinsley Oden, recruited him to the University of Texas (UT) at Austin's Texas Institute for Computational and Applied Mathematics (TICAM)—now called the Oden Institute for Computational Engineering and Sciences—and Ivo moved to Texas. In addition to his primary role as TICAM Senior Research Fellow, he served as a professor in both the Department of Mathematics and Department of Aerospace Engineering and Engineering Mechanics, and was appointed Robert B. Trull Chair in Engineering. In 2016, the University named then-90-year-old Ivo as the first recipient of the W.A. "Tex" Moncrief, Jr. Distinguished Faculty Fellowship in Computational Sciences. But despite these many titles, a more descriptive term for Ivo's role in Texas is the Grand Old Man of Finite Elements. He continued to work on the topics that had occupied his life and also pursued some new directions, including stochastic PDEs and uncertainty quantification in finite element computations.

While at TICAM, Ivo was elected to the European Academy of Sciences and the U.S. National Academy of Engineering. He also received the Gauss-Newton Medal from the International Association for Computational Mechanics and the AMS Leroy P. Steele Prize for Lifetime Achievement. Ivo ultimately spent 23 years at UT Austin before officially retiring (again!) in 2018 at the age of 92.

Although Ivo had no formal teaching responsibilities at UT Austin, he remained very committed to training the next generation of computational scientists. Soon after his arrival, he founded the TICAM Forum (later renamed the Babuška Forum): a seminar series for students in contemporary computational science and engineering that is still active today.

In 2020, Renata Babuška—Ivo's wife of 63 years and his greatest collaborator—passed away. Ivo then moved to Albuquerque, NM, to be close to his son Vit. Ivo and Renata are also survived by their daughter Lenka and several grandchildren.

Ivo was exceptionally generous with his time and knowledge and always eager to talk about the subjects he loved. Later in life, he gave back to the community through philanthropy as well. In fact, a very generous donation by Ivo last year allowed SIAM to establish the Ivo & Renata Babuška Prize:[3] a biennial award in the amount of $10,000 that recognizes "high-quality interdisciplinary work that targets any aspect of modeling and numerical solution of a specific engineering or scientific application, including mathematical modeling, numerical analysis, algorithms, and validation." The first iteration of the Ivo & Renata Babuška Prize will be presented at the 2025 SIAM Conference on Computational Science and Engineering. Ivo was equally charitable with other institutions that were closely connected to his career, including UMD, UT Austin, AMS, and several Czech scientific societies.

Ivo was a towering figure in a broad interdisciplinary field that combines mathematics, engineering, and computer science. He helped to shape and define the area that we now call computational science and engineering. He will be greatly missed by the numerous scientific communities that felt the impact of his work, and even more by his many friends and admirers. At the same time, let us celebrate the incredible legacy that he left behind.

Douglas N. Arnold is McKnight Presidential Professor of Mathematics at the University of Minnesota. He was a colleague of Ivo Babuška at the University of Maryland from 1979 to 1989, and a lifelong admirer and friend.

[3] https://go.siam.org/cXkmD0

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Astroids Under Water

The observer in Figure 1 is looking at a pebble at the bottom of a clear lake. As the eye E moves, so does the pebble's image P'. The image O' stays on one special cusped curve γ wherever E is above water. This γ is the envelope of refracted rays that are emitted by the pebble. After writing about this curve in my May *SIAM News* article ("Some Shallow Observations"),[1] I discovered—via a slightly boring calculation—that this curve is obtained by stretching astroid

$$x^{2/3}+y^{2/3}=a^{2/3} \qquad (1)$$

in the x-direction by the factor $1/\sqrt{1-c^2}$, where $c=v_{\text{water}}/v_{\text{air}}\approx 0.7$ is the speed of light in the water relative to that in the air and

$$a=Dc,$$

where D is the depth of the pebble.

