Math 231: Test 2A
Fall 2016
Instructor: Linda Green

- Calculators are NOT allowed.
- Please code true/false and multiple choice answers on a scantron. These are questions 1 - 13.
- Since you have test version A, please code the "Sequence Number" on the scantron as 111111 (all 1's).
- No partial credit for multiple choice / no work needs to be shown.
- For short answer questions, you MUST SHOW WORK for full and partial credit unless otherwise specified.
- Sign the honor pledge below after completing the exam.

First and last name . . . .

PID . . . .

UNC Email . . . .

KEY

Honor Pledge: I have neither given nor received unauthorized help on this exam.

Signature: . . . .

True means always true. False means sometimes or always false.

1. True or False: $\frac{d}{dx}x^{7x} = 7x \cdot x^{7x-1}$.

   Power rule does not apply, since variable in exponent. have to use logarithmic differentiation, and get different answer

   A. True
   B. False

2. True or False: If f is differentiable and $f'(x) > 0$ for all x in $(-\infty, \infty)$, then $f(x)$ has no local extreme points.

   A. True
   B. False

   If $f(x)$ had an extreme point, we would have $f'(x) = 0$ or $f'(x)$ DNE at that x-value

3. (2 pts) True or False: If $f(x)$ is a differentiable function on $(-\infty, \infty)$ and $f'(5) = 0$, then f has a local maximum or local minimum at $x = 5$.

   A. True
   B. False

   Consider $y = (x-5)^3$

4. (2 pts) True or False: If $f(x)$ is a polynomial, then $f(x)$ has an absolute minimum value on the interval $[1, 2]$.

   A. True
   B. False

   $f(x)$ is continuous, so the Extreme Value Thm guarantees an abs. max & min on a closed interval

5. (2 pts) True or False: The function $f(x) = \frac{1}{x}$ has an absolute maximum value on the interval $[-1, 1]$.

   A. True
   B. False

   as $x \to 0^+$ $f(x) \to \infty$

   Note $f(x)$ is not continuous on $[-1,1]$ so the Extreme Value Thm doesn't apply

6. (5 pts) A rectangle is morphing in such a way that its length is INCREASING at a rate of 3 cm per minute and the width is DECREASING at a rate of 2 cm per minute. At what rate will the area of the rectangle be changing when the length and width are both 5 cm?

$A = l \cdot w$

$\frac{dA}{dt} = l\frac{dw}{dt} + \frac{dl}{dt}w$

$= 5 \cdot (-2) + 3 \cdot 5$

$= 5$

A. the area will be decreasing at a rate of 15 cm^2 per minute
B. the area will be decreasing at a rate of 10 cm^2 per minute
C. the area will be increasing at a rate of 5 cm^2 per minute
D. the area will be increasing at a rate of 15 cm^2 per minute
E. the area will be increasing at a rate of 25 cm^2 per minute

7. (5 pts) Suppose the function $g(t) = \frac{t^3}{3} - 3t^2 + 8t + 5$ represents the motion of a particle going up and down at time t for $0 \le t \le 10$. Find the intervals for which the particle is speeding up.

$g'(t) = t^2 - 6t + 8$

Set $g'(t) = 0 \Rightarrow t^2 - 6t + 8 = 0$

$\Rightarrow (t-2)(t-4) = 0 \Rightarrow t = 2$ or $t = 4$

$g''(t) = 2t - 6$ Set $g''(t) = 0 \Rightarrow 2t - 6 = 0 \Rightarrow t = 3$

$g''(t)$ ⊖ ⊕

$g'(t)$ ⊕ ⊖ ⊕

0 2 3 4 10

both negative when $x \in (2,3)$
both positive when $x \in (4,10)$

A. $(3, 10)$
B. $(4, 10)$
C. $(0, 2) \cup (3, 4)$
D. $(0, 2) \cup (4, 10)$
E. $(2, 3) \cup (4, 10)$

$g(t) = \frac{t^3}{3} - 3t^2 + 8t + 5$

8. (5 pts) For the same function ~~$g(t) = \frac{t^3}{3} - 4t^2 + 15t + 2$~~ that represents the motion of a particle going up and down at time t. Which expression represents the total distance traveled by the particle between $t = 0$ and $t = 10$?

$g'(t) = 0$ at $t = 2, t = 4$

A. $|g'(10) - g'(0)|$
B. $|g'(10) - g'(3)| + |g'(3) - g'(0)|$
C. $|g(10) - g(0)|$
D. $|g(10) - g(4)| + |g(4) - g(2)| + |g(2) - g(0)|$
E. Cannot be determined from this information.

