

**Math 231: Test 1A**  
**Spring 2016**  
**Instructor: Linda Green**

---

key

- Calculators are NOT allowed.
- Please code true/false and multiple choice answers on a scantron. These are questions 1 - 13.
- Since you have test version A, please code the Section field on the scantron as 111111 (all 1's).
- No partial credit for multiple choice / no work needs to be shown.
- For short answer questions, you must show work for full and partial credit if specified.
- Sign the honor pledge below after completing the exam.

First and last name .....

PID .....

UNC Email .....

Honor Pledge: I have neither given nor received unauthorized help on this exam.

Signature: .....

1. (2 pts) True or False: If  $f$  and  $g$  are continuous at  $x = a$  with  $g(a) \neq 0$ , then  $h(x) = \frac{f(x) + \sin(x)}{g(x)}$  is continuous at  $x = a$ .

A. True

B. False

Combinations of continuous functions are continuous where defined

2. (2 pts) True or False: For  $f(x) = \frac{1}{x}$ , since  $f(-2) = -\frac{1}{2}$  and  $f(2) = \frac{1}{2}$ , there must be a number  $c$  between  $-2$  and  $2$  such that  $f(c) = 0$ .

A. True

B. False

$\frac{1}{x}$  is never 0  
The Intermediate Value Theorem doesn't apply since  $f$  is not continuous.

3. (2 pts) True or False: If  $f(x)$  is continuous at  $x = 2$  then  $f(x)$  is differentiable at  $x = 2$ .

A. True

B. False

For example,  $f(x) = |x|$  is continuous but not differentiable

4. (2 pts) True or False: If  $f(c)$  exists and  $f$  is continuous, then  $\lim_{x \rightarrow c} f(x)$  exists.

A. True

B. False

$f$  continuous means  $\lim_{x \rightarrow c} f(x) = f(c)$

5. (2 pts) True or False: If  $\lim_{x \rightarrow c} f(x) = 0$  and  $\lim_{x \rightarrow c} g(x) = 0$  then  $\lim_{x \rightarrow c} \frac{f(x)}{g(x)}$  is either  $0$ ,  $\infty$ ,  $-\infty$ , or DNE.

A. True

B. False

For example  $f(x) = x$   
 $g(x) = x$   
 $c = 0$   
 $\lim_{x \rightarrow 0} \frac{x}{x} = 1$

6. (5 pts) Find  $\lim_{x \rightarrow 4} \frac{x+7}{|x-4|}$

A. 1

B.  $\frac{7}{4}$

C.  $-\infty$

D.  $\infty$

E. DNE

$x+7 \rightarrow 11$   
 $|x-4| \rightarrow 0$  through positive numbers

$\frac{11}{\text{tiny}^+} \rightarrow \infty$

7. (5 pts) Find  $\lim_{x \rightarrow 5} \frac{\sqrt{5} - \sqrt{x}}{(x-5)}$

A. 0

B.  $\frac{1}{2}$

C.  $-\frac{\sqrt{5}}{10}$

D.  $2\sqrt{5}$

E. DNE

$$\lim_{x \rightarrow 5} \frac{\sqrt{5} - \sqrt{x}}{x-5} \cdot \frac{\sqrt{5} + \sqrt{x}}{\sqrt{5} + \sqrt{x}}$$

$$= \lim_{x \rightarrow 5} \frac{5-x}{(x-5)(\sqrt{5} + \sqrt{x})}$$

$$= \lim_{x \rightarrow 5} \frac{-1}{\sqrt{5} + \sqrt{x}} = \frac{-1}{\sqrt{5} + \sqrt{5}} = \frac{-1}{2\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = -\frac{\sqrt{5}}{10}$$

8. (5 pts) Find  $\lim_{x \rightarrow 0} \frac{x}{\tan(5x)}$

A. 0

B.  $\frac{1}{5}$

C. 1

D. 5

E. DNE

$$\lim_{x \rightarrow 0} \frac{x}{\frac{\sin(5x)}{\cos(5x)}} = \lim_{x \rightarrow 0} \frac{\cos(5x)x}{\sin 5x}$$

$$= \lim_{x \rightarrow 0} \frac{\cos(5x) \cdot x}{\sin 5x} \cdot \frac{5x}{5x} = \lim_{x \rightarrow 0} \frac{\cos(5x) \cdot \cancel{5x} \cdot \frac{x}{\cancel{5x}}}{1 \cdot 1 \cdot \frac{1}{5}} = \frac{1}{5}$$

