Math 231: Test 3A
Spring 2016
Instructor: Linda Green

- Calculators are NOT allowed.
- Please code true/false and multiple choice answers on a scantron. These are questions 1 - 12.
- Since you have test version A, please code the ”Sequence Number” on the scantron as 111111 (all 1’s).
- No partial credit for multiple choice / no work needs to be shown.
- For short answer questions, you MUST SHOW WORK for full and partial credit unless otherwise specified.
- Sign the honor pledge below after completing the exam.

First and last name ....Key..

PID ....

UNC Email ....

Honor Pledge: I have neither given nor received unauthorized help on this exam.

Signature: ....

Either answer is correct; compensated by 2 pt curve.

1. (2 pts) True or False: $\int_3^7 f(x)\,dx = \int_7^3 f(x)\,dx$

   A. True

   B. False

   $\int_3^7 f(x)\,dx = -\int_7^3 f(x)\,dx$

   f is differentiable on $(-4,4)$ and continuous on $[-4,4]$

2. (2 pts) True or False: If $f(-4) = -1$ and $f(4) = 9$, then $f'(x) > 1$ for some x value with $|x| < 4$.

   if we assume f' exists → A. True

   if we do not assume f' exists → B. False

   slope of secant line $= \dfrac{f(4) - f(-4)}{4-(-4)} = \dfrac{9-(-1)}{8} = \dfrac{10}{8} = \dfrac{5}{4}$

   by MVT, $f'(c) = \frac{5}{4}$ for some c between -4 & 4 & $\frac{5}{4} > 1$

3. (2 pts) True or False: Suppose f is a function whose second derivative f'' exists and is continuous. If $f'(2) = 0$ and $f''(2) < 0$, then f has a local minimum at $x = 2$.

   A. True

   B. False

   False by the Second derivative test – there is a local max not a local min.

   Note: $f''(2) < 0$ means concave down

4. (2 pts) True or False: For a continuous function $f(x)$, if $f'(x) < 0$ for $x < 0$ and $f'(x) > 0$ for $x > 0$, then f has an absolute minimum at $x = 0$.

   A. True

   B. False

   $(-)$ $(+)$

   0

   There are no other critical points where f could change direction

5. (2 pts) If $\lim_{x\to\infty} f(x) = 0$ and $\lim_{x\to\infty} g(x) = \infty$, then $\lim_{x\to\infty} f(x) \cdot g(x) = \lim_{x\to\infty} f'(x) \cdot g'(x)$, provided that this second limit exists.

   A. True

   B. False

   For example, take $f(x) = \frac{1}{x}$ $g(x) = x$

   $\lim_{x\to\infty} f(x)\cdot g(x) = \lim_{x\to\infty} \frac{1}{x}\cdot x = 1$ but $\lim_{x\to\infty} f'(x)\cdot g'(x) = \lim_{x\to\infty} -\frac{1}{x^2}\cdot 1 = 0$

6. (5 pts) On what interval is $f(x) = 3x^3 - 36x$ both increasing and concave up?

   A. $(-\infty, 2)$

   B. $(-2, 0)$

   C. $(0, 2)$

   D. $(2, \infty)$

   $f'(x) = 9x^2 - 36$ $\quad 9x^2 - 36 = 0 \Rightarrow 9(x^2-4) = 0 \Rightarrow 9(x-2)(x+2) = 0 \Rightarrow x = 2, -2$

   $f''(x) = 18x$ $\quad 18x = 0 \Rightarrow x = 0$

$f''(x)$ $(-)$ $(-)$ $(+)$ $(+)$

$f'(x)$ $(+)$ $(-)$ $(-)$ $(+)$

-2 $\quad 0 \quad$ 2

7. (5 pts) Express $\int_2^7 x\,dx$ as the limit of a Riemann sum using right endpoints.

