MATH 130

Final Exam Version A
December 2021

Name: ____

UNC Email Address: ____________________________________

Instructor: ____

- There are 26 questions on this test:
  - 20 are multiple choice and worth 4 points each;
  - 6 are free response and worth 10 points each.
- Calculators are NOT allowed. Answers are to be given in a form that could be typed into a calculator and use units as necessary. I do NOT need to simplify arithmetic / algebraic expressions.
- I am expected to SHOW ALL WORK on the free response questions; no credit will be given for correct answers without supporting work.
- Partial credit may be awarded on multiple choice questions, but it will be based on my answer only; work may be shown but will not be graded.
- Notation and clarity count. My job is to communicate mathematically and make what I am thinking clear.
- I will sign the Honor Pledge when I am finished or I will let my instructor know of any irregularities with this exam.

I have neither given nor received any unauthorized help on this test and I have conducted myself within the guidelines of the University Honor Code.

Pledge: ______________________________

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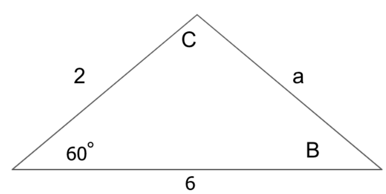
1. If the point $(2, 3)$ lives on the graph of $y = f(x)$, which point must live on the graph of $y = -2f(x - 1) + 2$?
   - ○ $(3, -4)$
   - ○ $(1, -2)$
   - ○ $(1, -4)$
   - ○ $(2, 3)$
   - ○ $(2, -4)$

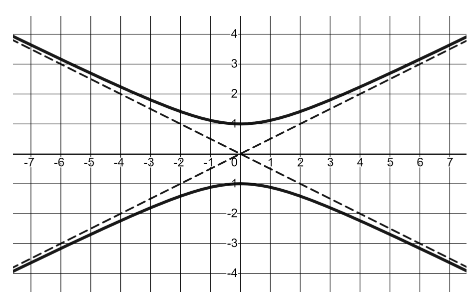
2. Match the function to the set of equations of its vertical asymptotes. Note, each set of equations can be selected more than once.

____________ $\sin x$	A. $\{x = \frac{\pi}{2} + \pi k,\ k \in \mathbb{Z}\}$
____________ $\cos x$	
____________ $\tan x$	B. $\{x = \pi k,\ k \in \mathbb{Z}\}$
____________ $\csc x$	
____________ $\sec x$	C. $\{\ \}$ (No asymptotes)
____________ $\cot x$	

3. Find the area of the following triangle.
   - ○ $\sqrt{3}$
   - ○ $3\sqrt{3}$
   - ○ $6\sqrt{3}$
   - ○ $\dfrac{\sqrt{3}}{3}$



4. What is the equation of the hyperbola shown?
   - ○ $\dfrac{x^2}{4} - y^2 = 1$
   - ○ $x^2 - \dfrac{y^2}{4} = 1$
   - ○ $\dfrac{x^2}{4} + y^2 = 1$
   - ○ $\dfrac{y^2}{4} - x^2 = 1$
   - ○ $y^2 - \dfrac{x^2}{4} = 1$
   - ○ $x^2 + \dfrac{y^2}{4} = 1$



5. Is the function $f(x) = 4x^3 \cos(x) \tan(x)$ even, odd, or neither?
   - ○ f is odd because $f(-x) = -f(x)$.
   - ○ f is even because $f(-x) = -f(x)$.
   - ○ f is odd because $f(-x) = f(x)$.
   - ○ f is even because $f(-x) = f(x)$.
   - ○ f is neither because $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$.