9. (5 pts) Find  $\lim_{t \rightarrow \infty} \frac{5}{t^2 e^t}$

A. 0

B. 5

C.  $-\infty$

D.  $\infty$

E. DNE

or  $t \rightarrow \infty$   $t^2 \rightarrow \infty$  and  $e^t \rightarrow \infty$  so  $t^2 e^t \rightarrow \infty$

$$\text{so } \frac{5}{t^2 e^t} \rightarrow 0$$

10. (5 pts) Find  $\lim_{x \rightarrow \infty} (\sin(x) + 5)$

A. 0

B. 5

C. 6

D.  $\infty$

E. DNE

$\sin(x) + 5$  oscillates through all values between 4 and 6 as  $x \rightarrow \infty$

so limit DNE

11. (5 pts) If  $f'(-4) = 3$  and  $f(-4) = 10$ , find the equation of the tangent line to  $y = f(x)$  at  $x = -4$ .

- A.  $y = 3x - 4$   
B.  $y = 3x + \frac{10}{3}$   
C.  $y = 3x + 10$   
 D.  $y = 3x + 22$

$$\begin{aligned} m &= 3 \quad \text{point } (-4, 10) \\ y &= 3x + b \\ 10 &= 3(-4) + b \Rightarrow b = 22 \\ y &= 3x + 22 \end{aligned}$$

- E. There is not enough information to determine the equation of the tangent line.

12. (5 pts) The function  $p(t) = t^3 + 2$  represents the depth of a submarine in meters at time  $t$  minutes, as the submarine descends directly down. Find the submarine's average velocity between  $t = 1$  and  $t = 3$ .

- A. 12 meters per minute  
 B. 13 meters per minute  
C. 15 meters per minute  
D. 27 meters per minute  
E. 29 meters per minute

$$\begin{aligned} \frac{p(3) - p(1)}{3 - 1} &= \frac{(3^3 + 2) - (1^3 + 2)}{2} \\ &= \frac{27 - 1}{2} = 13 \end{aligned}$$

13. (5 pts) As in the last problem, the function  $p(t) = t^3 + 2$  represents the depth of a submarine in meters at time  $t$  minutes, as the submarine descends directly down. Find the submarine's velocity at exactly  $t = 3$ .

- A. 12 meters per minute  
B. 13 meters per minute  
C. 15 meters per minute  
 D. 27 meters per minute  
E. 29 meters per minute

$$\begin{aligned} p'(t) &= 3t^2 \\ p'(3) &= 27 \end{aligned}$$

14. (8 pts) Each of the following limits represents the derivative of some function  $f(x)$  at some number  $a$ . State a formula for  $f$  and the value of  $a$ . You DO NOT need to find the limits or show work.

(a)  $\lim_{x \rightarrow -1} \frac{e^x - \frac{1}{e}}{x + 1}$

|                          |
|--------------------------|
| $f(x) = e^x$<br>$a = -1$ |
|--------------------------|

2 pts  
2 pts

(b)  $\lim_{h \rightarrow 0} \frac{(5+h)^2 - 25}{h}$

|                         |
|-------------------------|
| $f(x) = x^2$<br>$a = 5$ |
|-------------------------|

2 pts  
2 pts

15. (10 pts)  $y = x^3 \cos(x)$ . Find  $\frac{d^2y}{dx^2}$  at  $x = \pi$ . Show work.

2 pts for deriv of  $\cos x$   
2 pts for deriv of  $x^3$   
2 pts for other stuff  
✓✓✓✓✓

$$\begin{aligned} \frac{dy}{dx} &= x^3(-\sin(x)) + 3x^2 \cos x \\ &= -x^3 \sin x + 3x^2 \cos x \end{aligned}$$

$$\begin{aligned} \frac{d^2y}{dx^2} &= -x^3 \cos x - 3x^2 \sin x + 3x^2(-\sin x) + 6x \cos x \\ &= -x^3 \cos x - 6x^2 \sin x + 6x \cos x \end{aligned}$$

$$\begin{aligned} \frac{d^2y}{dx^2} \Big|_{x=\pi} &= -\pi^3 \cos \pi - 6\pi^2 \sin \pi + 6\pi \cos \pi \\ &= \boxed{\pi^3 - 6\pi} \end{aligned}$$

