MATH 130

Final Exam
2 May 2022

Name: Solutions

UNC Email Address: _____

Instructor: _____

- There are 25 questions on this test:
  - 15 are multiple choice and worth 4 points each;
  - 10 are free response and worth 6 points each.
- Calculators are NOT allowed. Answers are to be given in a form that could be typed into a calculator and use units as necessary. I do NOT need to simplify arithmetic / algebraic expressions.
- I am expected to SHOW ALL WORK on the free response questions; no credit will be given for correct answers without supporting work.
- Partial credit may be awarded on multiple choice questions, but it will be based on my answer only; work may be shown but will not be graded.
- Notation and clarity count. My job is to communicate mathematically and make what I am thinking clear.
- I will sign the Honor Pledge when I am finished or I will let my instructor know of any irregularities with this exam.

I have neither given nor received any unauthorized help on this test and I have conducted myself within the guidelines of the University Honor Code.

Pledge: _____

1. Which one of the following functions is an odd function?

   - ○ $x^2 + \sin(x)$
   - ○ $x^3 \sin(x)$
   - ● $\sin(x) + \tan(x)$
   - ○ $\sin(x) + \cos(x)$

underlined functions are odd.
Multiplying odds gives an even

2. What is the range of the function $f(x) = 3\cos(10x + \pi) - 2$?

   - ○ $[-3, -1]$
   - ○ $[-10, 10]$
   - ○ $[-3, 3]$
   - ● $[-5, 1]$

amplitude $\rightarrow [-3,3]$
shift down $\rightarrow [-3-2, 3-2] = [-5,1]$

3. What is the period of $y = \cot(10\pi x + 2)$?

   - ○ $\frac{\pi}{10}$
   - ● $\frac{1}{10}$
   - ○ $\frac{2}{5}$
   - ○ 2

cot: period π
here, $\frac{\pi}{10\pi} = \frac{1}{10}$

4. Which of the following statements is FALSE?

   - ○ Knowing two sides of a triangle and the measure of the angle in between them is sufficient information to compute its area. True $\frac{1}{2}ab\sin C = \text{Area}$
   - ○ Applying the Law of Sines requires us to know at least one side length of the triangle in question. True
   - ● If a triangle contains an angle larger than 90° we cannot use the Law of Sines to determine any side lengths.
   - ○ The Law of Cosines can be used to find the side lengths of some oblique triangles. True, given SSS or SAS, Law of cosines applies.

5. An equilateral triangle T has area $\sqrt{3}$. What is its perimeter?

   - ● 6
   - ○ $3\sqrt{3}$
   - ○ 2
   - ○ $\frac{\sqrt{3}}{3}$

each angle is 60°

Area $= \sqrt{3} = \frac{1}{2}\, a \cdot a \cdot \sin 60^\circ$

$\sqrt{3} = \frac{a^2}{2} \cdot \frac{\sqrt{3}}{2}$

$a^2 = 4$

$a = 2$

Perimeter $= 3a = 6$

6. Find the equation for the graph of $g(x)$ obtained from the function

$$f(x) = \frac{x}{3-x}$$

after: reflecting it across the x-axis, then shifting it up 4 units, then shifting it to the right by one unit.

1 2 3

- ● $g(x) = \dfrac{1-x}{4-x} + 4$
- ○ $g(x) = \dfrac{1-x}{4-x} - 4$
- ○ $g(x) = \dfrac{1+x}{x-2} + 4$
- ○ $g(x) = \dfrac{1+x}{x-2} - 4$

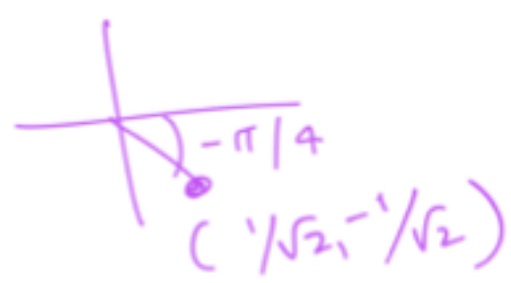
$\dfrac{x}{3-x} \xrightarrow{1} -\dfrac{x}{3-x} \xrightarrow{2} -\dfrac{x}{3-x} + 4$

