Math 231: Test 3A
Fall 2016
Instructor: Linda Green

- Calculators are NOT allowed.
- Please code true/false and multiple choice answers on a scantron. These are questions 1 - 11.
- Since you have test version A, please code the "Sequence Number" on the scantron as 111111 (all 1's).
- No partial credit for multiple choice / no work needs to be shown.
- For short answer questions, you MUST SHOW WORK for full and partial credit unless otherwise specified.
- Sign the honor pledge below after completing the exam.

First and last name .... Key ....

PID ....

UNC Email ....

Honor Pledge: I have neither given nor received unauthorized help on this exam.

Signature: ....

For the True False questions, assume all functions have first and second derivatives that exist and are continuous on $(-\infty, \infty)$. Remember that True means always true, and False means sometimes or always false.

1. (2 pts) If $f'(5) = 0$ and $f''(5) = 2$, then f has a local max at $x = 5$.

   A. True

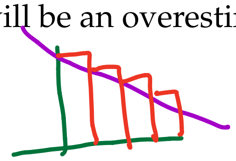
   B. False

   f has a local min since f is concave up at $x=2$

2. (2 pts) Suppose $f(x) > 0$ and and $f(x)$ is decreasing on $[0, 5]$. A Riemann sum using left endpoints will be an overestimate of $\int_0^5 f(x)\,dx$.

   A. True

   B. False



3. (2 pts) Suppose $F(x)$ is an antiderivative of $f(x)$. Then $\sin(F(x))$ is an antiderivative of $\cos(f(x))$.

   A. True

   B. False

   $\frac{d}{dx}\sin(F(x)) = \cos(F(x))\cdot F'(x)$
   $= \cos(F(x))\, f(x)$
   not $\cos(f(x))$

4. (2 pts) If $f(x) < g(x)$ on $[0, 10]$,then $\int_0^{10} f(x) - g(x)\,dx < 0$

   A. True

   B. False

   $\int_0^{10} f(x)\,dx < \int_0^{10} g(x)\,dx \Rightarrow \int_0^{10} f(x)\,dx - \int_0^{10} g(x)\,dx \le 0$
   $\Rightarrow \int_0^{10} f(x) - g(x)\,dx < 0$

5. (2 pts) If $f'(1) = 0$ and $f'(3) = 0$, then $f(c) = 0$ for some c between 1 and 3.

   A. True

   B. False

   $f''(c) = 0$ but not nec $f(c)$
   e.g. $f(x) = \cos(2\pi x) + 100$

6. (5 pts) For a differentiable function $f(x)$, $f(1) = 5$ and $f(2) = 5$ and $f(4) = 6.5$. Which of the following must be true about the derivative $f'(x)$?

   1. $f'(x) = 0.5$ for some x-value
   2. $f'(x) = 1.5$ for some x-value
   3. $f'(x) = 2$ for some x-value
   4. $f'(x) = 5.75$ for some x-value
   5. None of these have to be true.

   $\frac{f(4) - f(1)}{4-1} = \frac{1.5}{3} = 0.5$

   $\Rightarrow f'(c) = 0.5$ for some c between 1 & 4

7. (5 pts) If $\int_2^5 (2f(x) - 6)\,dx = -10$, then what is $\int_2^5 f(x)\,dx$?

A. -4

B. -2

C. 4

D. 8

E. 10

$-10 = \int_2^5 2f(x) - 6\,dx = 2\int_2^5 f(x)\,dx - \int_2^5 6\,dx$

$= 2\int_2^5 f(x)\,dx - 18$

$\Rightarrow 8 = 2\int_2^5 f(x)\,dx \Rightarrow \int_2^5 f(x)\,dx = 4$

8. (5 pts) Suppose that $g(x)$ is a continuous function. Some of the values of $g(x)$ are given in the table below.

x	-1	1	3	5
$g(x)$	-2	3	5	6

Use a Riemann sum and right endpoints to estimate $\int_{-1}^5 g(x)\,dx$.

A. 12

B. 14

C. 24

D. 28

E. 48

$2\cdot 3 + 2\cdot 5 + 2\cdot 6 = 2(3+5+6) = 28$

9. (5 pts) Express $\int_3^8 (4 - x)\,dx$ as the limit of a Riemann sum using right endpoints.

A. $\lim_{n\to\infty} \sum_{i=1}^{n} \left(4 - \frac{5i}{n}\right)$

B. $\lim_{n\to\infty} \sum_{i=1}^{n} \left(7 - \frac{5i}{n}\right)$

C. $\lim_{n\to\infty} \sum_{i=1}^{n} \left(1 - \frac{5i}{n}\right)\frac{5}{n}$

D. $\lim_{n\to\infty} \sum_{i=1}^{n} \left(3 - \frac{5i}{n}\right)\frac{5}{n}$

E. $\lim_{n\to\infty} \sum_{i=1}^{n} \left(4 - \frac{5i}{n}\right)\frac{5}{n}$

$\Delta x = \frac{5}{n}$

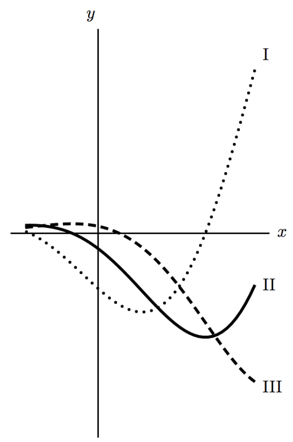
$x_i^* = 3 + \frac{5i}{n}$

$\lim_{n\to\infty} \sum_{i=1}^{n} (4 - x_i^*)\,\Delta x$

$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[4 - \left(3 + \frac{5i}{n}\right)\right] \frac{5i}{n}$