16. (6 pts) Find the vertical and horizontal asymptotes (if any) of  $f(x) = \frac{6x-12}{x^2-4}$ .

$$f(x) = \frac{6(x-2)}{(x-2)(x+2)}$$

V. A. at  $x = -2$

$$\lim_{x \rightarrow \infty} \frac{6x-12}{x^2-4} = 0$$

$$\lim_{x \rightarrow -\infty} \frac{6x-12}{x^2-4} = 0$$

H. A. at  $y = 0$

2 pts for  $\frac{\text{not}}{x=2}$  including

Horizontal Asymptote(s):  $y = 0$

2 pts

Vertical Asymptote(s):  $x = -2$

2 pts

17. (6 pts) Is it possible to find a value for  $c$  that makes the function  $g(x)$  below continuous? If so, find it. If not, indicate why not. Show work to justify your answer.

$$g(x) = \begin{cases} 3, & \text{if } x \leq 1 \\ cx^2, & \text{if } 1 < x < 2 \\ 10x - 5, & \text{if } 2 \leq x \end{cases}$$

If  $g(x)$  is continuous,

need  $\lim_{x \rightarrow 1^-} g(x) = \lim_{x \rightarrow 1^+} g(x)$

$$\Rightarrow \lim_{x \rightarrow 1^-} 3 = \lim_{x \rightarrow 1^+} cx^2$$

$$\Rightarrow 3 = c$$

and  $\lim_{x \rightarrow 2^-} g(x) = \lim_{x \rightarrow 2^+} g(x)$

$$\Rightarrow \lim_{x \rightarrow 2^-} cx^2 = \lim_{x \rightarrow 2^+} 10x - 5$$

$$\Rightarrow 4c = 15$$

$$\Rightarrow c = \frac{15}{4}$$

There is no value of  $c$  that works, since need  $c = 3$  and  $c = \frac{15}{4}$  at the same time.

18. (10 pts)  $y = \frac{xe^x}{3x^5 - \tan(x)}$ . Find  $y'$ . Show work. DO NOT simplify.

$$y' = \frac{(3x^5 - \tan x) \frac{d}{dx} xe^x - xe^x \frac{d}{dx} (3x^5 - \tan x)}{(3x^5 - \tan x)^2}$$

$$= \frac{(3x^5 - \tan x)(xe^x + e^x) - xe^x(15x^4 - \sec^2 x)}{(3x^5 - \tan x)^2}$$

4 pts  $\frac{d}{dx}(xe^x)$   
2 pts each for

19. (4 pts) If  $\lim_{x \rightarrow 2} \frac{A(x+B)}{x^2 - 6x + 8} = 5$ , find A and B.

$$\lim_{x \rightarrow 2} \frac{A(x+B)}{(x-2)(x-4)} = 5$$

Since  $\lim_{x \rightarrow 2} (x-2)(x-4) = 0$ , need  $\lim_{x \rightarrow 2} A(x+B) = 0$  also