6. Find the exact value of $\cos^{-1}\left(\cos\left(\frac{7\pi}{6}\right)\right)$.
   - ○ $-\frac{\pi}{6}$
   - ○ $\frac{\pi}{6}$
   - ○ $\frac{5\pi}{6}$
   - ○ $\frac{7\pi}{6}$

7. Which of the following is equivalent to $\frac{1+\cos(2\theta)}{2}$?
   - ○ $\sin^2(\theta)$
   - ○ $\cos^2(\theta)$
   - ○ $\tan^2(\theta)$
   - ○ $2\cos(\theta)\sin(\theta)$

8. Consider the points with polar coordinates:
   A $\left(-6, \frac{2\pi}{3}\right)$ B $\left(6, \frac{2\pi}{3}\right)$ C $\left(6, -\frac{\pi}{3}\right)$ D $\left(-6, -\frac{\pi}{3}\right)$
   Which correspond to the point with rectangular (Cartesian) coordinates $(-3, 3\sqrt{3})$?
   - ○ A and C
   - ○ A and D
   - ○ B and C
   - ○ B and D

9. Determine the graph of the curve given by parametric equations $x = 2\sin(3t),\ y = 1 + \cos(3t)$.
   - ○ A line
   - ○ A line segment
   - ○ A parabola
   - ○ A circle
   - ○ An ellipse

10. Consider the following function, $f(x) = \begin{cases} x^2+5 & \text{if } x < 2 \\ 3x-2 & \text{if } 2 \le x < 10 \\ 3 & \text{if } x \ge 10 \end{cases}$.

    What are the y-intercept(s) of the graph?

    - ○ -2
    - ○ 3
    - ○ 5
    - ○ All of the above.

11. Find the exact value of $\tan^{-1}\left(-\sqrt{3}\right)$.

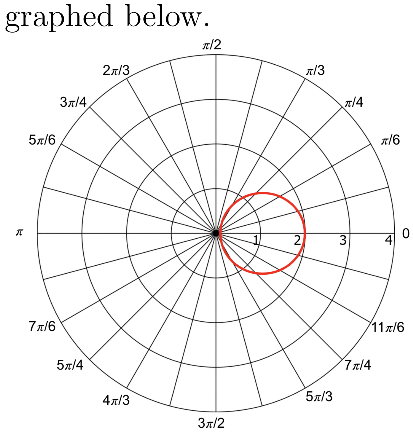
    - ○ $-\dfrac{\pi}{3}$
    - ○ $-\dfrac{\pi}{6}$
    - ○ $\dfrac{2\pi}{3}$
    - ○ $\tan^{-1}\left(-\sqrt{3}\right)$ is undefined.

12. Find the exact value of $\cos(195°)$

    - ○ $\dfrac{\sqrt{2}-\sqrt{6}}{4}$
    - ○ $\dfrac{\sqrt{2}+\sqrt{6}}{4}$
    - ○ $\dfrac{\sqrt{6}-\sqrt{2}}{4}$
    - ○ $\dfrac{-\sqrt{2}-\sqrt{6}}{4}$

13. Find the equation in polar coordinates of the function graphed below.

    - ○ $r = 2$
    - ○ $r\sin(\theta) = 2$
    - ○ $r\cos(\theta) = 2$
    - ○ $r = 2\sin(\theta)$
    - ○ $r = 2\cos(\theta)$



14. A dog runs around part of the circumference of a circle of radius 5 meters. His path makes an angle of 230° with the center of the circle. What distance has he run?

- ○ $\frac{5 \cdot 23\pi}{18}$ meters
- ○ $\frac{25 \cdot 23\pi}{36}$ meters
- ○ $25 \cdot 115$ meters
- ○ $5 \cdot 230$ meters
- ○ $\frac{5}{230}$ meters

15. Given a triangle with measurements $A = 40°$, $B = 80°$, and $a = 5$, What is b?

- ○ $\dfrac{3\sin(60°)}{\sin(40°)}$
- ○ $\dfrac{3\sin(80°)}{\sin(40°)}$
- ○ $\dfrac{5\sin(60°)}{\sin(40°)}$
- ○ $\dfrac{5\sin(80°)}{\sin(40°)}$

16. Find the equation of the directrix for the parabola $-8(x-2) = (y+1)^2$.

- ○ $x = 4$
- ○ $x = 0$
- ○ $x = -6$
- ○ $y = 7$
- ○ $y = -1$
- ○ $y = -3$

17. If $\sin\theta = 0.7$, what is the value of $\sin(\theta + \pi)$?

- ○ -0.7
- ○ -0.3
- ○ 0.3
- ○ 0.7
- ○ $\sqrt{0.91}$

18. Jamie is measuring the height of a monument on campus. She stands 200 feet away from the monument, and looks up at an angle of 70° to see the top of the monument. How tall is the monument?