$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[1 - \frac{5i}{n}\right] \frac{5i}{n}$

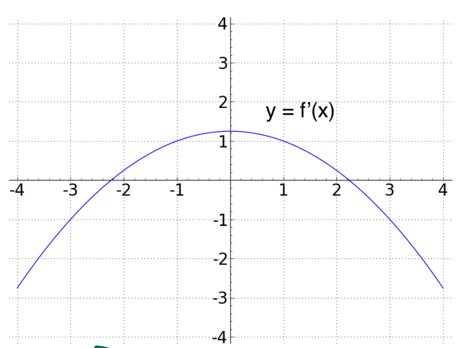
10. (5 pts) The figure shows the graphs of $y = f(x)$, $y = f'(x)$, and $y = f''(x)$. Which one is which?



The derivative of I is positive for large x, so the derivative of I is not II or III.
Therefore, I is f''
The derivative of II is I
so II is f'
so III is f

A. $f(x)$: I, $f'(x)$: III, $f''(x)$: II
B. $f(x)$: II, $f'(x)$: I, $f''(x)$: III
C. $f(x)$: II, $f'(x)$: III, $f''(x)$: I
D. $f(x)$: III, $f'(x)$: I, $f''(x)$: II
E. $f(x)$: III, $f'(x)$: II, $f''(x)$: I

11. (5 pts) Given the graph of the DERIVATIVE $f'(x)$ defined on the interval $[-4, 4]$ and drawn below, find the interval(s) on which the original function $f(x)$ is concave up.



$f(x)$ concave up
$\Leftrightarrow f''(x) > 0$
$\Leftrightarrow f'(x)$ increasing
so $(-4, 0)$

A. $(-4, 0)$
B. $(0, 4)$
C. $(-2.2, 2.2)$
D. $(-4, -2.2) \cup (2.2, 4)$
E. $f(x)$ is never concave up

12. (16 pts) Suppose $f(x)$ is a differentiable function defined for all x. The *derivative* and *second derivative* of $f(x)$ are given by:

$$f'(x) = 3(x+4)(x+2)^{1/3}$$

$$f''(x) = \frac{4x+10}{(x+2)^{2/3}}$$

$f''(x)=0$ where $4x+10=0$
$x = -\frac{5}{2}$

f'': $\ominus$ $\ominus$ $\oplus$ $\oplus$
f': $\oplus$ $\ominus$ $\ominus$ $\oplus$
-4 $\quad -\frac{5}{2}$ $\quad -2$

Answer the following questions, and write "None" if the requested feature does not occur.

(a) What are the critical number(s) for $f(x)$?

$x =$ $\boxed{-4, -2}$

(b) What are the critical number(s) for $f'(x)$?

$x =$ $\boxed{-5/2, -2}$

(c) At what x-values does $f(x)$ have local max(es)? local min(s)?

MAX at $x =$ $\boxed{-4}$ MIN at $x =$ $\boxed{-2}$

(d) At what x-values does $f(x)$ have inflection point(s)?

$x =$ $\boxed{-5/2}$

(e) On what interval(s) is $f(x)$ decreasing?

Answer: $\boxed{(-4, -2)}$

(f) On what interval(s) is $f(x)$ concave down?

Answer: $\boxed{(-\infty, -5/2)}$

(g) On what interval(s) is $f(x)$ both decreasing and concave down?

Answer: $\boxed{(-4, -5/2)}$

13. (16 pts) Suppose $f''(x) = \dfrac{3}{\sqrt{x}} + 5\cos(x)$, $f(0) = 0$, and $f(\pi) = 10$. Find $f(x)$.

$$f''(x) = 3x^{-1/2} + 5\cos(x)$$

$$f'(x) = \frac{3x^{1/2}}{\frac{1}{2}} + 5\sin(x) + C$$

$$f'(x) = 6x^{1/2} + 5\sin(x) + C$$

$$f(x) = 6x^{3/2} \cdot \frac{2}{3} - 5\cos(x) + Cx + D$$

$$f(x) = 4x^{3/2} - 5\cos(x) + Cx + D$$

$$0 = f(0) = 4 \cdot 0^{3/2} - 5\cos(0) + C \cdot 0 + D$$

$$0 = -5 + D \Rightarrow D = 5$$

$$10 = f(\pi) = 4 \cdot \pi^{3/2} - 5\cos(\pi) + C\pi + 5$$

$$10 = 4 \cdot \pi^{3/2} + 5 + C\pi + 5$$

$$\Rightarrow C\pi = -4\pi^{3/2} \Rightarrow C = \frac{-4\pi^{3/2}}{\pi} = -4\pi^{1/2} = -4\sqrt{\pi}$$

$$f(x) = 4x^{3/2} - 5\cos(x) - 4\sqrt{\pi}\,x + 5$$

Answer: $\boxed{f(x) = 4x^{3/2} - 5\cos(x) - 4\sqrt{\pi} + 5}$

14. (14 pts) Evaluate $\lim_{x\to\infty}\left(1+\frac{2}{x}\right)^{4x}$.