Circles Rolling on Circles

The astroid in (1) is a "hypocycloid" — more precisely, a path traced by a point of a circle of radius $a/4$ that rolls without slippage on the inside of a circle of radius a (see Figure 2). Interestingly, another kind of cycloid arises as the set of images of a distant object in a circular mirror (see Figure 3). As we walk around a shiny post, the Sun's reflection inside the post follows the nephroid (more precisely, the left half of it in Figure 3).

MATHEMATICAL CURIOSITIES

By Mark Levi

Cycloids arise in two entirely different settings mentioned here. Is this a coincidence or a manifestation of some more general fact? I do not know.

Back to Stretched Astroids

Returning to the claim that γ is a stretched astroid, the proof relies on two facts of independent interest (which I also mentioned in the May article): (i) The ratio of the depth of P' to that of P is

$$c\,\frac{\cos^3\alpha}{\cos^3\beta},$$

where α is the angle of the above-water ray with the vertical, with β defined similarly underwater, and c is as before. And (ii):

$$\frac{CP'}{CP}=c\,\frac{\cos^2\alpha}{\cos^2\beta},$$

where C is the crossing point (see Figure 1). From (i), (ii), and Snell's law, one gets parametric equations of γ; elimination of the parameter results in the claim.

This proof is disappointingly algebraic for such a geometrical result. It would be nice to find a more geometrical and insightful proof than the one outlined here.

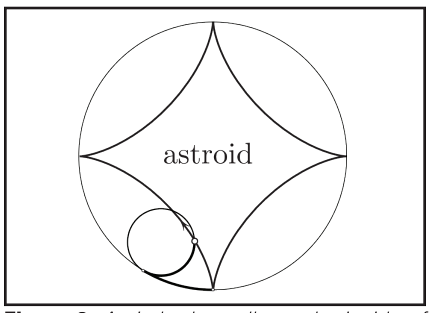

Figure 2. *A circle that rolls on the inside of a larger stationary circle—with a 1:4 ratio of radii—generates the astroid.*

The figures in this article were provided by the author.

Mark Levi (levi@math.psu.edu) is a professor of mathematics at the Pennsylvania State University.

[1] https://sinews.siam.org/Details-Page/some-shallow-observations

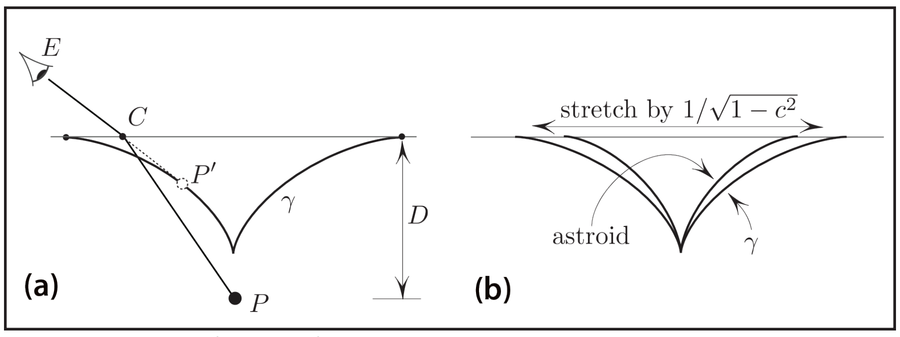

Figure 1. *The set γ of images of the pebble* **(1a)** *is the stretched astroid* **(1b)**.

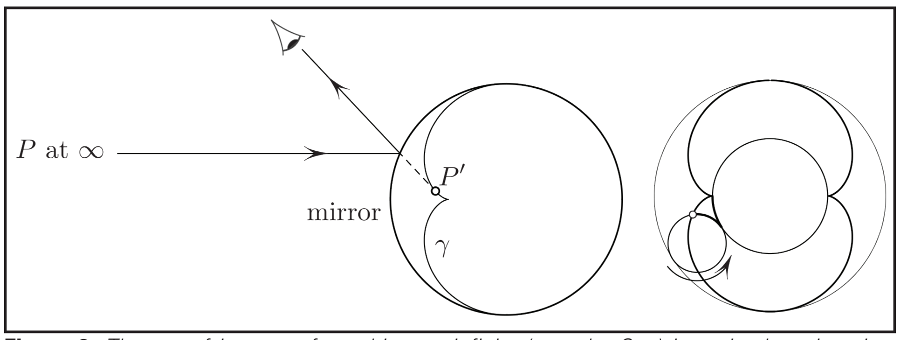

Figure 3. *The set of images of an object at infinity (say, the Sun) in a circular mirror is a nephroid — i.e., a (hyper) cycloid generated by a circle that rolls on the outside of a stationary circle with a 1:2 ratio of radii.*