A. $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{5i}{n}$

B. $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{25i}{n^2}$

C. $\lim_{n\to\infty} \sum_{i=1}^{n} 2 + \frac{5i}{n}$

D. $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{10}{n} + \frac{25i}{n^2}$

$\Delta x = \frac{7-2}{n} = \frac{5}{n}$

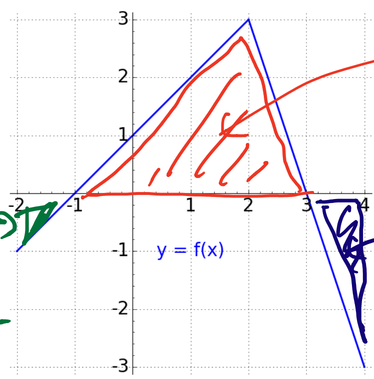
$x_i = 2 + \frac{5}{n} i$

$\lim_{n\to\infty} \sum_{i=1}^{n} \Delta x\, f(x_i)$

$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{5}{n}\left(2 + \frac{5}{n} i\right)$

$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{10}{n} + \frac{25}{n^2} i$

8. (5 pts) Use the graph of $y = f(x)$ to evaluate $\int_{-2}^{4} f(x)dx$.



area $= \frac{1}{2} \cdot 4 \cdot 3 = 6$

area $= \frac{1}{2} \cdot 1 \cdot 1 = \frac{1}{2}$

area $= \frac{1}{2} \cdot 1 \cdot 3 = \frac{3}{2}$

A. 2

B. 4

C. 6

D. 8

$6 - \frac{1}{2} - \frac{3}{2} = 4$

9. (5 pts) Estimate $\int_0^4 \frac{60}{x+1}\,dx$ using two rectangles and midpoints for sample points.

A. 45

B. 64

C. 90

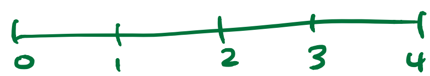
D. 112

E. 160

$\Delta x = 2 \quad x_1 = 1 \quad x_2 = 3$

$2 \cdot f(1) + 2 \cdot f(3) = 2 \cdot \frac{60}{2} + 2 \cdot \frac{60}{4}$

$= 90$



10. (5 pts) Suppose that we are using Newton's method to estimate $\sqrt{2}$ using the formula $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ and a simple "four function" calculator that does addition, subtraction, multiplication, and division. Which function should we use for $f(x)$?

A. $f(x) = \sqrt{x}$

B. $f(x) = x^2$

C. $f(x) = \sqrt{x} - \sqrt{2}$

D. $f(x) = x^2 - 2$

E. $f(x) = x^2 - 4$

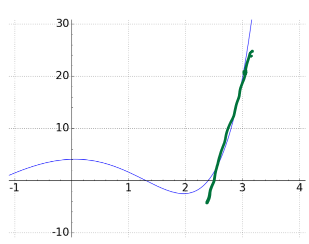
$x = \sqrt{2}$

$\Rightarrow x^2 = 2$

$\Rightarrow x^2 - 2 = 0$

use $f(x) = x^2 - 2$

11. (5 pts) Suppose that we wish to use Newton's method to estimate the RIGHT-MOST of the two x-intercepts shown in this graph. Which is the best choice for a starting value x_1?



A. 0

B. 1

C. 2

D. 3

The tangent line at x = 3 has an x-intercept very close to the one we are looking for, while the tangent line at x = 2 is horizontal or close to horizontal and won't work.
The tangent line at x = 1 will have an x-intercept close to the leftmost instead of the rightmost x-intercept, and the tangent line at x = 0 is also close to horizontal.

12. (5 points) The function $f(x) = \sin(x) - \cos(x) + x$ has inflection points at what x-values on the interval $[0, 2\pi]$?

A. $\frac{\pi}{4}$ and $\frac{3\pi}{4}$

B. $\frac{\pi}{4}$ and $\frac{5\pi}{4}$

C. $\frac{5\pi}{4}$ and $\frac{7\pi}{4}$

D. $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}$, and $\frac{7\pi}{4}$

E. No inflection points.

$f'(x) = \cos(x) + \sin(x) + 1$

$f''(x) = -\sin(x) + \cos(x)$

$f''(x) = 0 \iff \sin(x) = \cos(x)$

$\Rightarrow x = \pi/4$ or $5\pi/4$

13. (7 pts) Find the general antiderivative of $f(x) = \dfrac{3\sqrt{x} + 1}{x}$.