   ◯ $200 \sin 70°$ feet
   ◯ $200 \cos 70°$ feet
   ◯ $200 \tan 70°$ feet
   ◯ $200 \cot 70°$ feet

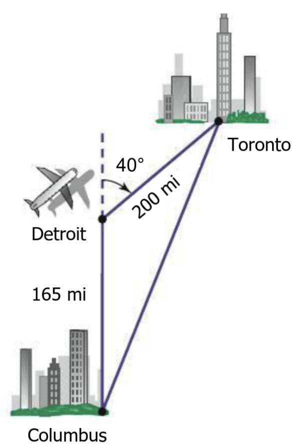
19. Find and simplify the difference quotient for $f(x) = x^2 - 1$.

   ◯ 1
   ◯ $2x + h$
   ◯ $2xh + h^2$
   ◯ $\frac{2xh+h^2-2}{h}$

20. Determine the center of the conic given by $x^2 + 2y^2 - 12x + 8y + 26 = 0$.

   ◯ $(6, -4)$
   ◯ $(-6, -4)$
   ◯ $(6, -2)$
   ◯ $(-6, -2)$

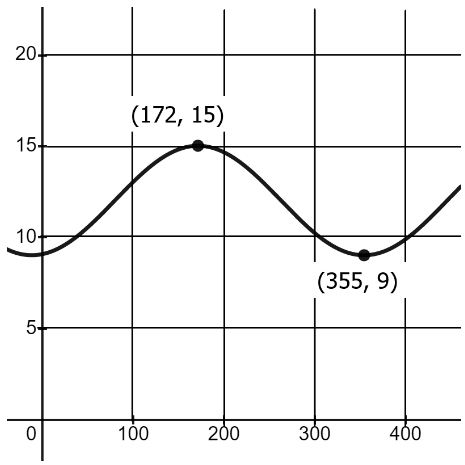
21. An airplane flies due north from Columbus to Detroit, a distance of 165 miles, and then turns through an angle of 40° and flies to Toronto, a distance of 200 miles. How far is it directly from Columbus to Toronto? Leave your answer in an exact form that could be typed into a calculator to get an approximation and draw a box around it.



22. Given $\sec\theta = \frac{7}{4}$, and $\frac{3\pi}{2} < \theta < 2\pi$, find $\tan\theta$.

23. Find all solutions to the equation $2\cos\left(5\theta + \dfrac{\pi}{3}\right) = -1$.

24. The number of hours of daylight over time follows a sinusoidal curve. At location A, the maximum amount of daylight, 15 hours, occurs on the summer solstice which is on day 172. The minimum amount of daylight at location A happens on the winter solstice, when they get 9 hours of daylight on day 355. This repeats annually with a period of 366 days.



(a) If we use the cosine function to model this curve, what is the phase shift in days? (include left or right in your answer)

(b) What is the average amount of daylight for the whole year? (This corresponds to the midline of the function.)

(c) Write an equation using cosine that gives the hours of daylight y as a function of time t in days. (The solution is graphed above.)

25. Find the inverse function for $f(x) = \dfrac{3x+2}{4x-1}$ and give the domain and range of f^{-1} in interval notation.

$f^{-1}(x) =$
Domain:
Range:

26. Establish the identity $\tan^2 x = \dfrac{\sin^2(2x)}{(1+\cos(2x))^2}$

(This page is intentionally left blank to be used for scratch work if necessary.)

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This page will NOT be graded and you do NOT need to fill out the unit circle. It is included in case you find it helpful.

Formulas

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Area: $\frac{1}{2}ab\sin C$

$$\begin{aligned}\sin(2\theta) &= 2\sin\theta\cos\theta\\ \cos(2\theta) &= \cos^2\theta - \sin^2\theta\\ &= 2\cos^2\theta - 1\\ &= 1 - 2\sin^2\theta\end{aligned}$$

$$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

Parabola:

$$(y-k)^2 = \pm 4a(x-h) \qquad (x-h)^2 = \pm 4a(y-k)$$

$$(y-k)^2 = 4\rho(x-h) \qquad (x-h)^2 = 4\rho(y-k)$$

Ellipse:

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 \qquad \frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

Hyperbola:

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1 \qquad \frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$