$\xrightarrow{3} -\dfrac{(x-1)}{3-(x-1)} + 4 = \dfrac{1-x}{4-x} + 4$

7. Find the exact value of $\csc^{-1}(-\sqrt{2})$. $= \theta$ in $[-\pi/2, \pi/2]$ with $\csc\theta = -\sqrt{2}$

$\sin\theta = -\dfrac{1}{\sqrt{2}}$

- ○ $\dfrac{3\pi}{4}$
- ○ $\dfrac{7\pi}{4}$
- ● $-\dfrac{\pi}{4}$
- ○ $\csc^{-1}(-\sqrt{2})$ is undefined



8. Find the exact value of $\cos^{-1}\left(\cos\left(\dfrac{9\pi}{8}\right)\right)$.

- ○ $\dfrac{\pi}{8}$
- ● $\dfrac{7\pi}{8}$
- ○ $-\dfrac{\pi}{8}$
- ○ $-\dfrac{9\pi}{8}$
- ○ $\cos^{-1}\left(\cos\left(\dfrac{9\pi}{8}\right)\right)$ is not defined

same cosine value, in $[0, \pi]$

9. Select one answer that gives the exact value of $\cos(195°)$.

- ○ $-\sqrt{\frac{2-\sqrt{3}}{4}}$
- ● $-\sqrt{\frac{2+\sqrt{3}}{4}}$
- ○ $\sqrt{\frac{2-\sqrt{3}}{4}}$
- ○ $\frac{\sqrt{2}-\sqrt{6}}{4}$
- ○ $\frac{\sqrt{6}-\sqrt{2}}{4}$
- ● $\frac{-\sqrt{2}-\sqrt{6}}{4}$

sum/difference

$\cos 195° = \cos(150° + 45°) = \cos(240° - 45°)$

$= \cos 150° \cos 45° - \sin 150° \sin 45° = \cos 240° \cos 45° + \sin 240° \sin 45°$

$= -\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = -\frac{1}{2} \cdot \frac{\sqrt{2}}{2} + -\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$

$= \frac{-\sqrt{6}-\sqrt{2}}{4} \qquad = \frac{-\sqrt{2}-\sqrt{6}}{4}$

Half: $\cos 195° = \cos(\frac{1}{2} 390°) = -\sqrt{\frac{1+\cos 390°}{2}}$

in Q3

$= -\sqrt{\frac{1+\sqrt{3}/2}{2}} = -\sqrt{\frac{2+\sqrt{3}}{4}}$

10. An alternate form of $\frac{1}{\cos(x)+1} + \frac{1}{\sec(x)+1}$ is:

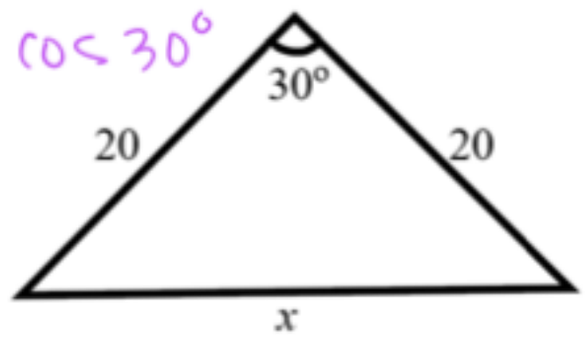
- ● 1
- ○ $\cos(x)$
- ○ $\sec(x)$
- ○ $\frac{2}{\cos(x)+1}$

$\frac{(\sec x + 1) + (\cos x + 1)}{\cos x \sec x + \cos x + \sec x + 1}$

$= \frac{\sec x + \cos x + 2}{\cos x + \sec x + 2}$

11. What is the value of x in the following diagram?

- ○ $20\sqrt{3}$
- ○ $\frac{20\sqrt{3}}{\sqrt{3}}$
- ○ $\sqrt{800}$
- ○ $\sqrt{800-400}$
- ● $\sqrt{800-400\sqrt{3}}$



$x^2 = 20^2 + 20^2 - 2 \cdot 20 \cdot 20 \cos 30°$

$x = \sqrt{800 - 800\frac{\sqrt{3}}{2}}$

$= \sqrt{800 - 400\sqrt{3}}$

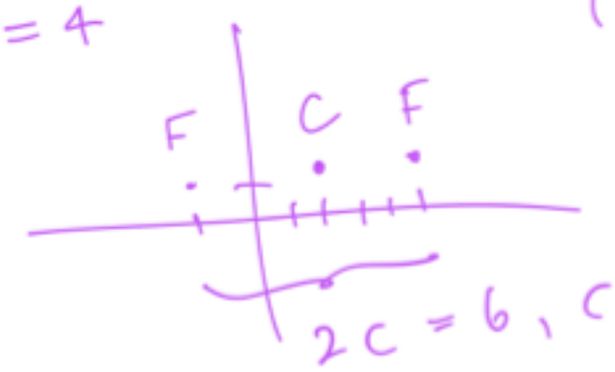
12. Find the equation for the ellipse with foci at $(5, 1)$ and $(-1, 1)$ that has 8 as the length of the major axis.

