

Math 130: Final Exam Version A
Spring 2019

- Calculators are NOT allowed.
- No partial credit for multiple choice / no work needs to be shown.
- For short answer questions, you must show work for full and partial credit.
- Sign the honor pledge below after completing the exam.
- Please put all work to be graded on the test itself.

First and last name

PID

UNC Email

I have not given or received unauthorized help on this exam.

Signature:

1. (3 points) Is $h(x) = x^2 \sin(x) - 4x$ an even or odd function?

- A. Even
- B. Odd
- C. Neither

2. (3 points) If $\tan(\theta) = -\frac{4}{3}$ and $0 < \theta < \pi$, what is $\sin(\theta)$.

- A. $-\frac{4}{5}$
- B. $-\frac{3}{5}$
- C. $\frac{3}{5}$
- D. $\frac{3}{4}$
- E. $\frac{4}{5}$

3. (3 points) **Select two** polar points that are equivalent to the Cartesian point $(-1, \sqrt{3})$.

- A. $(2, \frac{\pi}{3})$
- B. $(2, \frac{2\pi}{3})$
- C. $(2, \frac{5\pi}{6})$
- D. $(2, -\frac{5\pi}{6})$
- E. $(-2, \frac{2\pi}{3})$
- F. $(-2, \frac{\pi}{3})$
- G. $(-2, -\frac{\pi}{6})$
- H. $(-2, -\frac{\pi}{3})$

4. (3 points) Convert the following system of parametric equations to a Cartesian equation.

$$\begin{cases} x(t) = t + 3 \\ y(t) = -2t^2 \end{cases}$$

- A. $y = -2x^2$
- B. $y = -2x^2 + 6$
- C. $y = -2x^2 - 9$
- D. $y = -2(x + 3)^2$
- E. $y = -2(x - 3)^2$

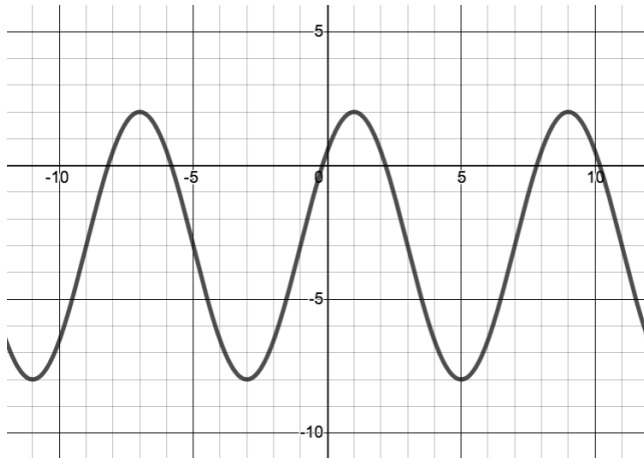
5. (5 points) Which of the following equations represents a circle? Select all that apply.

- A. $3y^2 + 24y - x^2 = 6$
- B. $x = 4 \sin(t) - 2, y = 4 \cos(t) + 3$
- C. $r = -5$
- D. $r = 7$
- E. $r = 3 \cos(\theta)$

6. (3 points) The minute hand of a clock is 5 inches long. What distance does the tip of the minute hand travel in 10 minutes?

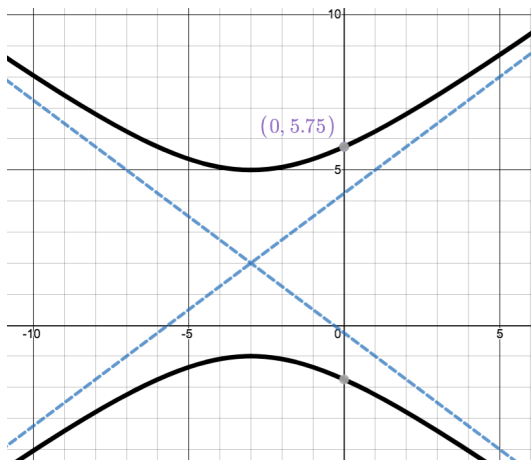
- A. $\frac{\pi}{5}$ inches
- B. $\frac{5\pi}{3}$ inches
- C. $\frac{25\pi}{6}$ inches
- D. 10π inches
- E. 300 inches

7. (3 points) Find the equation of this sinusoidal function.



- A. $y = 3 \cos(8x - 1) - 5$
- B. $y = 5 \cos(8(x - 1)) - 3$
- C. $y = 3 \cos\left(\frac{\pi}{4}x - 1\right) - 5$
- D. $y = 5 \cos\left(\frac{\pi}{4}x - 1\right) - 3$
- E. $y = 5 \cos\left(\frac{\pi}{4}(x - 1)\right) - 3$

8. (3 points) Find the equation of the curve graphed below.



- A. $\frac{(y-2)^2}{9} - \frac{(x+3)^2}{16} = 1$
- B. $\frac{(x+3)^2}{16} - \frac{(y-2)^2}{9} = 1$
- C. $\frac{(y-2)^2}{9} - \frac{(x+3)^2}{49} = 1$
- D. $\frac{(x+3)^2}{49} - \frac{(y-2)^2}{9} = 1$

9. (6 points) Evaluate $\cos(\frac{\pi}{6} + \sin^{-1}(-\frac{5}{7}))$ exactly and simplify your answer.

