Applied Project Ideas

APP 1

Example Text: *Nonlinear Dynamics And Chaos* by Steven Strogatz

Additional Materials: MATLAB and/or Python

Project: A dynamical system, loosely, is a system of physical quantities that evolves over time. This subject is a natural follow-up to MATH 383, where we now study the behavior of solutions to non-linear ordinary differential equations, often in a highly qualitative manner. The goal of this project is to learn the basics of dynamical systems from a qualitative and applied perspective, building up to a small project at the end. For such a project, we could either perform further exploratory study on a special topic (like studying the Lorenz system or the Cantor set) or work through parts of an accessible scientific paper in a topic of interest (like a paper on the spread of infection disease, such as the SIR model, or some type of weather modeling). We will read various parts of Strogatz's text, primarily portions of chapters discussing 1D and 2D flows (with a focus on stability analysis, phase portraits, and bifurcation theory) and chaos theory, along with different or additional topics depending on the interest level. We will also do some studying of basic finite difference schemes to allow us to perform some numerical experiments with the areas that we are studying.

Suggested Prerequisites: The preferred prerequisites are MATH 383 (where eigenvalues and eigenvectors were discussed) and some programming experience (ideally with MATLAB). However, a strong background in MATH 231, along with familiarity with Taylor series, separable ODEs, eigenvalues and eigenvectors, partial derivatives, the Jacobian matrix, and programming would suffice. MATH 547 would be useful, although not required. Point set topology could also prove useful in some places, but it is certainly not required.

Keywords: *dynamical systems, differential equations, chaos theory, calculus, programming*

APP 2

Example Text: *Spikes, Decisions, and Actions: The Dynamical Foundations of Neuroscience* by Wilson

Additional Materials: Python or/and MATLAB

Project: As the book title suggests, students will explore how dynamical systems and ordinary differential equations are used to model neurons firing (in the brain). The project will explore basic properties of dynamical systems, including phase portraits, bifurcations, nullclines, etc., and then apply these concepts to the biological foundations of neuron firing, such as potassium and sodium channels opening and closing. Two simple models will be investigated in detail (Hodgkin-Huxley and FitzHugh-Nagumo), and interested students will have the opportunity to analyze more recent models of neuron firing. Students will have the opportunity to explore these dynamical systems using numerical software such as Python or MATLAB. This project can also be adapted to focus more on the dynamical systems aspect, or focus more on the biological component.

Suggested Prerequisites: Math 383 (basic biology/anatomy helps), Math 383L is a plus

Keywords: *dynamical systems, neuroscience, matlab, python, numerical analysis*

APP 3

Example Text: *Spectral methods in MATLab* by Trefethen

Additional Materials: MATLab or/and python

Project: Students will learn about spectral methods (the Fourier transform), and how they can be used to find accurate approximations to solutions of a wide-range of ordinary and partial differential equations. The directed reading will be on the (very readable) text "Spectral Methods in MATLAB". The book includes a number of exercises, and much of the work will be in implementing code in either MATLAB or Python. Students will be exposed to a number of numerical packages, and will gain experience in solving differential equations numerically. Math 383 is a necessary prerequisite, and Math 383L is an added bonus (but not necessary).

Suggested Prerequisites: Math 383

Keywords: *spectral methods, numerical analysis, Fourier transform, matlab, programming*

APP 4

Example Text: *Mathematics of Social Choice* by Christoph Börgers

Project: The focus for this project will be a mathematical understanding of voting. While many democratic countries, states, cities, towns, etc. all use voting as a way to construct their government, voting often looks different depending on the laws, the rules, or the reasons for voting. We often imagine the scenario of two candidates competing for a seat to represent us, but careful examination of your local ballot will show a variety of elections that employ voting methods following different rules. Our particular interest will be understanding these situations and more importantly exploring ranked-choice voting, where voters cast ballots ranking candidates, and methods for producing a winner from these types of ballots. For crafting policy around voting we need a mathematical understanding of these methods and how to compare, interpret, and justify the outcomes. A good goal of the project is an understanding of Arrow's Impossibility Theorem, what it actually means, and to give the beginning math major a unique avenue into thinking like a mathematician in the context of social science, economics, and politics. The intrepid student may wish to do some coding and simulations with the methods we discuss, or leap into related subjects like weighted voting.

Suggested Prerequisites: This project is best for a student who has completed Math 381 for experience writing proofs and exposure to sets, functions, and basic combinatorics.

Keywords: *voting, social choice, ranked-choice voting, Arrow's Impossibility Theorem, political science, economics, social science*

APP 5

Example Text: *Simulating Hamiltonian Dynamics* by Leimkuhler and Reich

Project: The double pendulum is a famous example a chaotic dynamics. This project will simulate the double pendulum with an added restriction on the maximum angle between the two parts. These simulations will be used to explore the chaotic dynamics of the restricted system, and how the dynamics depend on the physical parameters of the double pendulum.

Suggested Prerequisites: Differential equations (Math 383), and even better with the lab. Numerical analysis (Math 566) would be helpful as well.

Keywords: *dynamical systems, numerical analysis, physics, chaos, simulation*

APP 6

Example Text: *Computational Optimal Transport* by Peyre and Cuturi

Project: Optimal transport is a very active area of research in both theoretical and applied math (a recent Fields medal was awarded for work on optimal transport). In short, optimal transport is concerned with problems about, you guessed it, transporting goods from one location to another in the most optimal way. In practice, this means actually moving goods, while in theory, this means finding a "matching" between two probability distributions. In this project we'll learn about optimal transport from a very computational standpoint. The focus will be on using computers and algorithms to solve/construct solutions to optimal transport problems. Along the way we'll dip into some probability theory, some algorithm design, and more!

Suggested Prerequisites: Basic probability. Analysis (Math 521) is preferable but not necessary.

Keywords: *probability, optimal transport, computational, algorithms, measure theory*