$$y = \left(1+\frac{2}{x}\right)^{4x}$$

$$\ln y = \ln\left(1+\frac{2}{x}\right)^{4x}$$

$$\ln y = 4x\ln\left(1+\frac{2}{x}\right)$$ ✓✓

$$\lim_{x\to\infty}\ln y = \lim_{x\to\infty} 4x\ln\left(1+\frac{2}{x}\right) = \lim_{x\to\infty}\frac{4\ln\left(1+\frac{2}{x}\right)}{x^{-1}}$$ ✓

$$\overset{L'H}{=} \lim_{x\to\infty}\frac{4\cdot\frac{1}{1+\frac{2}{x}}\cdot\frac{-2}{x^2}}{-\frac{1}{x^2}} = \lim_{x\to\infty}\frac{8}{1+\frac{2}{x}} = 8$$ ✓✓ ✓✓ ✓✓ ✓

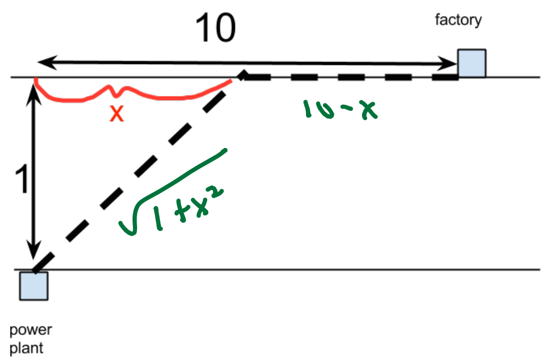
−2 if 4 ends up incorrectly in denominator

so $\lim_{x\to\infty}\ln y = 8 \Rightarrow \lim_{x\to\infty} y = \boxed{e^8}$ ✓✓

Answer: e^8

15. (14 pts) Pick ONE of the two questions to answer. For credit, you must use calculus in your solution.

(a) On one side of a river 1 mile wide is an electric power station; on the other side, 10 miles upstream, is a factory. It costs \$300 per mile to run cable over land and \$500 per mile under water. What value of x in the diagram will give the cheapest way to run cable from the station to the factory? Hint: find distances in terms of x, then convert distances to costs.



(b) You wish to make a cylinder with a base and sides but no top. The cylinder must have surface area of 100 cm^2. Give the ~~dimensions of the cylinder~~ radius (r~~adius and height~~), in order to maximize volume.

a) $C(x) = 500\sqrt{1+x^2} + 300(10-x)$ on $[0,10]$

6 pts eqn

$C(x) = 500\sqrt{1+x^2} + 3000 - 300x$

$C'(x) = 500 \cdot \frac{1}{2}(1+x^2)^{-1/2} \cdot 2x - 300$

4 pts derivs

$C'(x) = \dfrac{500x}{\sqrt{1+x^2}} - 300 = 0$

$\Rightarrow \dfrac{500x}{\sqrt{1+x^2}} - 300 = 0 \Rightarrow \dfrac{500x}{\sqrt{1+x^2}} = 300$

$\Rightarrow 500x = 300\sqrt{1+x^2} \Rightarrow 5x = 3\sqrt{1+x^2}$

$\Rightarrow 25x^2 = 9(1+x^2) \Rightarrow 16x^2 = 9 \Rightarrow x = \pm\frac{3}{4}$

Answer: $\boxed{x = 3/4}$

4 pts solving for x

$C'(x)$ ⊖ ⊕ at $\frac{3}{4}$

not required for full credit

B) $V = \pi r^2 h$ ✓✓

$100 = A = \pi r^2 + 2\pi r h$ ✓✓

$\Rightarrow h = \dfrac{100 - \pi r^2}{2\pi r}$

$\Rightarrow V = \pi r^2 \cdot \dfrac{(100 - \pi r^2)}{2\pi r}$ ✓✓

$\Rightarrow V = \frac{r}{2}(100 - \pi r^2)$

$\Rightarrow V = 50r - \frac{\pi}{2} r^3$

4 pts derivs

$V'(r) = 50 - \frac{3\pi}{2} r^2 = 0$

$\Rightarrow 50 = \frac{3\pi}{2} r^2$

not required for full credit

$V'(r)$ ⊕ ⊖

$\Rightarrow r^2 = \dfrac{100}{3\pi}$

$\Rightarrow r = \dfrac{10}{\sqrt{3\pi}}$

4 pts solving for r