Photos from the 2023 SIAM Conference on Applications of Dynamical Systems

Victoria Booth of the University of Michigan (right) accepts the J.D. Crawford Prize from Mason Porter of the University of California, Los Angeles—chair of the SIAM Activity Group on Dynamical Systems—at the 2023 SIAM Conference on Applications of Dynamical Systems (DS23), which took place in Portland, Ore., this May. Booth was honored for her exceptional mathematical biology research on the formulation, analysis, and interpretation of dynamical systems models for sleep-wake cycles. The other prize recipient at DS23 was Yoshiki Kuramoto of Kyoto University in Japan, who delivered the 2023 Jürgen Moser Lecture virtually. SIAM photo.

The Red Sock Award at the 2023 SIAM Conference on Applications of Dynamical Systems, which was held in Portland, Ore., in May, recognized exceptional poster presentations by students and postdoctoral researchers. From left to right: Maarten Droste of Vrije Universiteit Amsterdam, Indranil Ghosh of Massey University, Andrea Lama of Scuola Superiore Meridionale, Lucia Layritz of the Technical University of Munich, and Frits Veerman on behalf of Daphne Nesenberend (both of Leiden University) accept their crimson footwear from the appropriately attired James A. Yorke of the University of Maryland, College Park, whose recognizable fashion is the namesake for this prize. Each winner also received $100.00. SIAM photo.

During the 2023 SIAM Conference on Applications of Dynamical Systems, which took place this May in Portland, Ore., members of the classes of 2022 and 2023 MGB-SIAM Early Career Fellows gathered for a dinner that allowed them to meet, converse, and discuss their respective experiences. The MGB-SIAM Early Career Fellowship reflects a joint commitment by Mathematically Gifted & Black (MGB) and SIAM to promote the Fellows' long-term engagement with SIAM and continued success within the wider applied mathematics and computational science community. SIAM photo.

A lunchtime mentoring session during the 2023 SIAM Conference on Applications of Dynamical Systems, which was held this May in Portland, Ore., allowed attendees to connect and discuss common issues. Alethea Barbaro of Delft University of Technology and Korana Burke of the University of California, Davis organized the session, which attracted nearly 200 participants. Attendees broke into smaller interest groups with particular themes, such as thriving at various career stages, applying for different positions, contending with burnout and maintaining work-life balance, and navigating applied mathematics and computational science as a member and/or ally of an underrepresented group. SIAM photo.

Gilbert Strang

Continued from page 4

how do you fit those points in an efficient way? How do you pass a surface through the known data in a stable manner? This was previously an open problem, but eventually a nonlinear way was found.

Computer science departments usually handle deep learning and artificial intelligence (AI), but I wanted math to have a role. So I started a course at MIT called Matrix Methods in Data Analysis, Signal Processing, and Machine Learning[11] and wrote *Linear Algebra and Learning From Data*,[12] which published in 2019. The math of deep learning is really interesting and has a lot of open problems. It's been fun and even appears in the sixth edition of *Introduction to Linear Algebra*, which published earlier this year.

SN: How do you keep new versions of *Introduction to Linear Algebra* unique and up to date?

GS: Each new edition is a major job because I don't just make tiny changes. The books sort of form themselves in my mind. As I begin to think about what the chapters should be, I get ideas about topics and phrasing. I can fortunately reuse some of the old exercises each time, since creating new problem sets for every section of *Introduction to Linear Algebra* would be a big project. I also have to think about which topics are the right ones at any given moment in time. In the most recent edition, I altered the introduction and changed the ending to be about deep learning and AI.

