

Math 233: Test 3A

Fall 2017

Instructor: Linda Green

- Please code your name and PID on your scantron.
- Since you have test version A, please code your scantron PAGE NUMBER as 1.
- If you have a registration appointment during the exam, please bring your test and laptop up to the front and fill in these times:
Registration started at: _____ Registration competed at: _____
- Calculators are NOT allowed.
- For short answer questions, you must show work for full and partial credit. Please put all work to be graded on the test, not on scrap paper, since only pages with QR codes will be graded.
- No partial credit for multiple choice / no work needs to be shown.
- Sign the honor pledge below after completing the exam.

First and last name

PID

UNC Email

Honor Pledge: I have neither given nor received unauthorized help on this exam.

Signature:

$$\begin{aligned}\cos(30^\circ) &= \frac{\sqrt{3}}{2} \\ \cos(45^\circ) &= \frac{\sqrt{2}}{2} \\ \cos(60^\circ) &= \frac{1}{2}\end{aligned}$$

$$\begin{aligned}\sin(30^\circ) &= \frac{1}{2} \\ \sin(45^\circ) &= \frac{\sqrt{2}}{2} \\ \sin(60^\circ) &= \frac{\sqrt{3}}{2}\end{aligned}$$

$$\sqrt{2} \approx 1.41$$

$$\sqrt{3} \approx 1.73$$

1. (2 pts) True or False: It is possible to rewrite the integral as follows:

$$\int_{\theta=0}^{2\pi} \int_{r=0}^{\cos \theta} r^2 \sin \theta \, dr \, d\theta = \int_{\theta=0}^{2\pi} \sin \theta \, d\theta \int_{r=0}^{\cos \theta} r^2 \, dr$$

- A. True
B. False

2. (2 pts) True or False: It is possible to change the order of integration as follows:

$$\int_{\theta=0}^{\pi} \int_{\phi=0}^{\pi/3} \int_{\rho=1}^2 \rho^3 \cos \phi \sin \phi \sin \theta \, d\rho \, d\phi \, d\theta = \int_{\rho=1}^2 \int_{\phi=0}^{\pi/3} \int_{\theta=0}^{\pi} \rho^3 \cos \phi \sin \phi \sin \theta \, d\theta \, d\phi \, d\rho$$

- A. True
B. False

3. (2 pts) True or False: The value of the integral $\int_{x=1}^3 \int_{y=4}^7 \int_{z=2}^7 \sin^2(z^2y + x) \, dz \, dy \, dx$ is between 0 and 30.

- A. True
B. False

4. (2 pts) True or False: $\int_{\theta=-\pi/4}^{\pi/4} \int_{r=0}^5 \int_{z=0}^{\ln(r)} e^{r^2} \sin \theta \, dz \, dr \, d\theta = 0$. Hint: you can figure this out without doing any tricky integration.

- A. True
B. False

5. (2 pts) True or False: The vector field $\vec{F}(x, y) = \langle 2y, 2y \rangle$ is a conservative vector field.

- A. True
B. False

6. (4 pts) The table below contains data about the values of a continuous function $f(x, y)$. Use a Riemann sum with 2 intervals in the x -direction and 2 in the y -direction to estimate $\int_{x=0}^8 \int_{y=2}^6 f(x, y) \, dy \, dx$. Use midpoints for sample points.

$x \backslash y$	2	3	4	5	6
0	0	-3	-8	-15	-24
1	5	3	0	-6	-14
2	6	5	2	-3	-11
3	7	7	4	-1	-9
4	9	8	5	0	-7
5	10	9	6	1	-6
6	11	10	7	2	-4
7	12	11	8	4	-3
8	13	12	9	5	-2