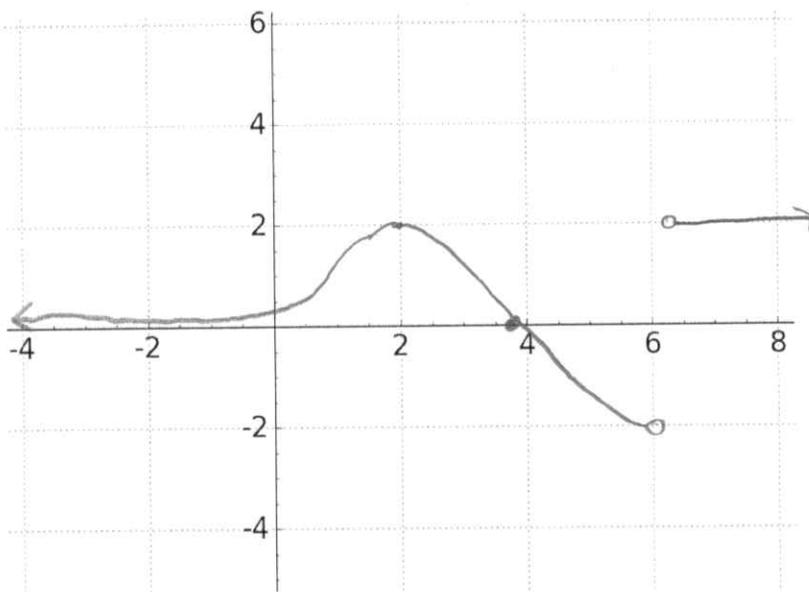
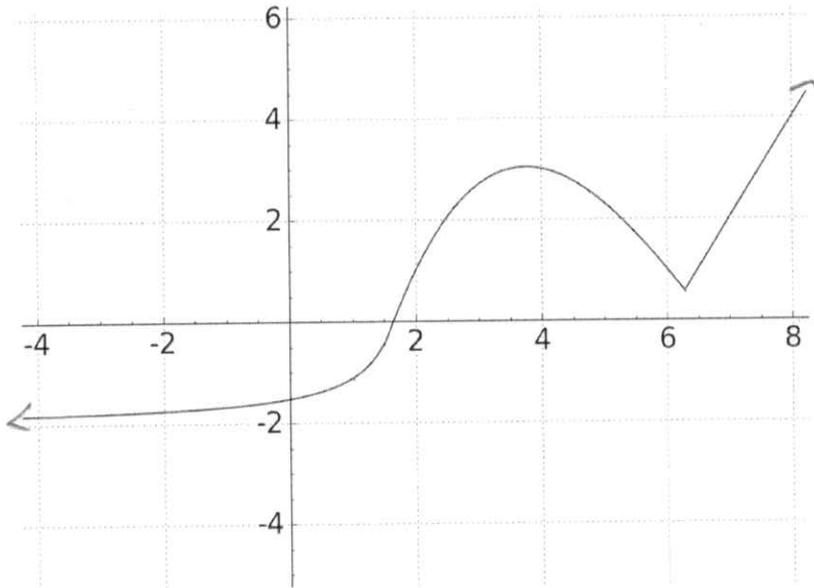
$$\Rightarrow 2+B=0 \Rightarrow B=-2$$

now  $\lim_{x \rightarrow 2} \frac{A(x-2)}{(x-2)(x-4)} = 5 \Rightarrow \frac{A}{2-4} = 5 \Rightarrow A = -10$

A = -10

B = -2

20. (6 pts) The graph of  $y = f(x)$  is drawn below. On the blank axes, sketch a graph of  $y = f'(x)$ .



1 pt flat line  
1 pt height of  $\approx 2$

1 pt jump w/  
open circles

1 pt approach 0  
when  $x \rightarrow -4$

1 pt 0 at  $x \approx 4$

1 pt pos when  $x < 4$   
neg when  $4 < x < 6$



11. (5 pts) The function  $p(t) = t^3 + 2$  represents the depth of a submarine in meters at time  $t$  minutes, as the submarine descends directly down. Find the submarine's velocity at exactly  $t = 3$ .

- A. 12 meters per minute
- B. 13 meters per minute
- C. 15 meters per minute
- D. 27 meters per minute
- E. 29 meters per minute

$$p'(t) = 3t^2$$
$$p'(3) = 27$$

12. (5 pts) As in the last problem, the function  $p(t) = t^3 + 2$  represents the depth of a submarine in meters at time  $t$  minutes, as the submarine descends directly down. Find the submarine's average velocity between  $t = 1$  and  $t = 3$ .

- A. 12 meters per minute
- B. 13 meters per minute
- C. 15 meters per minute
- D. 27 meters per minute
- E. 29 meters per minute

$$\frac{p(3) - p(1)}{3 - 1} = \frac{(3^3 + 2) - (1^3 + 2)}{3 - 1}$$
$$= \frac{29 - 3}{2} = 13$$

13. (5 pts) If the tangent line to  $y = f(x)$  at  $x = -4$  is  $y = 3x + 21$ , then what is the value of the function  $f(-4)$ ?

- A. 3
- B. 7
- C. 9
- D. 21

$$f(-4) = 3(-4) + 21 = -12 + 21 = 9$$

- E. There is not enough information to determine  $f(-4)$ .

14. (8 pts) Each of the following limits represents the derivative of some function  $f(x)$  at some number  $a$ . State a formula for  $f$  and the value of  $a$ . You DO NOT need to find the limits or show work.

