Math 233: Test 1B
Spring 2014
Instructor: Linda Green

1. Calculators are allowed.
2. You must show work for full and partial credit, except where otherwise noted.
3. Give exact values instead of decimal approximations, except where otherwise noted.
4. Sign the honor pledge after completing the exam.

   I have neither given nor received unauthorized help on this exam.

   Key

5 min

1. (20 pts) Consider the three points $A = (0,2,3)$, $B = (-2,1,4)$, and $C = (1,5,5)$.

   (a) Find parametric equations for the line through A and B.

   (b) Find the area of the triangle with vertices A, B, and C.

   (c) Find an equation for the plane P through A, B, and C.

   (d) Find the angle that the plane P makes with the x-y plane. Give your answer in degrees to the nearest tenth.

2 pts correct vector
1 pt correct point
2 pts form of eqn

a) $\vec{AB} = \langle -2, -1, 1 \rangle$

point $A = (0,2,3)$

$$\boxed{\begin{aligned} x &= 0 - 2t \\ y &= 2 - t \\ z &= 3 + t \end{aligned}}$$

2 pts vectors
2 pts cross product
1 pt $\frac{1}{2}\sqrt{\ }$

b) $\vec{AB} = \langle -2, -1, 1 \rangle$
$\vec{AC} = \langle 1, 3, 2 \rangle$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2 & -1 & 1 \\ 1 & 3 & 2 \end{vmatrix} = \begin{vmatrix} -1 & 1 \\ 3 & 2 \end{vmatrix} \vec{i} - \begin{vmatrix} -2 & 1 \\ 1 & 2 \end{vmatrix} \vec{j} + \begin{vmatrix} -2 & 1 \\ 1 & 3 \end{vmatrix} \vec{k}$$

$$= -5\vec{i} + 5\vec{j} - 5\vec{k}$$

area $= \frac{1}{2} \| \vec{AB} \times \vec{AC} \| = \frac{1}{2}\sqrt{75} = \frac{5}{2}\sqrt{3}$

2 pts correct normal
2 pts correct pt
1 point form of eqn

c) $\vec{n} = \vec{AB} \times \vec{AC} = -5\vec{i} + 5\vec{j} - 5\vec{k}$

point $A = (0, 2, 3)$

eqn: $-5(x-0) + 5(y-2) - 5(z-3)$

or $-5x + 5y - 5z = -5$

or $-x + y - z = -1$

1 pt normal to P
1 pt normal to xy-plane
1 pt $\vec{a} \cdot \vec{b} = \|\vec{a}\| \|\vec{b}\| \cos\theta$
2 pt computations

d) $\vec{n} = \langle -1, 1, -1 \rangle$ $\quad$ $\vec{n}$ for xy plane is $\langle 0, 0, 1 \rangle$

$$\cos\theta = \frac{\langle -1,1,-1 \rangle \cdot \langle 0,0,1 \rangle}{\| \langle -1,1,-1 \rangle \circ \langle 0,0,1 \rangle \|}$$

$$= \frac{-1}{\sqrt{3}\sqrt{1}} = \frac{-1}{\sqrt{3}}$$

$$\theta = \cos^{-1}\left(-\frac{1}{\sqrt{3}}\right) = 125.3^\circ$$

2. (10 pts) Find the tangent vector and the unit tangent vector for the curve

$$\vec{r}(t) = < 3t^2, \sin(t) - t\cos(t), \cos(t) + t\sin(t) >$$

at the point $(3\pi^2, \pi, -1)$.

$\vec{r}\,'(t) = \langle 6t, \cos t - \cos t + t\sin t, -\sin t + \sin t + t\cos t \rangle$

$= \langle 6t, t\sin t, t\cos t \rangle$

at $(3\pi^2, \pi, -1)$ $\quad 3t^2 = 3\pi^2$ so $t = \pi$

$\vec{r}\,'(\pi) = \langle 6\pi, \pi\sin\pi, \pi\cos\pi \rangle$

$= \langle 6\pi, 0, -\pi \rangle$

unit tangent vector

$$\frac{\vec{r}\,'(t)}{\|\vec{r}\,'(t)\|} = \frac{\langle 6\pi, 0, -\pi \rangle}{\sqrt{(6\pi)^2 + 0^2 + \pi^2}} = \frac{\langle 6\pi, 0, -\pi \rangle}{\pi\sqrt{37}}$$

$$= \left\langle \frac{6}{\sqrt{37}}, 0, -\frac{1}{\sqrt{37}} \right\rangle$$

11:48

3. (12 pts) Which of the following represent lines in $\mathbb{R}^3$? Circle all correct answers. No work needed.

(a) All (x, y, z) such that $x = -2t$, $y = 3t + 1$, and $z = 4t + 6$ for $t \in \mathbb{R}$. Line

(b) $< t, t^2, t^3 >$ for $t \in \mathbb{R}$ not a line (a curve)

(c) All (x, y, z) such that $5x + 4y + 3z = 2$ and $x + 4y - 7z = 17$ line

(d) All (x, y, z) such that $5y + 4 = 8z - 7$ not a line (a plane)

(e) All (x, y, z) such that $5(x - 3) - 6(y + 2) + 3z = 0$ not a line (a plane)

(f) $\vec{r}(t) = e^t\vec{i} + \vec{j} + 2e^t\vec{k}$ for $t \in \mathbb{R}$. piece of a line - actually a ray

either yes or no was accepted here b/c it is a ray not exactly a line

11:48

4. Consider the line $x+5=\frac{y}{2}=\frac{z}{3}-1$ and the plane $-x+2y-z=7$.

(a) (5 pts) Verify that the line and the plane do not intersect.