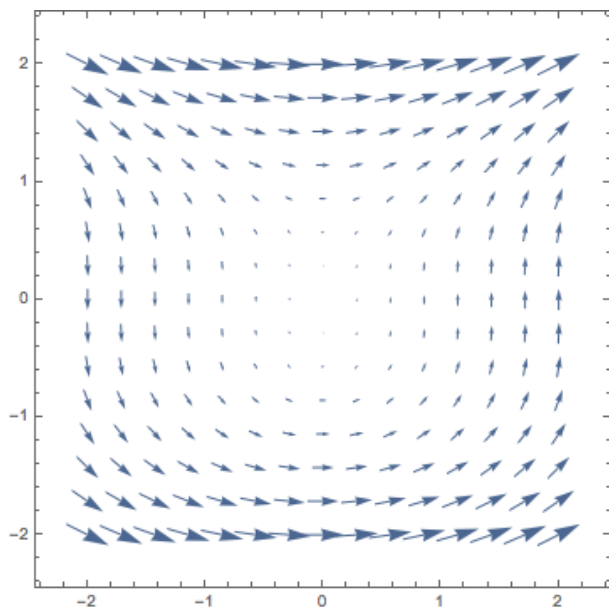
- A. 14
B. 48
C. 56
D. 112
E. 448

7. (4 pts) Which of the following integrals is equivalent to

$$\int_{x=-1}^1 \int_{y=0}^{\sqrt{1-x^2}} \int_{z=0}^{4-x^2-y^2} xy \, dz \, dy \, dx$$

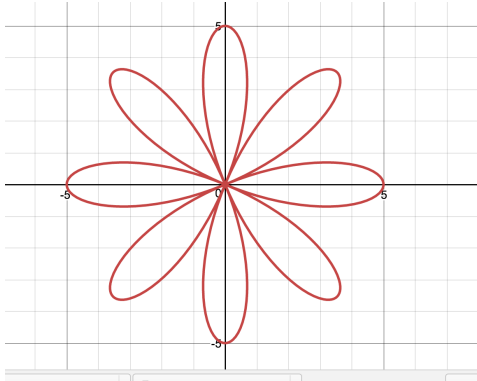
- A. $\int_{\theta=0}^{2\pi} \int_{r=0}^{\sqrt{1-r^2 \cos^2(\theta)}} \int_{z=0}^{4-r^2} r^2 \sin \theta \cos \theta \, dz \, dr \, d\theta$
- B. $\int_{\theta=0}^{\pi} \int_{r=0}^{\sqrt{1-r^2 \cos^2(\theta)}} \int_{z=0}^{4-r^2} r^3 \sin \theta \cos \theta \, dz \, dr \, d\theta$
- C. $\int_{\theta=0}^{\pi} \int_{r=0}^1 \int_{z=0}^{4-r^2} r^2 \sin \theta \cos \theta \, dz \, dr \, d\theta$
- D. $\int_{\theta=0}^{\pi} \int_{r=0}^1 \int_{z=0}^{4-r^2} r^3 \sin \theta \cos \theta \, dz \, dr \, d\theta$
- E. $\int_{\theta=0}^{\pi} \int_{r=0}^1 \int_{z=0}^4 r^3 \sin \theta \cos \theta \, dz \, dr \, d\theta$

8. (4 pts) Match the vector field with the equation.



- A. $\vec{F}(x, y) = \langle y^2, y \rangle$
- B. $\vec{F}(x, y) = \langle x^2, y \rangle$
- C. $\vec{F}(x, y) = \langle -x^2, y \rangle$
- D. $\vec{F}(x, y) = \langle y^2, x \rangle$
- E. $\vec{F}(x, y) = \langle y^2, -x \rangle$

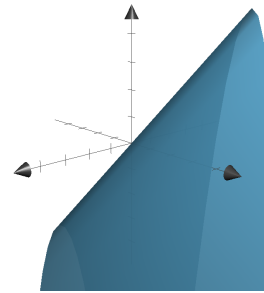
9. (12 pts) A flower is given by the following equation: $r = 5 \cos(4\theta)$. Find the area of the flower.



10. (6 pts) Fill in the bounds of integration for the integral

$$\int \int \int_E y \, dV$$

where E is the solid region in the first octant bounded by $z = y - 2x^2$ and $y = 18$. Use the specified order of integration. DO NOT evaluate the integral. (The first octant is where $x \geq 0, y \geq 0, z \geq 0$.)



$$\int \int \int y \, dz \, dy \, dx$$

11. (14 pts) Consider the solid that lies above the cone $z^2 = x^2 + y^2$ and below the sphere $x^2 + y^2 + z^2 = 6$. Suppose the density of the solid is given by $\rho(x, y, z) = z\sqrt{x^2 + y^2 + z^2}$. Find the mass of the region.

12. (10 pts) Evaluate the integral.

$$\int_{x=0}^1 \int_{y=\sqrt{x}}^1 e^{y^3} dy dx$$