

Math 233: Test 3A

Fall 2017

Instructor: Linda Green

- Please code your name and PID on your scantron.
- Since you have test version A, please code your scantron PAGE NUMBER as 1.
- If you have a registration appointment during the exam, please bring your test and laptop up to the front and fill in these times:
Registration started at: _____ Registration completed at: _____
- Calculators are NOT allowed.
- For short answer questions, you must show work for full and partial credit. Please put all work to be graded on the test, not on scrap paper, since only pages with QR codes will be graded.
- No partial credit for multiple choice / no work needs to be shown.
- Sign the honor pledge below after completing the exam.

First and last name *Key*

PID

UNC Email

Honor Pledge: I have neither given nor received unauthorized help on this exam.

Signature:

$$\cos(30^\circ) = \frac{\sqrt{3}}{2}$$

$$\sin(30^\circ) = \frac{1}{2}$$

$$\cos(45^\circ) = \frac{\sqrt{2}}{2}$$

$$\sin(45^\circ) = \frac{\sqrt{2}}{2}$$

$$\cos(60^\circ) = \frac{1}{2}$$

$$\sin(60^\circ) = \frac{\sqrt{3}}{2}$$

$$\sqrt{2} \approx 1.41$$

$$\sqrt{3} \approx 1.73$$

1. (2 pts) True or False: It is possible to rewrite the integral as follows:

fixed answer will be a number

$$\int_{\theta=0}^{2\pi} \int_{r=0}^{\cos \theta} r^2 \sin \theta \, dr \, d\theta = \int_{\theta=0}^{2\pi} \sin \theta \, d\theta \int_{r=0}^{\cos \theta} r^2 \, dr \quad \text{← final answer will have } \theta's \text{ in it}$$

Can't split up integral this way because θ *'s are in the bounds of integration*

- A. True B. False

(A)

B. False

Note all bounds of integration are numbers.

B. False

This is Fubini's theorem

3. (2 pts) True or False: The value of the integral $\int_{x=1}^3 \int_{y=4}^7 \int_{z=2}^7 \sin^2(z^2y + x) \, dz \, dy \, dx$ is between 0 and 30.

(A) True

B. False

$$0 \leq \sin^2(z^2y + x) \leq 1, \text{ so } 0 \leq \iiint_E \sin^2(z^2y + x) \, dV \leq 1 \cdot \text{vol}(E) = 30$$

4. (2 pts) True or False: $\int_{\theta=-\pi/4}^{\pi/4} \int_{r=0}^5 \int_{z=0}^{\ln(r)} e^{r^2} \sin \theta \, dz \, dr \, d\theta = 0$. Hint: you can figure this out without doing any tricky integration.

(A) True

B. False

$$\int_{\theta=-\pi/4}^{\pi/4} \sin \theta \, d\theta \int_{r=0}^5 \int_{z=0}^{\ln(r)} e^{r^2} \, dz \, dr$$

Can rewrite as

$$\int_{\theta=-\pi/4}^{\pi/4} \sin \theta \, d\theta = -\cos \theta \Big|_{-\pi/4}^{\pi/4} = \left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right) = 0$$

5. (2 pts) True or False: The vector field $\vec{F}(x, y) = < 2y, 2y >$ is a conservative vector field.

A. True

(B) False

If $\vec{F} = \nabla f$ then would have

6. (4 pts) The table below contains data about the values of a continuous function $f(x, y)$. Use a Riemann sum with 2 intervals in the x-direction and 2 in the y-direction to estimate $\int_{x=0}^8 \int_{y=2}^6 f(x, y) \, dy \, dx$. Use midpoints for sample points.

$x \backslash y$	2	3	4	5	6
0	0	-3	-8	-15	-24
1	5	3	0	-6	-14
2	6	5	2	-3	-11
3	7	7	4	-1	-9
4	9	8	5	0	-7
5	10	9	6	1	-6
6	11	10	7	2	-4
7	12	11	8	4	-3
8	13	12	9	5	-2