9. (5 pts) Find the derivative of the function. $r(z) = \ln(\arctan(Mz))$

Note r is a function of z, so M is a constant

A. $\dfrac{M}{Mz(1+M^2z^2)}$

B. $\dfrac{M}{\arctan(Mz)}$

C. $\dfrac{M}{(1+z^2)\arctan(Mz)}$

D. $\dfrac{M}{(1+M^2z^2)\arctan(Mz)}$

E. $\dfrac{M+z}{(1+M^2z^2)\arctan(Mz)}$

$$\frac{dr}{dz} = \frac{1}{\arctan(Mz)} \cdot \frac{1}{1+(Mz)^2} \cdot M$$

by the chain Rule

10. (5 pts) Find the derivative of the function. $p(x) = 2^{\sec(5x)}$

A. $5\ln 2 \cdot 2^{\sec(5x)}$

B. $5\ln 2 \cdot 2^{\sec(5x)\tan(5x)}$

C. $5\ln 2 \cdot 2^{\sec(5x)} \sec(5x)\tan(5x)$

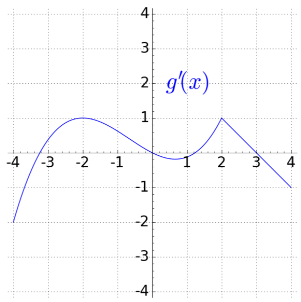
D. $5\ln 2 \cdot 2^{\sec(5x)} \sec(x)\tan(x)$

E. $5\ln 2 \cdot 2^{\sec(5x)\tan(5x)}[\sec(5x)\tan^2(5x) + \sec^3(5x)]$

$$p'(x) = \ln 2 \cdot 2^{\sec(5x)} \cdot \sec(5x)\tan(5x) \cdot 5$$

by the chain Rule

A function $g(x)$ is defined on the interval $[-4, 4]$. The DERIVATIVE $g'(x)$ is graphed below.



11. (5 pts) Using the graph of the DERIVATIVE $g'(x)$ above, find the interval(s) on which the function $g(x)$ is increasing.

g(x) is increasing where $g'(x) > 0$

   A. $(-1, 2)$
   B. $(-2, -1) \cup (-0.5, 2)$
   C. $(-3.4, 0) \cup (1.2, 3)$
   D. $(-4, -2) \cup (0.5, 2)$
   E. Cannot be determined from this information.

12. (5 pts) Using the graph of the DERIVATIVE $g'(x)$ above, find all critical points of the function $g(x)$ on the interval $(-4, 4)$.

critical #'s are where $g'(x) = 0$ or $g'(x)$ DNE

   A. $x = 2$
   B. $x = -2, 0.5$
   C. $x = -2, 0.5, 2$
   D. $x = -3.4, 0, 1.2, 3$
   E. Cannot be determined from this information.

13. (5 pts) Using the graph of the DERIVATIVE $g'(x)$ above, find the interval(s) on which the function $g(x)$ is negative.

   A. $(-1, 2)$
   B. $(-2, -1) \cup (-0.5, 2)$
   C. $(-3.4, 0) \cup (1.2, 3)$
   D. $(-4, -2) \cup (0.5, 2)$
   E. Cannot be determined from this information.

Cannot determine if position is positive or negative from derivative, since g(x) & g(x)+1000 & g(x)−2000 all have the same derivative, even though g(x)+1000 is likely >0 & g(x)−2000 is likely <0

14. The diameter of a circle is measured to be 80 meters with a possible error of 0.3 meters. Use differentials to approximate the **error** and the **relative error** in the calculated area of the circle.

$A = \pi r^2$ $\qquad r = \frac{D}{2}$

A = area
r = radius
D = diameter

$A = \pi \left(\frac{D}{2}\right)^2$

$A = \frac{\pi D^2}{4}$ ✓✓

Alternatively, can use
$dA = 2\pi r\, dr$ ✓✓✓✓✓
& $dr = \frac{0.3}{2} = 0.15$ ✓
& $r = 40$ ✓

$dA = A'(D)\, dD$ ✓✓

$dA = \frac{2\pi D}{4}\, dD$ ✓✓

$dA = \frac{\pi D}{2}\, dD$ $\qquad$ when $D = 80$ & $dD = 0.3$

$dA = \pi \cdot \frac{80}{2} \cdot 0.3 = \pi \cdot 40 \cdot 0.3 = 12\pi$ ✓✓

$\frac{dA}{A} = \frac{12\pi}{\frac{\pi \cdot 80^2}{4}} = \frac{48}{6400} = \frac{6}{800} = \frac{3}{400} = 0.0075$ ✓✓

error = 12π m^2	relative error = 0.0075

Use logarithmic differentiation
Since there is a variable in the base
and in the exponent.

15. Find the derivative of $f(x) = (4 + 7x)^{3x}$.

$$y = (4+7x)^{3x}$$

$$\ln y = \ln(4+7x)^{3x}$$

$$\ln y = 3x \ln(4+7x)$$

$$\frac{1}{y}\frac{dy}{dx} = 3\ln(4+7x) + 3x \cdot \frac{1}{4+7x} \cdot 7$$

$$\frac{dy}{dx} = y\left[3\ln(4+7x) + \frac{21x}{4+7x}\right]$$

$$\frac{dy}{dx} = (4+7x)^{3x}\left[3\ln(4+7x) + \frac{21x}{4+7x}\right]$$

use implicit differentiation

16. Find the slope of the tangent line to the curve

$$\sin(x - y) = y^2 + xy$$

at the point $(\pi, 0)$.

