Math 231: Test 2A
Spring 2016
Instructor: Linda Green

- Calculators are NOT allowed.
- Please code true/false and multiple choice answers on a scantron. These are questions 1 - 13.
- Since you have test version A, please code the Section field on the scantron as 111111 (all 1's).
- No partial credit for multiple choice / no work needs to be shown.
- For short answer questions, you MUST SHOW WORK for full and partial credit unless otherwise specified.
- Sign the honor pledge below after completing the exam.

First and last name .... Key ....

PID ....

UNC Email ....

Honor Pledge: I have neither given nor received unauthorized help on this exam.

Signature: ....

A bug is moving left and right. Let $s(t)$ represent the position of a bug in feet to the right of the center of the room, where the center of the room is at 0 feet, 2 feet right of the center would be $s(t) = 2$, and 2 feet left of the center would be $s(t) = -2$. Let t be time in seconds.

Suppose $s'(t) < 0$ and $s''(t) > 0$ for $0 < t < 6$. True False Questions 1- 5 are related to the bug's motion while $0 < t < 6$.

1. (2 pts)True or False: The bug must be left of the center of the room.

   A. True

   B. False

   No info is given about the position $s(t)$

2. (2 pts)True or False: The bug must be moving left.

   A. True

   B. False

   $s'(t) < 0$

3. (2 pts)True or False: The bug must be slowing down.

   A. True

   B. False

   $s'(t)$ and $s''(t)$ have opposite signs

4. (2 pts)True or False: The bug's velocity must be decreasing.

   A. True

   B. False

   $s''(t) > 0$ so $s'(t)$ is increasing

5. (2 pts)True or False: The bug must have negative acceleration.

   A. True

   B. False

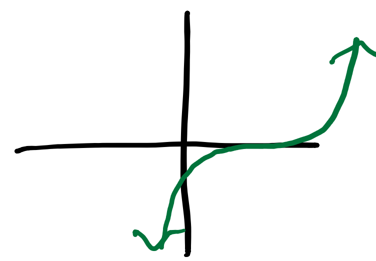
   $s''(t) > 0$

6. (2 pts) True or False: If f is a differentiable function on $(0, 10)$ and $f'(3) = 0$, then f has a local maximum or a local minimum at $x = 3$.

   A. True

   B. False

Consider $f(x) = (x-3)^3$



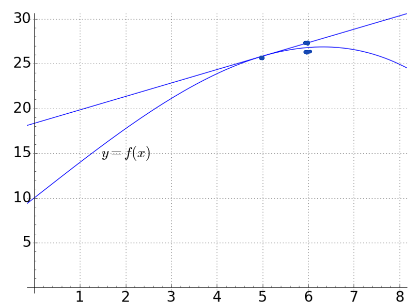
7. (2 pts) True or False: $\frac{d}{dx} log_2(5) = \frac{1}{ln(2) \cdot 5}$.

   A. True

   B. False

$\log_2(5)$ is a constant

8. (2 pts) The figure below shows the graph of $y = f(x)$ and the graph of the tangent line at $x = 5$. Suppose we use the differential df to approximate the change in f as x changes from 5 to 6.



df is change in height of tangent line which is greater that Δf, the change in height of the function

True or False: $|df| > |\Delta f|$.

   A. True

   B. False

9. (2 pts) True or False: If $f(x) = ax + b$ for some constants a and b, then the linearization of f is equal to f.

   A. True

   B. False

$f(x) = ax+b$ means $f(x)$ is a <u>line</u>

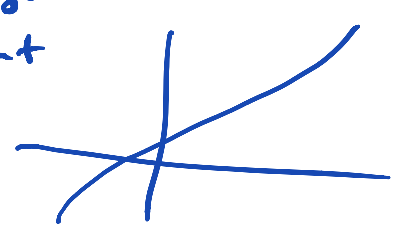
so tangent line is the same line as $f(x)$

10. (2 pts) True or False: A strictly increasing function cannot have a local maximum.