SN: You self-publish all of your books though Wellesley-Cambridge Press. What inspired you to found your own publishing company?

GS: I founded Wellesley-Cambridge Press in 1986, so that's 37 years ago. I just thought I would have the adventure. I had already written two books, but my most active years of book writing came after Wellesley-Cambridge's establishment. The easy thing is to send the manuscript off to a publisher and let them handle it, but I thought I would just stay with my books and see them through: choose the covers, select the paper, organize the printing, and so on. That really is my life's adventure and I have enjoyed it all. People can contact me about everything, and they certainly do.

I still ask friends from SIAM for help with the covers, and SIAM keeps copies of all the books at its headquarters in Philadelphia. I'm in touch with SIAM so much — it really is a great society.

SN: When did you first become involved with SIAM?

GS: The first thing I did for SIAM was serve as Vice President for Education. Out of the blue, Ed Block asked me to take on that role and invited me to come to Philadelphia to talk about it. He met me at the airport, along with then-SIAM President Bob O'Malley, which was memorable. Ed was an unusual person: super active, always willing to do things, and exactly what SIAM needed. He truly was the founder of SIAM. And Jim Crowley was the best possible executive director for so many years.[13]

SN: A few years later, you were elected SIAM President. What do you remember most from your tenure?

GS: Oh, I recall lots of things. It's all been rewarding and was just fun to do with good people. My presidency was during a very active period in Washington, D.C., and we tried to boost funding for applied math. Mathematics had fallen behind, but Philippe Tondeur—the new director of the National Science Foundation's Division of Mathematical Sciences at the time—came in with lots of energy. It was an exciting time to see the budget grow and watch mathematics take its rightful place.

The Annual Meetings were also particularly memorable. I was happy to be part of starting the SIAM Activity Group on Computational Science and Engineering[14] — I remember initial conversations about that at the Annual Meeting in Toronto, Canada, in 1998.[15] Some people felt that SIAM was already a society for computational science and therefore didn't need an activity group on the topic. But I'm glad that we created one because it means that we have a big meeting at different times from the Annual Meeting.

SN: What makes SIAM so important to the research community?

GS: SIAM is the right society for people who use mathematics in all kinds of ways, and I hope that we can keep making more connections with deep learning, AI, and statistics. Data science is just growing and growing, and it's good that SIAM now has an activity group[16] and a journal[17] on this subject, and that it frequently appears in *SIAM News*. Interest in data science and statistics has now—quite correctly—moved front and center, which is a big change and a good change.

As other societies like the American Statistical Association continue to grow, we'll want to stay connected with them. We also need to focus on what students are learning and the jobs that they're getting.

SN: Do you have any specific plans for retirement?

GS: I've been so busy that I don't have any set plans, but I'm hoping to travel more now that COVID is ending. I don't get to the theater as much as I should, so maybe I will now. I go for walks to the library and do plenty of reading, and of course there's lots of email. I still receive and reply to many messages about my books and videos. I might even end up writing some more, who knows?

I'll tell you a possibly crazy idea. I have been wondering whether high school algebra could use some new thinking. I'll try to find out and make some connections with local teachers to see if there's anything I can contribute. I might just call up the local high school and ask, but I haven't done so yet.

Of course, I have Wellesley-Cambridge Press and the textbooks, and I stay connected with distribution. And SIAM will still be a big part of everything! I have had a really happy friendship with SIAM for all of this time and I look forward to continuing it. I feel quite fortunate about everything; I'm very lucky.

Lina Sorg is the managing editor of SIAM News.

Gilbert (Gil) Strang poses in front of 121 cupcakes with his favorite -1, 2, -1 matrix in icing on top. Students in his undergraduate linear algebra class provided the cupcakes several years ago in honor of his birthday. Photo courtesy of Gil Strang.