- ○ $\frac{(x-1)^2}{9} + \frac{(y-2)^2}{16} = 1$
- ○ $\frac{(x-2)^2}{5} + \frac{(y-1)^2}{16} = 1$
- ○ $\frac{(x-1)^2}{4} + \frac{(y-2)^2}{7} = 1$
- ● $\frac{(x-2)^2}{16} + \frac{(y-1)^2}{7} = 1$
- ○ $\frac{(x-2)^2}{4} + \frac{(y-1)^2}{7} = 1$
- ○ $\frac{(x-2)^2}{16} + \frac{(y-1)^2}{9} = 1$

$2a = 8$

$a = 4$

$(h,k) = (2,1)$



$2c = 6, c = 3$

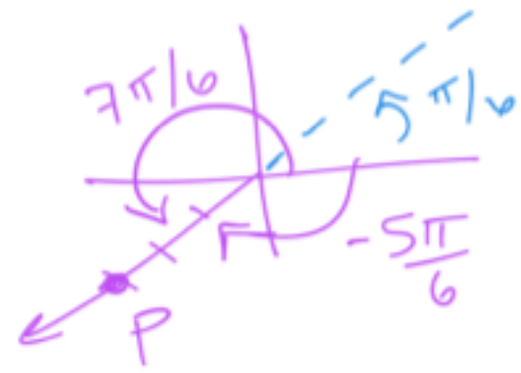
$a^2 = b^2 + c^2$

$16 - 9 = b^2 = 7$

$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

13. Choose the TWO points in polar coordinates that coincide with $\left(3, -\frac{5\pi}{6}\right)$.

- ○ $\left(3, \frac{5\pi}{6}\right)$
- ○ $\left(-3, \frac{5\pi}{6}\right)$
- ○ $\left(3, \frac{\pi}{6}\right)$
- ● $\left(-3, \frac{\pi}{6}\right)$
- ● $\left(3, \frac{7\pi}{6}\right)$
- ○ $\left(-3, \frac{7\pi}{6}\right)$



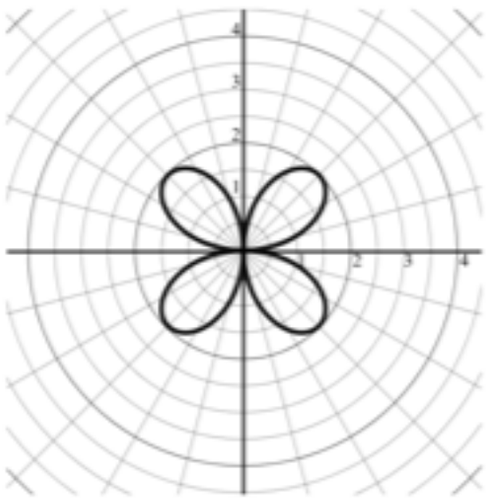
14. Find and simplify the difference quotient of $f(x) = x^2 - 5$.

- ○ 1
- ● $2x + h$
- ○ $1 - \frac{10}{h}$
- ○ $2x + h - \frac{10}{h}$

$$\frac{(x+h)^2 - 5 - (x^2-5)}{h} = \frac{x^2+2xh+h^2-5-x^2+5}{h} = \frac{2xh+h^2}{h} = 2x+h$$