   A. True

   B. False

You can get higher by moving right



11. (5 pts) $s(t) = 5t^2 - 10t$ represents the depth of a submarine in meters at time t minutes for $0 \le t \le 3$ as it moves up and down in the water (no sideways motion). What is the total distance traveled by the submarine during the first three minutes?

turns around when $s'(t) = 0$

$s'(t) = 10t - 10 = 0 \Rightarrow t = 1$

A. 5 m

B. 10 m

C. 15 m

D. 20 m

E. 25 m

t	$s(t)$
0	0
1	-5
3	15

> 5, > 20

$20 + 5 = 25$

12. (5 pts) f and g are differentiable functions, with the following values and derivatives.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	2	3	4	-3
2	-2	2	5	2
3	3	4	1	5
4	4	3	6	5
5	9	0	1	2

Let $h(x) = f(g(2x+1))$. Find $h'(1)$.

A. -24

B. -12

C. 10

D. 30

E. 60

$$h'(x) = f'(g(2x+1))\,g'(2x+1)\cdot 2$$
$$= f'(g(3))\,g'(3)\cdot 2$$
$$= f'(4)\,g'(3)\cdot 2$$
$$= 6\cdot 5\cdot 2 = 60$$

13. (10 pts) Find the absolute maximum and absolute minimum values of $f(x) = 2x^3+3x^2-12x$ on $[0,2]$, and the points at which these values are achieved.

$f'(x) = 6x^2+6x-12$

$6(x^2+x-2) = 0$

$6(x+2)(x-1) = 0$

$\cancel{x = -2}$ or $x = 1$

not in interval

x	$f(x)$
0	0
1	-7
2	4

consider all 3 points

Absolute MAXimum value(s): $\boxed{4}$

Absolute MAXimum point(s): $\boxed{(2,4)}$

Absolute minimum value(s): $\boxed{-7}$

Absolute minimum point(s): $\boxed{(1,-7)}$

14. (10 pts) Find $\dfrac{dy}{dx}$ for $y = \sqrt{\arctan(5^x)}$. You do not need to simplify.

$$\frac{dy}{dx} = \frac{1}{2}\left(\arctan\left(5^x\right)\right)^{-1/2} \cdot \frac{1}{1+(5^x)^2} \ln 5 \cdot 5^x$$

15. (10 pts) Find the slope of the tangent line of the curve $x^3 - 4x^2y + y^2 = 1$ at the point $(1, 0)$.

$$\frac{d}{dx}\left(x^3 - 4x^2y + y^2\right) = \frac{d}{dx}(1)$$

4 pts deriv of each of 4 terms: $x^3, -4x^2y, y^2, 1$

2 pts $\frac{dy}{dx}$ in correct places

$$3x^2 - 4x^2\frac{dy}{dx} - 8xy + 2y\frac{dy}{dx} = 0$$

$$-4x^2\frac{dy}{dx} + 2y\frac{dy}{dx} = 8xy - 3x^2$$

$$\frac{dy}{dx}\left(2y - 4x^2\right) = 8xy - 3x^2$$

$$\frac{dy}{dx} = \frac{8xy - 3x^2}{2y - 4x^2}$$ ✓✓

at $(x, y) = (0, 1)$.

$$\frac{dy}{dx} = \frac{-3}{-4} = \frac{3}{4}$$ ✓✓

16. (10 pts) Evaluate $\frac{dy}{dx}$ at $x = 2$ if $y = x^{g(x)}$ and $g(2) = 3$ and $g'(2) = -5$. You do not have to simplify your answer.