[11] https://ocw.mit.edu/courses/18-065-matrix-methods-in-data-analysis-signal-processing-and-machine-learning-spring-2018/video_galleries/video-lectures

[12] https://math.mit.edu/~gs/learningfromdata

[13] https://sinews.siam.org/Details-Page/executive-director-jim-crowley-retires-after-25-years-of-siam-leadership

[14] https://www.siam.org/membership/activity-groups/detail/computational-science-and-engineering

[15] https://archive.siam.org/meetings/an98

[16] https://www.siam.org/membership/activity-groups/detail/data-science

[17] https://www.siam.org/publications/journals/siam-journal-on-mathematics-of-data-science-simods

Class of 2024 nominations will be accepted until October 18, 2023

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Pictured: Members of the 2022 and 2021 Classes of SIAM Fellows and SIAM President, Sven Leyffer. The 2023 Class of SIAM Fellows will be recognized at the 2024 SIAM Annual Meeting.

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Call for Nominations for the 2023 Ostrowski Prize

The aim of the Ostrowski Foundation is to promote the mathematical sciences. Every second year, it provides a prize for recent outstanding achievements in pure mathematics and the foundations of numerical mathematics. The value of the prize for 2023 is 100.000 Swiss francs.

The prize has been awarded every two years since 1989. The most recent winners are Yitang Zhang in 2013, Peter Scholze in 2015, Akshay Venkatesh in 2017, Assaf Naor in 2019, and Tim Austin in 2021. See **https://www.ostrowski.ch/index_e.php** for the complete list and further details.

The jury invites nominations for candidates for the 2023 Ostrowski Prize. Nominations should include a CV of the candidate, a letter of nomination, and two-three letters of reference.

The chair of the jury for 2023 is Cameron Stewart of the University of Waterloo in Canada. Nominations should be sent to **cstewart@uwaterloo.ca** by **September 30th, 2023**.

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Two-day SIAM Hackathon at CSE23 Explores Critical Problems in Industry

By Tjeerd Jan Heeringa

The 2023 SIAM Conference on Computational Science and Engineering[1] (CSE23), which took place from February 26 to March 3, ventured outside of the U.S. for the first time and made its way to Amsterdam, the Netherlands. With more than 2,000 attendees, the meeting marked the largest gathering of mathematicians in the Netherlands to date. It also introduced an exciting special event: the first-ever SIAM Hackathon.[2]

The hackathon was held on February 25 and 26 (just before CSE23) in the Van der Valk Hotel Amsterdam, which was close to the CSE23 venue (though participation in the hackathon was independent from participation in the conference). Wolfgang Bangerth (Colorado State University), David Gleich (Purdue University), Dirk Hartmann (Siemens Digital Industries Software), Jeffrey Sachs (Merck), and Jesse van Doren, (Hackathon Op Maat) comprised the Organizing Committee. During the two-round, team-based competition, approximately 150 participants from all over the world tackled a wide variety of industry-based challenges that were provided by six contributing companies: Advanced Micro Devices[3] (AMD), Advanced Semiconductor Materials Lithography[4] (ASML), Amazon Web Services[5] (AWS), KUKA,[6] Institut Roche,[7] and Siemens.[8]

In the first round, teams of up to five individuals each worked on one of the challenges and competed with one another to advance to the second round. The six winning teams from the initial round (one for each company's proposed problem) then vied for the first-place spot.

Each company submitted a challenge[9] that was specific to its operations, required limited prerequisite knowledge, and was solvable with only a laptop. These problems varied in both research area and application. AMD sought a fast surrogate model for the wave equation, while ASML wanted a better model or algorithm for the accurate estimation of heat transfer between the hydrogen buffer gas and the surrounding components or boundaries of their new Extreme Ultra Violette lithography machine. AWS requested a way to stream high-fidelity simulation data over bandwidth-limited channels to consumer-grade hardware, and KUKA desired a novel strategy that would solve their warehouse optimization difficulties. Institut Roche asked teams to create a procedure that anonymizes clinical data and minimizes the introduced noise to prevent the destruction of clinically relevant trends or patterns. Finally, Siemens hoped to improve building efficiency while ensuring occupant comfort, with the ultimate goal of reducing investments and operational costs while minimizing carbon footprints.