15. For each graph provided, select ONE equation in polar coordinates that gives that graph.

B



A. $r = 2\sin(\theta)$

B. $r = 2\sin(2\theta)$

C. $r = 2\sin(4\theta)$

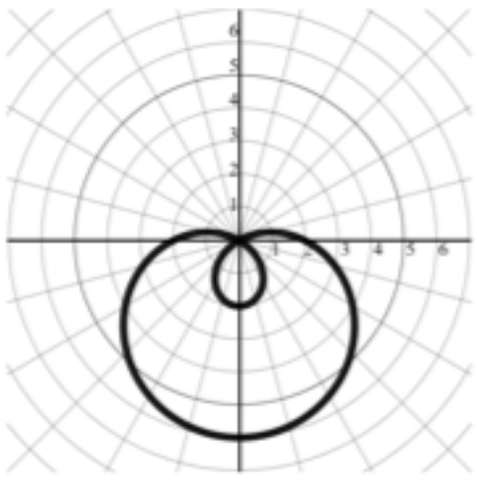
petal curves

D. $r = 2 - 4\sin(\theta)$ limaçon w/ inner loop

E. $r = 2 + 2\sin(\theta)$ cardioid

F. $r = 4 - 2\sin(\theta)$ limaçon w/o inner loop

D



Q II, IV Q I, IV

16. If $\cot(\theta) = -10$ and $\sec(\theta) > 0$, then determine the values of $\sin(\theta), \cos(\theta), \tan(\theta), \sec(\theta)$, and $\csc(\theta)$.

θ in Quadrant IV

$\cot\theta = -10 = x/y$

Let $x = 10, y = -1$

$r^2 = 100 + 1 = 101$

$r = \sqrt{101}$

$\sin\theta = \frac{y}{r} = -\frac{1}{\sqrt{101}}$

$\cos\theta = \frac{x}{r} = \frac{10}{\sqrt{101}}$

$\tan\theta = y/x = -1/10$

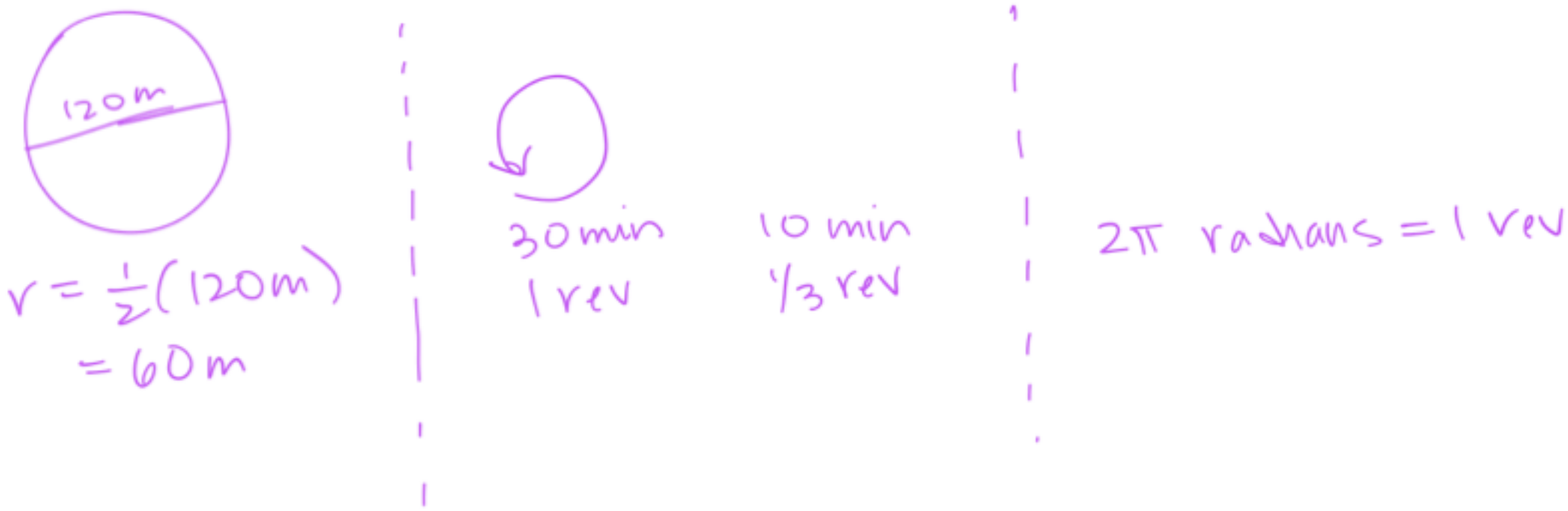
$\sec\theta = r/x = \sqrt{101}/10$

$\csc\theta = r/y = -\sqrt{101}$

17. The London Eye is a Ferris wheel on the bank of the Thames river. Its diameter is 120 meters and it takes 30 minutes to complete one full revolution.

(a) How far has a London Eye rider traveled in 10 minutes?