$$y = x^{g(x)}$$

$$\ln y = \ln x^{g(x)}$$

$$\ln y = g(x) \ln x$$

$$\frac{1}{y} \frac{dy}{dx} = g(x) \cdot \frac{1}{x} + g'(x) \ln x$$

$$\frac{dy}{dx} = y \left(g(x) \cdot \frac{1}{x} + g'(x) \ln x \right)$$

$$\frac{dy}{dx} = x^{g(x)} \left(g(x) \cdot \frac{1}{x} + g'(x) \ln x \right)$$

$$\left.\frac{dy}{dx}\right|_{x=2} = 2^3 \left(3 \cdot \frac{1}{2} - 5 \cdot \ln 2 \right)$$

$$= 12 - 40 \ln 2$$

17. (10 pts) Find the linearization of $f(x) = \ln(2x)$ at the point $(0.5, 0)$. Use it to approximate $\ln(1.2) = \ln(2 \cdot 0.6)$

$L(x) = f'(a)(x-a) + f(a)$ ✓✓

$f'(x) = \frac{1}{2x} \cdot 2 = \frac{1}{x}$ ✓✓

$f'(0.5) = \frac{1}{0.5} = 2$

$f(0.5) = \ln(1) = 0$

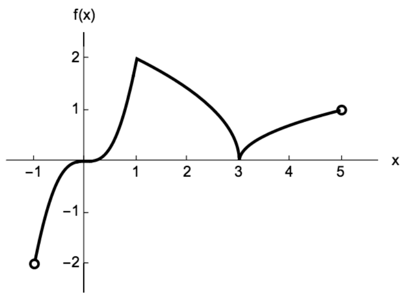
$\boxed{L(x) = 2(x-0.5) + 0}$ ✓ ✓ ✓

use $x = 0.6$ ✓✓

$\ln(1.2) = f(0.6) \approx L(0.6) = 2(0.6) - 1 = 1.2 - 1$

$= \boxed{0.2}$ ✓

18. (10 pts) The figure shows a graph of $y = f(x)$. No work needs to be shown on this problem.



(a) Find the x-value(s) of all critical points (or write NONE).

$x = 0, x = 1, x = 3$

(b) Find the x-value(s) of all local minimum points (or write NONE).

$x = 3$

(c) Find the x-value(s) at which f attains its global minimum (or write NONE).

NONE

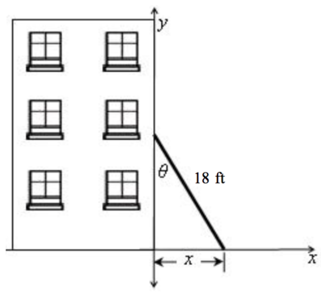
(d) Find the interval(s) on which f is increasing. (write your answer in interval notation or write NONE)

$(-1, 1) \cup (3, 5)$

(e) Find the interval(s) on which the DERIVATIVE of f is increasing. (write your answer in interval notation or write NONE)

$(0, 1)$

19. (10 pts) A 18-ft ladder leaning against a wall begins to slide. How fast is the angle between the ladder and the wall changing at the instant of time when the bottom of the ladder is 9 ft from the wall and sliding away from the wall at the rate of 4 ft/sec?



$$\sin\theta = \frac{x}{18}$$

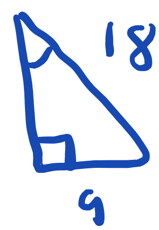
$$\cos\theta \frac{d\theta}{dt} = \frac{1}{18}\frac{dx}{dt}$$

$$\frac{d\theta}{dt} = \frac{1}{18\cos\theta}\cdot\frac{dx}{dt}$$

$$\frac{d\theta}{dt} = \frac{1}{18\frac{\sqrt{3}}{2}}\cdot 4$$

$$= \frac{4}{9\sqrt{3}} = \frac{4\sqrt{3}}{27}\ \frac{\text{rad}}{\text{sec}}$$

when $x=9$



$$\sin\theta = \frac{9}{18} = \frac{1}{2}$$

$$\Rightarrow \theta = \frac{\pi}{6}$$

$$\Rightarrow \cos\theta = \frac{\sqrt{3}}{2}$$

$$\frac{dx}{dt} = 4$$