The use of existing real-world scenarios during the hackathon allowed participants to become more familiar with potential employers, learn about the types of problems that large companies face on an everyday basis, and network with both company representatives and other attendees. At the same time, the organizations in question were able to advertise themselves and showcase the challenges that their employees are actively attempting to solve.

Individuals who signed up for the SIAM Hackathon received a description of the six challenges in advance and had to select their top three options. The organizers then evenly distributed the competitors based on their preferences. At the beginning of the first day, participants who were assigned to the same problem sorted themselves into teams; at least five groups worked on each challenge. To help create teams of people with complementary work styles and skillsets, everyone completed several tests that assigned them a classification based on their self-evaluation. The organizers encouraged individuals to collaborate with people who had different classifications to ensure diversity and balance in each group.

The newly formed teams then received more details about their respective challenges and immediately got to work. They had just over 24 hours to put together a cohesive solution and prepare a five-minute pitch. Various activities throughout the event's duration—including a Mario Kart competition, Nerf gun fights, cardio sessions, virtual reality ping-pong, and even an opportunity to make music by poking electrodes into fruits—helped promote camaraderie and alleviate stress. Two buffet lunches and a dinner kept everyone well fed, and participants were free to return to their hotel rooms to sleep if they wished (onsite bean bag chairs also provided a comfortable setting for the occasional power nap).

After the initial 24-hour period, each group pitched their solution to the other teams who had tackled the same challenge. Representatives of the partner companies that supplied the problems judged the pitches based on a predetermined set of criteria, and the teams with the best solutions for each individual challenge won the first round. These six finalist teams had just 15 minutes to prepare and adapt their previous presentation into a new pitch for the second round of the competition. They delivered these updated pitches to all attendees—not just those who were involved with their respective challenge—which meant that they had to clearly explain the initial concept of their task as well.

Members of the winning team of the SIAM Hackathon—which took place just before the 2023 SIAM Conference on Computational Science and Engineering (CSE23) in Amsterdam, the Netherlands—pose after successfully presenting their final pitch on February 26. From left to right: Daniel Fink (challenge organizer from KUKA), Tjeerd Jan Heeringa (University of Twente), Wouter van Harten (Radboud University), Ikrom Akramov (Technical University of Hamburg), and Wil Schilders (Eindhoven University of Technology and Organizing Committee co-chair of CSE23). SIAM photo.

The company representatives judged the second round of pitches and asked questions after each presentation. Though there were no specific guidelines for evaluation, they were not allowed to vote for the winner of their own challenge. Once all of the pitches were complete, the judges retreated for a short deliberation and announced the overall winning team.

Our team, which we dubbed "Robo Go'round," was assigned to the KUKA challenge[10] about cross-vehicle route planning in a warehouse. In addition to myself (a Ph.D. candidate at the University of Twente in the Netherlands), the team consisted of Wouter van Harten (a Ph.D. candidate at Radboud University in the Netherlands) and Ikrom Akramov (a Ph.D. candidate at the Technical University of Hamburg in Germany). The KUKA scenario asked us to imagine a large, automatized warehouse in which self-driving robotic carts need to pick up boxes at one point and drop them off at another location. The robots must avoid colliding with both each other and other objects. Because the boxes are elevated, the robots can drive underneath them when they are not carrying one (if they are carrying a box, however, then the two boxes would bump into each other). The robots are also usually constrained to only move along certain paths. These types of problems—known as multi-agent pickup and delivery problems in the literature—are especially difficult due to the exponential scaling of their complexity with the number of boxes, robots, and possible paths. Typically, finding even an "okay-ish" solution is a significant achievement.

KUKA provided participating teams with code to visualize both the warehouse and the movement of the robots and boxes inside it. They also supplied a small set of examples that characterized difficult applications. In one example called the *swap problem*, two robots start on opposite ends of a hallway and have to pass each other to take their boxes to the other side.[11] Since the robots are blocking each other's paths, they must work together. The optimal solution allows one robot to move out of the way so the other can pass. For a simple setup, one can easily determine whether a robot has to move out of the way and where it should go. However, this discernment is no longer possible in larger, more complex situations (see Figure 1).