$s = r\theta = 60\text{m} \cdot \left(\frac{1}{3}\text{rev}\right) = 60\text{m} \cdot \frac{2\pi}{3} = \boxed{40\pi \text{ meters}}$



(b) How fast, in km/hr, is a rider moving on the London Eye?

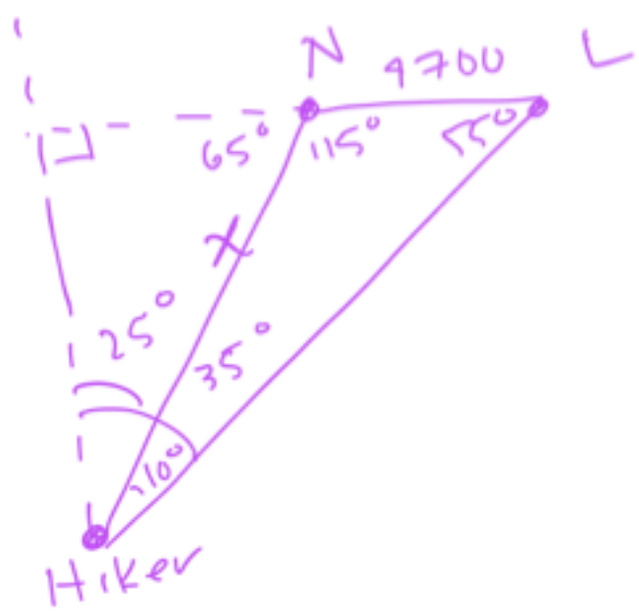
$v = r\omega = 60\text{m} \cdot \frac{2\pi}{30\text{ min}}$

$= 4\pi$ meters per minute

$= 4\pi \frac{\text{m}}{\text{min}} \cdot \frac{60\text{ min}}{1\text{ hr}} \cdot \frac{1\text{ km}}{1000\text{ m}}$

$= \boxed{\frac{24\pi}{100}\text{ km/hr} = .24\pi\text{ km/h}}$

18. A climber entering the Himalayan Zone in Nepal and facing due North observes two mountain peaks, Nuptse and Lhotse, at bearings of $N25°E$ and $N35°E$, respectively. It is known that these two peaks are $4700m$ apart along an East-West line. How far (measured along the ground only) is the climber from Nuptse? Leave your answer into a form that could be evaluated by a calculator.



$$\frac{x}{\sin 55°} = \frac{4700\,m}{\sin 10°}$$

$$x = \boxed{\frac{4700 \sin 55°}{\sin 10°}\ m}$$

$90 - 25 = 65$

$180 - 65 = 115$

$180 - (115 + 10) = 55$

19. Establish the identity $\dfrac{\sec^2 x - \tan^2 x + \tan x}{\sec x} = \sin x + \cos x$. Many answers possible. A few are shown here.

① $$\frac{\sec^2 x - \tan^2 x + \tan x}{\sec x} = \frac{1 + \tan x}{\sec x} = \frac{1}{\sec x} + \frac{\tan x}{\sec x} = \cos x + \frac{\sin x}{\cos x} \cdot \frac{\cos x}{1} = \cos x + \sin x$$

② $$\sec x - \frac{\sin^2 x}{\cos^2 x} \cdot \frac{\cos x}{1} + \frac{\sin x}{\cos x} \cdot \frac{\cos x}{1} = \sec x - \frac{\sin^2 x}{\cos x} + \sin x = \frac{1 - \sin^2 x}{\cos x} + \sin x = \frac{\cos^2 x}{\cos x} + \sin x = \cos x + \sin x$$

③ $$\sin x + \cos x = (\sin x + \cos x)\frac{\sec x}{\sec x} = \frac{\frac{\sin x}{\cos x} + 1}{\sec x} = \frac{\tan x + 1}{\sec x}$$

$$= \frac{\tan x + \sec^2 x - \tan^2 x}{\sec x}, \text{ since } \tan^2 x + 1 = \sec^2 x$$