(a)  $\lim_{h \rightarrow 0} \frac{(3+h)^3 - 27}{h}$

|                         |
|-------------------------|
| $f(x) = x^3$<br>$a = 3$ |
|-------------------------|

2 pts  
2 pts

(b)  $\lim_{x \rightarrow -1} \frac{e^x - \frac{1}{e}}{x + 1}$

|                          |
|--------------------------|
| $f(x) = e^x$<br>$a = -1$ |
|--------------------------|

2 pts  
2 pts

15. (10 pts)  $y = x^3 \cos(x)$ . Find  $\frac{d^2y}{dx^2}$  at  $x = \pi$ . Show work.

$$\begin{aligned} \frac{dy}{dx} &= x^3(-\sin x) + 3x^2 \cos x \\ &= -x^3 \sin x + 3x^2 \cos x \end{aligned}$$

2 pts for deriv of  $\cos x$   
2 pts for deriv of  $x^3$   
2 pts for other stuff  
✓✓✓✓✓

$$\begin{aligned} \frac{d^2y}{dx^2} &= -x^3 \cos x - 3x^2 \sin x + 3x^2(-\sin x) + 6x \cos x \\ &= -x^3 \cos x - 6x^2 \sin x + 6x \cos x \end{aligned}$$

$$\left. \frac{d^2y}{dx^2} \right|_{x=\pi} = -\pi^3 \cos \pi - 6\pi^2 \sin \pi + 6\pi \cos \pi = \boxed{\pi^3 - 6\pi}$$

16. (6 pts) Find the vertical and horizontal asymptotes (if any) of  $f(x) = \frac{4x+8}{x^2-4}$ .

$$f(x) = \frac{4(x+2)}{(x+2)(x-2)}$$

V.A. at  $x=2$   
but not at  $x=-2$

$$\text{Since } \lim_{x \rightarrow -2} \frac{4(x+2)}{(x+2)(x-2)} = \frac{4}{-2-2} = -1$$

To find H.A., take

$$\lim_{x \rightarrow \infty} \frac{4x+8}{x^2-4} = 0$$

$$\lim_{x \rightarrow -\infty} \frac{4x-8}{x^2-4} = 0$$

2 pts for not including  $x=-2$

Horizontal Asymptote(s):  $y = 0$

2 pts

Vertical Asymptote(s):  $x = 2$

2 pts

17. (6 pts) Is it possible to find a value for  $c$  that makes the function  $g(x)$  below continuous? If so, find it. If not, indicate why not. Show work to justify your answer.

$$g(x) = \begin{cases} 3, & \text{if } x \leq 1 \\ cx^2, & \text{if } 1 < x < 2 \\ 10x - 5, & \text{if } 2 \leq x \end{cases}$$

For  $g$  to be continuous, need

$$\lim_{x \rightarrow 1^-} g(x) = \lim_{x \rightarrow 1^+} g(x)$$

$$\Rightarrow \lim_{x \rightarrow 1^-} 3 = \lim_{x \rightarrow 1^+} cx^2$$

$$\Rightarrow 3 = c \quad \checkmark$$

$$\text{AND } \lim_{x \rightarrow 2^-} g(x) = \lim_{x \rightarrow 2^+} g(x)$$

$$\Rightarrow \lim_{x \rightarrow 2^-} cx^2 = \lim_{x \rightarrow 2^+} 10x - 5$$

$$\Rightarrow 4c = 15 \Rightarrow c = \frac{15}{4} \quad \checkmark$$

It is not possible, since  $3 \neq \frac{15}{4}$

$3 \neq \frac{15}{4}$   $\checkmark$

18. (10 pts)  $y = \frac{xe^x}{3x^5 - \tan(x)}$ . Find  $y'$ . Show work. DO NOT simplify.

$$y' = \frac{(3x^5 - \tan x) \frac{d}{dx} x e^x - x e^x \frac{d}{dx} (3x^5 - \tan x)}{(3x^5 - \tan(x))^2}$$

$$= \frac{(3x^5 - \tan x)(x e^x + e^x) - x e^x (15x^4 - \sec^2 x)}{(3x^5 - \tan(x))^2}$$

19. (4 pts) If  $\lim_{x \rightarrow 4} \frac{A(x+B)}{x^2 - 6x + 8} = 9$ , find A and B.