Our solution strategy is inspired by the modern roundabout that is omnipresent in the Netherlands. Instead of addressing the full situation in one go, we identify highways and roundabouts to divide the large problem into smaller pieces. The robots on the roundabouts have the right of way and can move freely to their respective offramps. We can then solve each optimization problem from the offramp to the destination. For instance, a roundabout exists in Figure 1 via the grid point $n02 \rightarrow n06 \rightarrow n36 \rightarrow n34 \rightarrow n14 \rightarrow n12 \rightarrow n02$. We enforce this traffic rule for the robots by removing certain travel directions (shown with gray arrows); doing so reduces the problem to a collection of smaller subproblems. The offramps $(n13, n23)$ and $(n12, n22)$ yield almost trivial instances, and the offramp $(n00, n01, n10, n11, n02)$ yields instances that are similar to the aforementioned swap example. This strategy earned us the top prize at the hackathon.[12]

In conclusion, the first-ever SIAM Hackathon marked a significant milestone by uniting mathematicians from around the world to tackle real-world industry challenges from leading companies. The event showcased the ingenuity and problem-solving skills of the participants, fostered valuable connections between academia and industry, and facilitated innovative thinking at the forefront of scientific research. By hosting this unique event, SIAM created new avenues for collaboration and knowledge exchange — ultimately paving the way for future advancements at the intersection of mathematics, computation, and real-world applications.

Tjeerd Jan Heeringa is a Ph.D. candidate in the Department of Applied Mathematics at the University of Twente in the Netherlands. He holds master's degrees from the University of Twente in applied mathematics, technical computer science, and control theory.

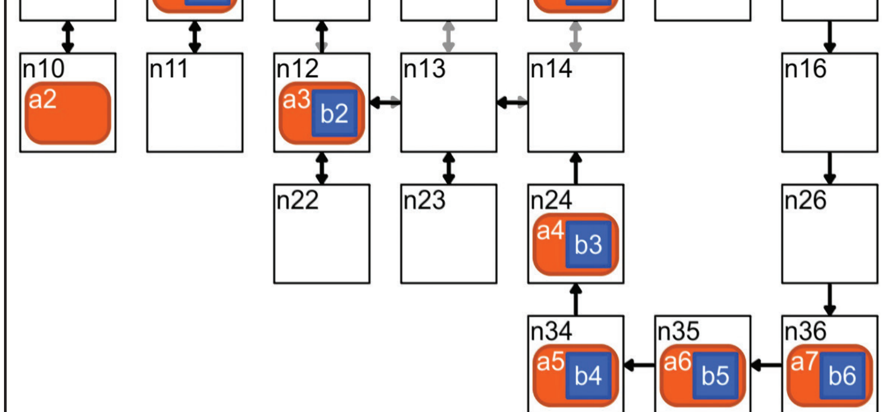

Figure 1. *A somewhat small instance of the KUKA warehouse optimization problem that we used to test the scaling of our solution algorithm. The arrows indicate the allowed directions of motion and each block represents a square meter. The robots (in orange) must work together to deliver the boxes (in blue) to their intended locations. Figure reflects the output of the code that KUKA provided as part of their proposed challenge.*

[1] https://www.siam.org/conferences/cm/conference/cse23
[2] https://www.youtube.com/watch?v=a6RuXmKf4uM
[3] https://www.amd.com
[4] https://www.asml.com
[5] https://aws.amazon.com
[6] https://www.kuka.com
[7] https://institut.roche.com
[8] https://www.siemens.com
[9] https://go.siam.org/4zOJO8

[10] https://www.kuka.com/en-us/company/iimagazine/2023/03/siam-hackathon-2023

[11] An animation in the online version of this article illustrates the swap.
[12] https://www.4tu.nl/ami/News/News/SIAM-2023-hackathon-winners