20. Find the inverse function of $g(x) = \dfrac{2x-1}{x-2}$.

$$x = \frac{2y-1}{y-2}$$

$$x(y-2) = 2y-1$$

$$xy - 2x = 2y - 1$$

$$xy - 2y = 2x - 1$$

$$y(x-2) = 2x - 1$$

$$y = \boxed{g^{-1}(x) = \frac{2x-1}{x-2}}$$

21. Find all solutions to $\sin(2\theta) = \sin(\theta)$ **on the interval** $[0, 2\pi)$.

$$2\sin\theta\cos\theta = \sin\theta$$

$$2\sin\theta\cos\theta - \sin\theta = 0$$

$$\sin\theta(2\cos\theta - 1) = 0$$

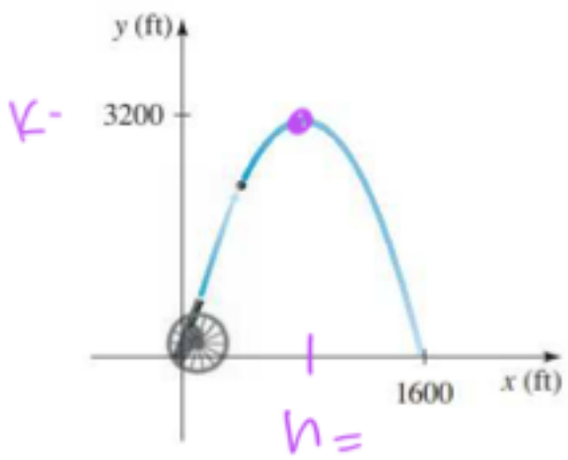
$$\sin\theta = 0 \quad \text{or} \quad \cos\theta = 1/2$$

$\theta = 0, \theta = \pi$

$\theta = \pi/3, \theta = 5\pi/3$

$$\boxed{\{0, \pi/3, \pi, 5\pi/3\}}$$

22. A cannon fires a cannonball as shown in the figure. The path of the cannonball is a parabola with vertex at the highest point of the path. If the cannonball lands 1600 ft from the cannon and the highest point it reaches is 3200 ft above the ground, find an equation for the path of the cannonball. Place the origin at the location of the cannon.



$h = \frac{1600}{2} = 800$

General form: $-4a(y-k) = (x-h)^2$

$-4a(y-3200) = (x-800)^2$

Use a point on the graph to find a:

$(x,y) = (0,0)$, e.g: $4a(3200) = 640000$

$4a = \frac{640000}{3200} = 200$

$$\boxed{-200(y-3200) = (x-800)^2}$$

23. Identify the type of conic section of the following equation, find the center, the vertex or vertices, and the focus or foci.

$$9x^2 - y^2 - 18x + 8y - 88 = 0$$

Hyperbola since x^2, y^2 are both present with opposite signs.

$9x^2 - 18x - y^2 + 8y = 88$

$9(x^2 - 2x + 1^2) - (y^2 - 8y + 4^2) = 88 + 9 - 16$

$9(x-1)^2 - (y-4)^2 = 81$

$\frac{(x-1)^2}{9} - \frac{(y-4)^2}{81} = 1$

$\boxed{\text{Center } (1,4)}$

$\boxed{\text{vertices}}$ $(1 \pm 3, 4)$: $\boxed{(4,4) \text{ and } (-2,4)}$

$c^2 = a^2 + b^2 = 9 + 81 = 90$ $\quad c = \sqrt{90} = 3\sqrt{10}$

$\boxed{\text{foci } (1 \pm 3\sqrt{10}, 4)}$

24. For the function $f(x) = -2\sec\left(\frac{\pi}{3}x - 3\right)$ determine all the x-values of its vertical asymptotes. Use k as your integer variable.

$\sec\theta$ DNE for $\cos\theta = 0$, i.e. $\theta = \pi/2 + \pi k$

f has VA at $\frac{\pi}{3}x - 3 = \frac{\pi}{2} + \pi k$, where k is any integer

$\frac{\pi}{3}x = 3 + \frac{\pi}{2} + \pi k$

$$\boxed{x = \frac{9}{\pi} + \frac{3}{2} + 3k}$$

25. A curve is given by the equations $x = 2t^2 + 3$, $y = 1 - t$.

(a) Eliminate the parameter to give a Cartesian (rectangular) equation of this curve.

$y = 1 - t$, $t = 1 - y$

$x = 2t^2 + 3 = 2(1-y)^2 + 3$

$$\boxed{x = 2(1-y)^2 + 3}$$

$\frac{1}{2}(x-3) = (y-1)^2$

parabola with vertex $(3,1)$

(b) Sketch the curve and include its orientation.

The parabola opens right; as t increases, $y = 1 - t$ decreases

t	(x, y)
-1	$(5, 2)$
0	$(3, 1)$
1	$(5, 0)$