A. 14

B. 48

C. 56

(D) 112

E. 448

*area of each subrectangle
is $4 \cdot 2 = 8$*

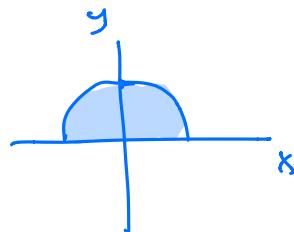
$$\left[f(2,3) + f(2,5) + f(6,3) + f(6,5) \right] \Delta A$$

$$= (5 - 3 + 10 + 2) \cdot 8 = 14 \cdot 8 = 112$$

7. (4 pts) Which of the following integrals is equivalent to

$$\int_{x=-1}^1 \int_{y=0}^{\sqrt{1-x^2}} \int_{z=0}^{4-x^2-y^2} xy \, dz \, dy \, dx$$

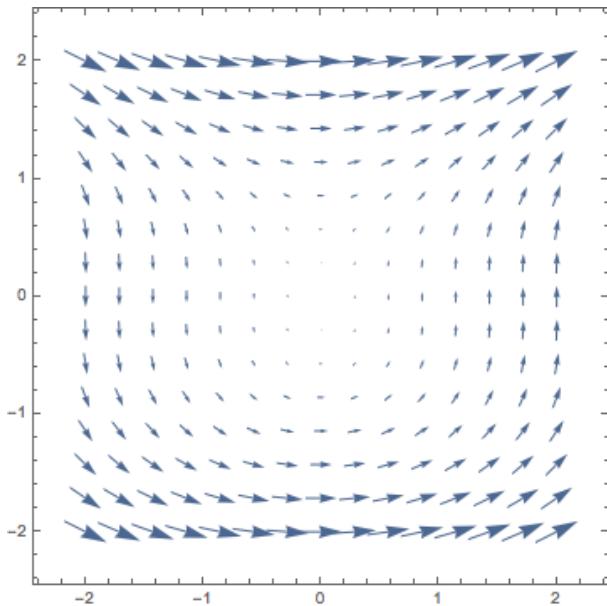
- A. $\int_{\theta=0}^{2\pi} \int_{r=0}^{\sqrt{1-r^2 \cos^2(\theta)}} \int_{z=0}^{4-r^2} r^2 \sin \theta \cos \theta \, dz \, dr \, d\theta$
- B. $\int_{\theta=0}^{\pi} \int_{r=0}^{\sqrt{1-r^2 \cos^2(\theta)}} \int_{z=0}^{4-r^2} r^3 \sin \theta \cos \theta \, dz \, dr \, d\theta$
- C. $\int_{\theta=0}^{\pi} \int_{r=0}^1 \int_{z=0}^{4-r^2} r^2 \sin \theta \cos \theta \, dz \, dr \, d\theta$
- D. $\int_{\theta=0}^{\pi} \int_{r=0}^1 \int_{z=0}^{4-r^2} r^3 \sin \theta \cos \theta \, dz \, dr \, d\theta$
- E. $\int_{\theta=0}^{\pi} \int_{r=0}^1 \int_{z=0}^4 r^3 \sin \theta \cos \theta \, dz \, dr \, d\theta$



$$\begin{aligned} z &= 4 - x^2 - y^2 \\ \Leftrightarrow z &= 4 - r^2 \end{aligned}$$

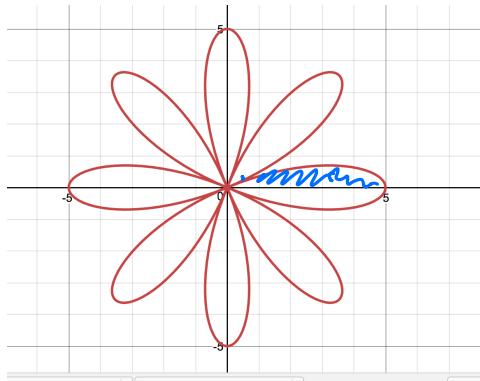
$$\begin{aligned} x &= r \cos \theta & y &= r \sin \theta \\ dv &= r \, dr \, d\theta \end{aligned}$$

8. (4 pts) Match the vector field with the equation.