$$\frac{d}{dx}\sin(x-y) = \frac{d}{dx}(y^2+xy)$$

$$\cos(x-y)\left[1-\frac{dy}{dx}\right] = 2y\frac{dy}{dx} + x\frac{dy}{dx} + y$$

$$\cos(x-y) - \cos(x-y)\frac{dy}{dx} = 2y\frac{dy}{dx} + x\frac{dy}{dx} + y$$

$$\cos(x-y) - y = 2y\frac{dy}{dx} + x\frac{dy}{dx} + \cos(x-y)\frac{dy}{dx}$$

$$\cos(x-y) - y = \left[2y + x + \cos(x-y)\right]\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{\cos(x-y)-y}{2y+x+\cos(x-y)}$$

when $x=\pi, y=0$

$$\frac{dy}{dx} = \frac{\cos(\pi-0)-0}{2\cdot 0+\pi+\cos(\pi-0)}$$

$$= \frac{-1}{\pi-1} = \frac{1}{1-\pi}$$

slope = $\frac{1}{1-\pi}$

17. Find the absolute maximum and minimum values of $f(x) = xe^{-2x}$ on the interval $[0, 5]$. As needed, you can use the following facts:

- $f'(x) = e^{-2x} - 2xe^{-2x}$
- $f''(x) = 4xe^{-2x} - 4e^{-2x}$
- $e \approx 3$

Write your final answer as an exact value, not an approximation or decimal.

Find critical numbers:

$f'(x) = 0$

$e^{-2x} - 2x e^{-2x} = 0$ ✓

$e^{-2x}(1-2x) = 0$

e^{-2x} never 0

$1 - 2x = 0$

$x = \frac{1}{2}$ ✓✓

Find values of fn on endpts & critical #'s

x	$f(x) = xe^{-2x}$
✓ 0	$\boxed{0}$ abs min
✓ $\frac{1}{2}$	$\frac{1}{2}e^{-2\cdot\frac{1}{2}} = \frac{1}{2}e^{-1} = \boxed{\frac{1}{2e}}$ abs max
✓ 5	$5e^{-10} = \frac{5}{e^{10}}$ ← very small but >0

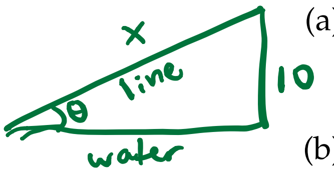
abs MAX value = $\frac{1}{2e}$ ✓✓

abs min value = 0 ✓✓

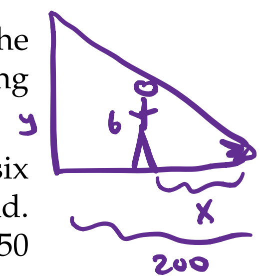
(a)

18. Choose ONE of the following problems to work. You only need to complete one.

(b)



(a) A fish is reeled in at a rate of 2 feet per second from a point 10 feet above the water. At what rate is the angle between the line and the water changing when there is a total of 20 feet of line out?



(b) A flood lamp is installed on the ground 200 feet from a vertical wall. A six foot tall man is walking towards the wall at the rate of 10 feet per second. How fast is the tip of his shadow moving down the wall when he is 50 feet from the wall?

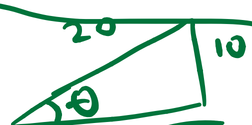
$\frac{dx}{dt} = -2$ ft/sec want $\frac{d\theta}{dt}$

$\sin\theta = \frac{10}{x}$

$\frac{d}{dt}(\sin\theta) = \frac{d}{dt}\left(\frac{10}{x}\right)$

$\cos\theta \frac{d\theta}{dt} = -\frac{10}{x^2}\frac{dx}{dt}$

When $x = 20$,



$\sqrt{20^2 - 10^2} = \sqrt{300} = 10\sqrt{3}$

$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{10\sqrt{3}}{20} = \frac{\sqrt{3}}{2}$

$\frac{\sqrt{3}}{2}\frac{d\theta}{dt} = \frac{-10}{20^2}(-2)$

$\Rightarrow \frac{d\theta}{dt} = \frac{20}{20^2} \cdot \frac{2}{\sqrt{3}} = \frac{1}{10\sqrt{3}}$

$= \frac{\sqrt{3}}{30}$ radians/second

It is increasing at a rate of $\frac{\sqrt{3}}{30}$ radians/sec.

$\frac{dx}{dt} = 10$ want $\frac{dy}{dt}$

$\frac{y}{200} = \frac{6}{x}$

$\frac{1}{200}\frac{dy}{dt} = -\frac{6}{x^2}\frac{dx}{dt}$

$\frac{1}{200}\frac{dy}{dt} = -\frac{6}{150^2} \cdot 10$

$\Rightarrow \frac{dy}{dt} = -\frac{6 \cdot 200 \cdot 10}{22500}$

$\Rightarrow \frac{dy}{dt} = \frac{-120}{225} = \frac{-24}{45} = \frac{-8}{15}$ ft/sec

It is moving down the wall at a rate of $\frac{8}{15}$ ft/sec.