Answer:

10. (6 points) Find **all** solutions to the equation: $\cos(2x) + \sin^2(x) = \cos(x)$

Answer:

11. (6 points) Find and simplify the difference quotient for $f(x) = x^2 - 5x + 3$.

Answer:

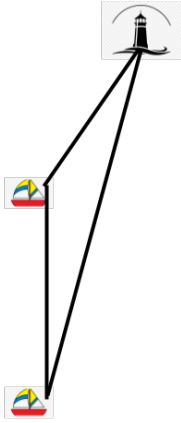
12. (6 points) Find the exact value of $\sin(15^\circ)$. Simplify your answer.

Answer:

13. (6 points) A tiny but horrible alien is standing at the top of the Eiffel Tower (which is 324 meters tall) and he is threatening to destroy the city of Paris! He can see Agent J standing on the ground and the angle of depression from the alien to Agent J is 80° . How far away is Agent J from the base of the Eiffel Tower? Write your answer in a form that could be typed into a calculator to get a numerical answer.

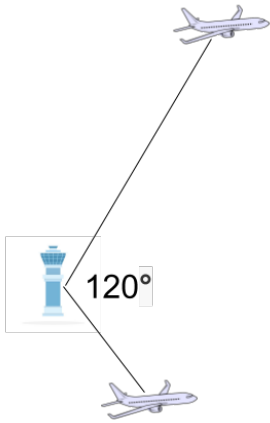
Answer:

14. (6 points) A boat sailing due north at a rate of 14 mph records the bearing of a lighthouse as $N8^\circ E$. Two hours later, the bearing of the same lighthouse is $N40^\circ E$. Find the distance d from the boat to the lighthouse at the time of the second sighting. Leave your answer in a form that can be typed into a calculator to get a numerical answer.



Answer:

15. (6 points) In tracking the relative location of two aircraft, a controller determines that the distance from the station to the first aircraft is 8 miles and the distance to the second aircraft is 7 miles. If the angle between the controller's sight lines to the two aircraft is 120° , how far apart are the two planes? **Simplify your answer.**



Answer:

16. (6 points) Prove the identity $\frac{\cos(x)}{1 - \sin(x)} = \sec(x) + \tan(x)$

17. (6 points) A window above a doorway is constructed in the shape of the top half on an ellipse, as shown in the figure. The window is 2 feet tall at its highest point and 8 feet wide at the bottom. Find the height of the window 1 foot from the center of the base.



Equation:

18. (8 points) For the function $g(x) = \frac{3 - 5x}{2x + 7}$.

(a) Find $g^{-1}(x)$

$$g^{-1}(x) = \boxed{\phantom{\frac{3 - 5x}{2x + 7}}}$$

(b) Find the domain and range of $g(x)$ and $g^{-1}(x)$. Write your answers in interval notation.

Domain of $g(x)$

Range of $g(x)$

Domain of $g^{-1}(x)$

Range of $g^{-1}(x)$

19. (6 points) Find the vertex, focus, and directrix for the parabola given by the equation $x^2 + 6x + 8y - 23 = 0$

Vertex:

Focus:

Directrix:

20. (6 points) True or False and justify your answer.

(a) $\sin(3\theta) = 3 \sin \theta \cos \theta$

A. True B. False

Justification

(b) $\cos^{-1}(\cos(193^\circ)) = 193^\circ$

A. True B. False

Justification

FORMULAS

$$\cos^2(\theta) + \sin^2(\theta) = 1$$

$$\tan^2(\theta) + 1 = \sec^2(\theta)$$

$$\cot^2(\theta) + 1 = \csc^2(\theta)$$

$$\sin(A + B) = \sin(A) \cos(B) + \cos(A) \sin(B)$$

$$\sin(A - B) = \sin(A) \cos(B) - \cos(A) \sin(B)$$

$$\cos(A + B) = \cos(A) \cos(B) - \sin(A) \sin(B)$$

$$\cos(A - B) = \cos(A) \cos(B) + \sin(A) \sin(B)$$

$$\sin(2\theta) = 2 \sin(\theta) \cos(\theta)$$

$$\sin \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{2}}$$

$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$$

$$\cos \frac{\theta}{2} = \pm \sqrt{\frac{1 + \cos \theta}{2}}$$

$$c^2 = a^2 + b^2 - 2ab \cos(C)$$

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$$

$$4p(y - k) = (x - h)^2$$

$$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$$

$$\frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1$$

$$4p(x - h) = (y - k)^2$$

$$\frac{(x - h)^2}{b^2} + \frac{(y - k)^2}{a^2} = 1$$

$$\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$$

$$c^2 = a^2 - b^2$$

$$c^2 = a^2 + b^2$$

$$x = r \cos(\theta)$$

$$y = r \sin(\theta)$$

$$r^2 = x^2 + y^2$$

$$\tan(\theta) = \frac{y}{x}$$