- A. $\vec{F}(x, y) = \langle y^2, y \rangle \leftarrow \text{doesn't depend on } x$
- B. $\vec{F}(x, y) = \langle x^2, y \rangle \leftarrow \text{for } x \text{ large, } y \text{ zero, pointing right}$
- C. $\vec{F}(x, y) = \langle -x^2, y \rangle \leftarrow \text{always pointing left since } -x^2 < 0$
- D. $\vec{F}(x, y) = \langle y^2, x \rangle \leftarrow \text{for } x \text{ positive, pointing up, for } y \text{ large pointing right}$
- E. $\vec{F}(x, y) = \langle y^2, -x \rangle \leftarrow \text{for } x \text{ positive, pointing down}$

9. (12 pts) A flower is given by the following equation: $r = 5 \cos(4\theta)$. Find the area of the flower.



$$\text{area} = 8 \cdot \text{area(petal)}$$

$$\text{for petal } 0 \leq r \leq 5 \cos 4\theta$$

to find bounds on θ ,
find θ values where $r=0$

$$0 = 5 \cos(4\theta) \Rightarrow \cos(4\theta) = 0$$

$$\Rightarrow 4\theta = \pm \frac{\pi}{2}$$

$$\Rightarrow \theta = \pm \frac{\pi}{8}$$

$$\begin{aligned} \text{area of petal} &= \int_{\theta=-\pi/8}^{\pi/8} \left[5 \cos 4\theta \right]_0^r r dr d\theta \\ &= \int_{\theta=-\pi/8}^{\pi/8} \left[\frac{r^2}{2} \right]_0^{5 \cos 4\theta} d\theta \\ &= \int_{\theta=-\pi/8}^{\pi/8} \frac{25 \cos^2 4\theta}{2} d\theta \\ &= \frac{25}{2} \int_{\theta=-\pi/8}^{\pi/8} \left(\frac{1}{2} + \frac{1}{2} \cos 8\theta \right) d\theta \\ &= \frac{25}{2} \left[\frac{1}{2}\theta + \frac{1}{16} \sin 8\theta \right]_{-\pi/8}^{\pi/8} \\ &= \frac{25}{2} \left[\left(\frac{1}{2} \cdot \frac{\pi}{8} + \frac{1}{16} \sin \pi \right) - \left(\frac{1}{2} \cdot (-\pi/8) + \frac{1}{16} \sin(-\pi) \right) \right] \\ &= \frac{25}{2} \cdot \frac{\pi}{8} = \frac{25\pi}{16} \end{aligned}$$

7 pts set-up

* 4 pts bounds

+ 1 pt integrating 1

+ 1 pt $r dr d\theta$

+ 1 pt multiplying by
8 or 16 appropriately

4 pts integration

+ 1 pt integrating r

* 1 pt trig id

* 1 pt integrating each term
after trig id

$$\text{area of flower} = 8 \cdot \frac{25\pi}{16} = \boxed{\frac{25\pi}{2}}$$

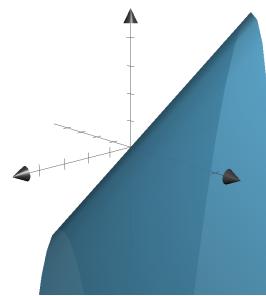
1 pt final answer

10. (6 pts) Fill in the bounds of integration for the integral

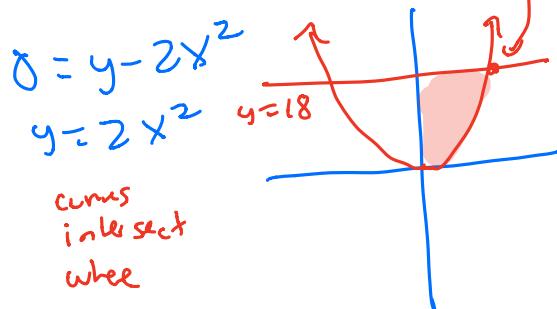
$$\int \int \int_E y \, dV$$

where E is the solid region in the first octant bounded by $z = y - 2x^2$ and $y = 18$. Use the specified order of integration. DO NOT evaluate the integral. (The first octant is where $x \geq 0, y \geq 0, z \geq 0$.)

1 pt each blank
no partial credit



$$\int_{x=0}^3 \int_{y=2x^2}^{18} \int_{z=0}^{y-2x^2} y \, dz \, dy \, dx$$



$\partial = y - 2x^2$
 $y = 2x^2$
 $y = 18$
 $y = 2x^2$
 $\Rightarrow 18 = 2x^2$
 $\Rightarrow x^2 = 9$
 $\Rightarrow x = \pm 3$

curve intersects where
 surface intersects
 x-y plane & to find
 boundary of curve of
 projection onto
 xy plane

upper half of

11. (14 pts) Consider the solid that lies above the cone $z^2 = x^2 + y^2$ and below the sphere $x^2 + y^2 + z^2 = 6$. Suppose the density of the solid is given by $\rho(x, y, z) = z \sqrt{x^2 + y^2 + z^2}$. Find the mass of the region.