$f(x) = (3x^{1/2} + 1)x^{-1}$

$f(x) = 3x^{-1/2} + x^{-1}$ ✓✓

$F(x) = \dfrac{3x^{1/2}}{1/2}$ ✓✓ $+ \ln|x|$ ✓✓ $+ C$ ✓

-1 if no +C

-1 if ln(x) instead of ln|x|

Answer: $\boxed{6\sqrt{x} + \ln|x| + C}$

14. (12 pts) Evaluate $\lim_{x\to 0^+}(e^{2x}+4x)^{1/x}$.

1^{∞} indeterminate form

set $y = (e^{2x}+4x)^{1/x}$

$\ln y = \ln (e^{2x}+4x)^{1/x}$ ✓✓

$\ln y = \frac{1}{x} \ln (e^{2x}+4x)$

$\ln y = \frac{\ln(e^{2x}+4x)}{x}$ ✓✓

$\lim_{x\to 0^+} \ln y = \lim_{x\to 0^+} \frac{\ln(e^{2x}+4x)}{x}$ $\quad \frac{0}{0}$

$\underset{L'H}{=} \lim_{x\to 0^+} \frac{\frac{1}{e^{2x}+4x} \cdot 2e^{2x}+4}{1}$ ✓ ✓ ✓ ✓

$= \lim_{x\to 0^+} \frac{2e^{2x}+4}{e^{2x}+4x} = \frac{6}{1} = 6$ ✓✓

$\Rightarrow \lim_{x\to 0^+} y = \lim_{x\to 0^+} e^{\ln y}$

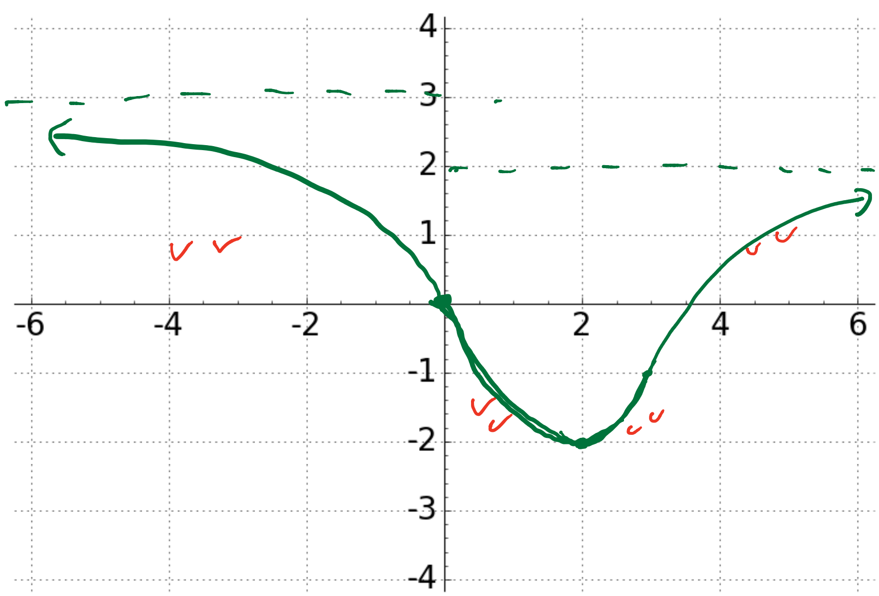
$= e^6$ ✓✓

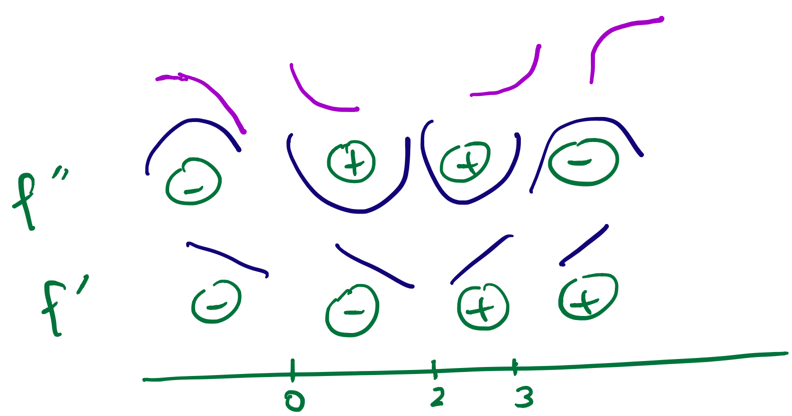
Answer: $\boxed{e^6}$

15. (12 pts) Sketch a graph of a function $f(x)$ with the following properties.