$$\frac{A(x+B)}{x^2 - 6x + 8} = \frac{A(x+B)}{(x-2)(x-4)}$$

Since  $\lim_{x \rightarrow 4} (x-2)(x-4) = 0$

to get  $\lim_{x \rightarrow 4} \frac{A(x+B)}{x^2 - 6x + 8}$  to be any thing other than  $\infty, -\infty, \text{DNE}$

need  $\lim_{x \rightarrow 4} A(x+B) = 0$  also. So  $A(4+B) = 0 \Rightarrow B = -4$

Now  $\lim_{x \rightarrow 4} \frac{A(x-4)}{(x-2)(x-4)} = \frac{A}{4-2} = \frac{A}{2} = 9 \Rightarrow A = 18$

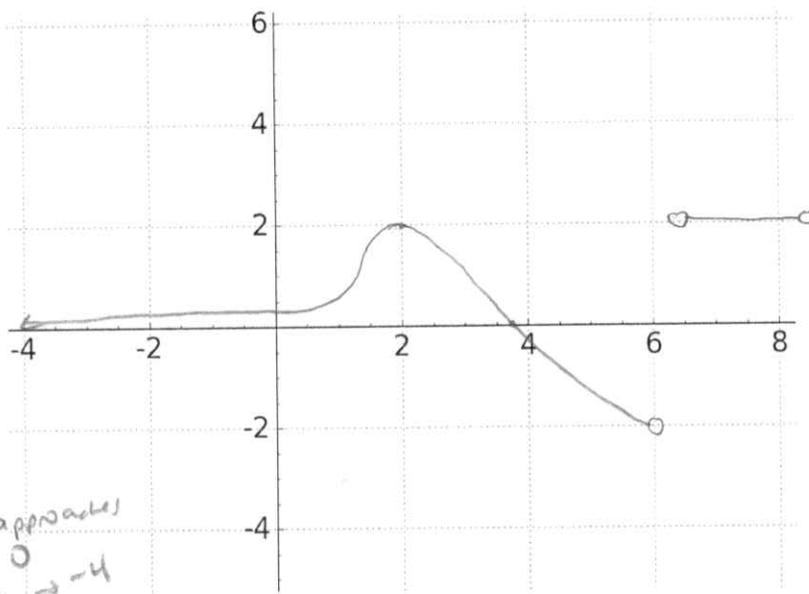
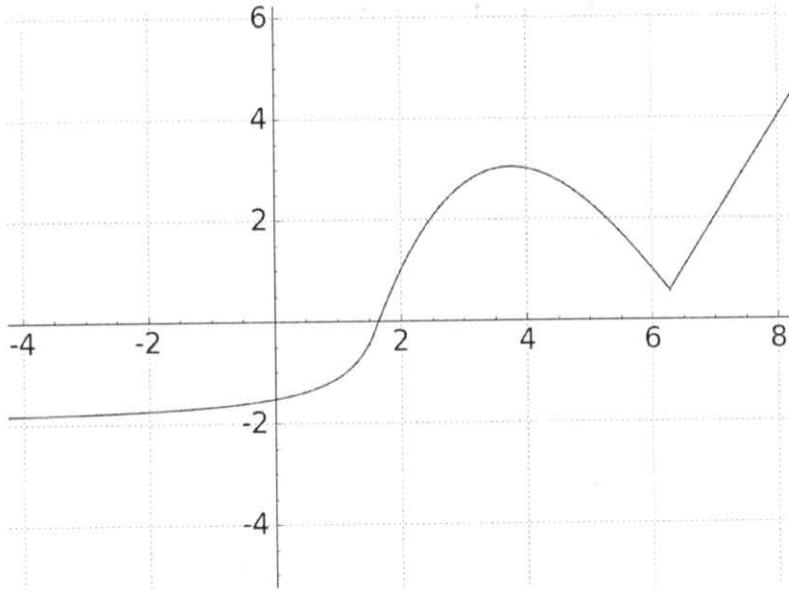
$$A = 18$$

✓✓

$$B = -4$$

✓✓

20. (6 pts) The graph of  $y = f(x)$  is drawn below. On the blank axes, sketch a graph of  $y = f'(x)$ .



1 pt approaches 0 as  $x \rightarrow -4$

1 pt flat line  
1 pt height of  $\approx 2$

1 pt jump at  $x = 6$  with open hole

1 pt  
pos. slope  $x < 4$   
neg. slope  $4 < x < 6$

1 pt 0 at  $x \approx 4$