$$\iiint \rho(x, y, z) dV$$

using spherical coords

$$\int_{\theta=0}^{2\pi} \int_{\phi=0}^{\pi/4} \int_{\rho=0}^{\sqrt{6}} \rho^2 \cos\phi \rho^2 \sin\phi d\rho d\phi d\theta$$

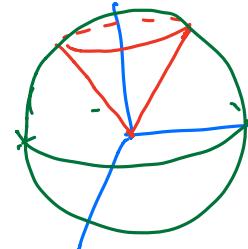
$$= \int_{\theta=0}^{2\pi} \int_{\phi=0}^{\pi/4} \int_{\rho=0}^{\sqrt{6}} \rho^4 \cos\phi \sin\phi d\rho d\phi d\theta$$

$$= \int_{\theta=0}^{2\pi} d\theta \int_{\phi=0}^{\pi/4} \cos\phi \sin\phi d\phi \int_{\rho=0}^{\sqrt{6}} \rho^4 d\rho$$

$$= \Theta \Big|_0^{2\pi} - \frac{\cos^2 \phi}{2} \Big|_0^{\pi/4} \int_0^{\sqrt{6}} \rho^5 d\rho$$

$$= 2\pi \left(-\frac{(\frac{\sqrt{2}}{2})^2}{2} + \frac{1^2}{2} \right) \frac{(\sqrt{6})^5}{5}$$

$$= 2\pi \left(-\frac{1}{4} + \frac{1}{2} \right) \frac{36\sqrt{6}}{5} = 2\pi \cdot \frac{1}{4} \cdot \frac{36\sqrt{6}}{5} = 18\pi\sqrt{6}/5$$



$$\begin{aligned}\rho(x, y, z) &= z \sqrt{x^2 + y^2 + z^2} \\ &= \rho \cos\phi \rho \\ &= \rho^2 \cos\phi\end{aligned}$$

$$dV = \rho^2 \sin\phi d\rho d\phi d\theta$$

$$0 \leq \rho \leq \sqrt{6}$$

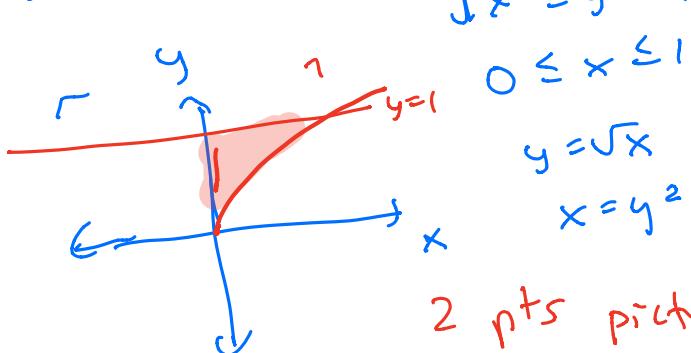
$$0 \leq \phi \leq \pi/4$$

$$0 \leq \theta \leq 2\pi$$

12. (10 pts) Evaluate the integral.

$$\int_{x=0}^1 \int_{y=\sqrt{x}}^1 e^{y^3} dy dx$$

Since $\int e^{y^3} dy$ cannot be computed directly,
try changing the order of integration



$$\sqrt{x} \leq y \leq 1$$

$$0 \leq x \leq 1$$

$$y = \sqrt{x}$$

$$x = y^2$$

2 pts picture

(no pt taken off if
rest is correct w/o
picture)

4 pts bonds of
integration

4 pts computing

$$\int_{y=0}^1 \int_{x=0}^{y^2} e^{y^3} dx dy$$

$$= \int_{y=0}^1 e^{y^3} x \Big|_0^{y^2} dy$$

$$= \int_{y=0}^1 \left(e^{y^3} y^2 - 0 \right) dy$$

$$u = y^3 \quad du = 3y^2 dy$$

$$\frac{1}{3} du = y^2 dy$$

$$= \int_0^1 e^u \frac{1}{3} du$$

$$y=0 \Rightarrow u=0 \\ y=1 \Rightarrow u=1$$

$$= \frac{1}{3} e^u \Big|_0^1 = \frac{1}{3} e^1 - \frac{1}{3} e^0 = \boxed{\frac{1}{3} e - \frac{1}{3}}$$