- $f(0) = 0$.
- $\lim_{x\to\infty} f(x) = 2$
- $\lim_{x\to-\infty} f(x) = 3$
- $f'(x) < 0$ for $x < 2$ and $f'(x) > 0$ for $x > 2$
- $f''(x) < 0$ for $x < 0$ and for $x > 3$ and $f''(x) > 0$ for $0 < x < 3$
- f has an absolute minimum value of -2

8 pts total
2 pts for shape of each of 4 sections as shown below





16. (12 pts) Suppose $f''(x) = 4x + \cos(x)$, $f'(0) = 2$, and $f(0) = 5$. Find $f(x)$.

$$f'(x) = \frac{4x^2}{2} + \sin(x) + C$$

$$f'(x) = 2x^2 + \sin(x) + C$$

$$2 = f'(0) = 2 \cdot 0^2 + \sin(0) + C \Rightarrow C = 2$$

$$f'(x) = 2x^2 + \sin(x) + 2$$

$$f(x) = \frac{2x^3}{3} - \cos(x) + 2x + D$$

−1 if sign error

$$5 = f(0) = \frac{2 \cdot 0^3}{3} - \cos(0) + 2 \cdot 0 + D$$

$$\Rightarrow 5 = -1 + D \Rightarrow D = 6$$

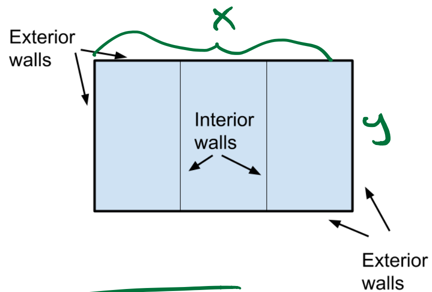
$$f(x) = \frac{2x^3}{3} - \cos(x) + 2x + 6$$

Answer: $\boxed{\frac{2x^3}{3} - \cos(x) + 2x + 6}$

17. (12 pts) Pick ONE of the two questions to answer.

(A) Find the x and y coordinates of the point(s) on the graph of $y = 3\sqrt{x}$, closest to the point $(5, 0)$.

(B) You need to make a retangular enclosure with an area of 6000 m^2 that is divided into 3 sections by walls parallel to one of its sides. The external walls cost \$3 per meter in length and the interior walls cost \$2 per meter in length. What are the dimensions that minimize the cost of the walls?



A) $d = \sqrt{(x-5)^2 + (y-0)^2}$ ✓✓ ✓✓

$d^2 = (x-5)^2 + y^2$

$d^2 = (x-5)^2 + 9x$ ✓✓

$d^2 = x^2 - 10x + 25 + 9x$

$f(x) = d^2 = x^2 - x + 25$ interval: $[0, \infty)$

$f'(x) = 2x - 1 = 0 \Rightarrow x = \frac{1}{2}$ ✓✓ ✓✓

f' (−) (+) $\frac{1}{2}$

Answer: $\left(\frac{1}{2}, 3\sqrt{\frac{1}{2}}\right)$ ✓✓

B)

$C(x) = 3x + 3x + 3y + 3y + 2y + 2y$

$C(x) = 6x + 10y$ ✓✓✓✓

$xy = 6000 \Rightarrow y = \frac{6000}{x}$

$C(x) = 6x + \frac{60{,}000}{x}$ ✓✓ x in $[0, \infty)$

$C'(x) = 6 - \frac{60{,}000}{x^2}$ ✓✓

$C'(x)$ DNE when $x = 0$

$C'(x) = 0$ when $6 = \frac{60{,}000}{x^2}$

$\Rightarrow x^2 = 10{,}000 \Rightarrow x = \pm 100$

$\Rightarrow x = 100$ ✓✓

$\Rightarrow y = 60$ ✓✓

Answer: $x = 100$, $y = 60$

C' (−) (+) 100

do not need to justify that answer is a minimum for full credit.