(b) (10 pts) Find the (shortest) distance between the line and the plane. Hint: First, find any point Q on the line and any point P on the plane. Next, find the vector $\vec{PQ}$. Next, project this vector onto the normal vector for the plane.

On actual test, this question was phrased differently and divided into three parts.

a) (I) $x+5=\frac{y}{2}$

II $x+5=\frac{z}{3}-1$

III $-x+2y-z=7$

solve

I $y=2x+10$

II $z=3x+18$

5 pts

substitute into III

III $-x+2(2x+10)-(3x+18)=7$

III $\cancel{-x}+\cancel{4x}-\cancel{3x}+20-18=7$

$2=7 \Rightarrow \Leftarrow$

no solution

b) Point P on plane $(0,0,-7)$

Point Q on line: $(-5,0,3)$

6 pts

$\vec{PQ}=\langle -5,0,10\rangle$

$\vec{n}=\langle -1,2,-1\rangle$

$\text{comp}_{\vec{n}}\vec{PQ}=\frac{\vec{n}\circ\vec{PQ}}{\|\vec{n}\|}=\frac{5+0-10}{\sqrt{6}}=\frac{-5}{\sqrt{6}}$

distance $=\left|\frac{-5}{\sqrt{6}}\right|=\frac{5}{\sqrt{6}}$

or use formula: for distance btwn point and plane

$$D=\frac{|-1(-5)+2(0)-1(3)-7|}{\sqrt{1^2+2^2+1^2}}=\frac{5}{\sqrt{6}}$$

4 pts

5. (12 pts) Match the equations with the graphs. No work needed.

1 min

(a) $x^2 - y^2 + z^2 = 0$ $\rightarrow$ 6

(b) $-x^2 + y + z^2 = 1$ $\rightarrow$ 4

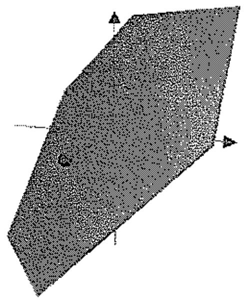
(c) $x - y + z = 1$ $\rightarrow$ 1

(d) $-x^2 + y^2 + z^2 = 1$ $\rightarrow$ 5

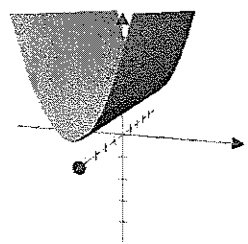
(e) $-y^2 + z = 1$ $\rightarrow$ 2

(f) $-x^2 - y^2 + z = 1$ $\rightarrow$ 3

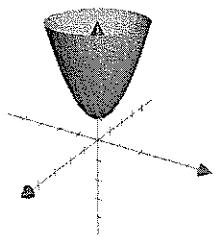
1. c



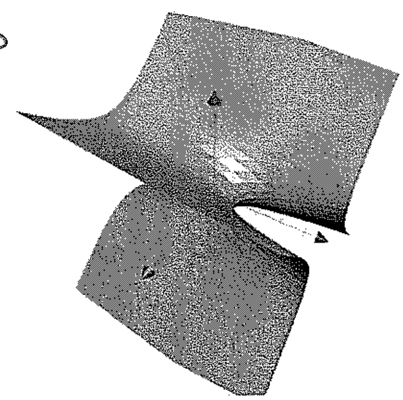
2. e



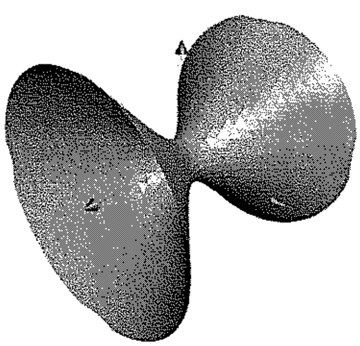
3. f



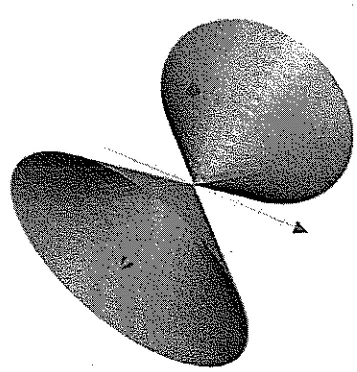
4. b



5. d



6. a



6. (10 pts) True or false. True means always true. False means sometimes or always false. No work needed.

   (a) T F $\vec{b} \circ (\vec{a} \times \vec{b}) = 0$

   (b) T F $\vec{a} \circ \vec{a} = 0$

   (c) T F The cross product of two unit vectors is a unit vector.

   (d) T F The dot product of two unit vectors is a unit vector.

   (e) T F $\|\vec{a} \times \vec{b}\| \leq \|\vec{a}\|\, \|\vec{b}\|$

answer is false here; $\|\vec{v} \times \vec{w}\| = \|\vec{v}\|\,\|\vec{w}\| \cos\theta$ for unit vectors $\vec{v}$ & $\vec{w}$; if $\theta \neq 90^\circ$, then $\|\vec{v} \times \vec{w}\| < 1$ so not unit vector

12:14