Math 233: Test 1A
Fall 2017

- Please code your name and PID on your scantron.
- Since you have test version A, please code your scantron PAGE NUMBER as 1.
- Calculators are NOT allowed.
- For short answer questions, you must show work for full and partial credit.
- No partial credit for multiple choice / no work needs to be shown.
- Sign the honor pledge below after completing the exam.

First and last name ..... Key .....

PID .....

UNC Email .....

Honor Pledge: I have neither given nor received unauthorized help on this exam.

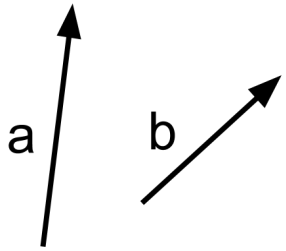
Signature: .....

$\cos(30°) = \frac{\sqrt{3}}{2}$ $\sin(30°) = \frac{1}{2}$
$\cos(45°) = \frac{\sqrt{2}}{2}$ $\sin(45°) = \frac{\sqrt{2}}{2}$
$\cos(60°) = \frac{1}{2}$ $\sin(60°) = \frac{\sqrt{3}}{2}$

1. (2 pts) True or False: For any two distinct lines in space, there is a unique plane that contains them.
   A. True
   B. False

Lines could be skew

2. (2 pts) True or False: The vector $\vec{a}\times\vec{b}$ points up out of the page. (Assume the paper is held horizontally.)



Right hand rule

   A. True
   B. False

3. (2 pts) True or False: The dot product of two unit vectors is 1.
   A. True
   B. False

For example, vectors could be $\perp$ with dot product 0.

4. (2 pts) True or False: $(\vec{a}\times\vec{b})\circ\vec{a}=0$
   A. True
   B. False

$\vec{a}\times\vec{b} \perp \vec{a}$ and perpendicular vectors have a dot product of 0

5. (2 pts) True or False: True or False: If $\vec{a}\times\vec{b}$ is parallel to $\vec{d}$ and $\vec{a}\times\vec{c}$ is parallel to $\vec{d}$, then $a\times(\vec{b}+\vec{c})$ is parallel to $\vec{d}$. (Assume none of the vectors are $\vec{0}$.)
   A. True
   B. False

$\vec{a}\times(\vec{b}+\vec{c}) = \underbrace{\vec{a}\times\vec{b}}_{\parallel \text{ to } \vec{d}} + \vec{a}\times\vec{c}$ $\parallel$ to $\vec{d}$

Sum of vectors that are $\parallel$ to $\vec{d}$ is $\parallel$ to $\vec{d}$

6. (4 pts) Which of the following expressions represents the same line as $x-3=\frac{y}{2}=1-z$?
   A. $x+2y-z=2$ — not a line
   B. $x=3t+1, y=2, z=t-1$ — wrong direction vector
   C. $x=3t, y=6t, z=-3t$
   D. $x=2t+6, y=4t+6, z=-2t-2$
   E. $x=2t+1, y=4t-2, z=-2t+5$

$x-3=t$ $\quad x=3+t$

$\frac{y}{2}=t$ $\quad y=2t$

$1-z=t$ $\quad z=1-t$

line has direction vector $\langle 1,2,-1\rangle$ and contains the point $(3,0,1)$

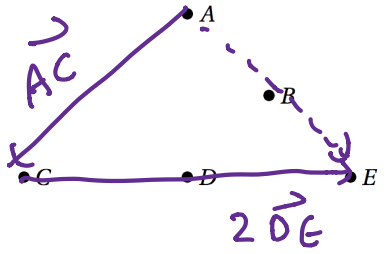
C: if $(x,y,z)=(3,0,1)$ then $t=1$ & $t=0$ ⇒ ⨉

D: if $(x,y,z)=(3,0,1)$ then $t=-3/2$ works!

E: if $(x,y,z)=(3,0,1)$ then $t=1$ & $t=\frac{1}{2}$ ⇒ ⨉

7. (4 pts) Consider the points in the plane shown below. Assume that points that look equally spaced are equally spaced. For example $\overline{CD}$ is the same length as $\overline{DE}$. Which vector is the same as $\vec{AC} + 2\vec{DE}$?

   A. $\vec{DA}$
   B. $\vec{DB}$
   C. $\vec{DE}$
   D. $\vec{DF}$
   E. $\vec{DG}$

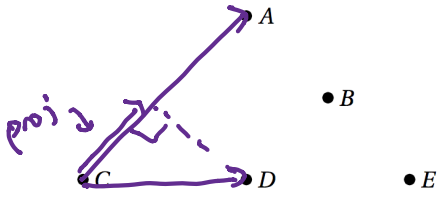


F G

$\vec{AE}$ which equals $\vec{DG}$

8. (4 pts) Consider the points in the plane shown below. Which vector is the same as $proj_{\vec{CA}}\vec{CD}$? (Careful, we want $proj_{\vec{CA}}\vec{CD}$ not $proj_{\vec{CD}}\vec{CA}$.)

   A. $\vec{DA}$
   B. $\vec{DB}$
   C. $\vec{DE}$
   D. $\vec{DF}$
   E. $\vec{DG}$

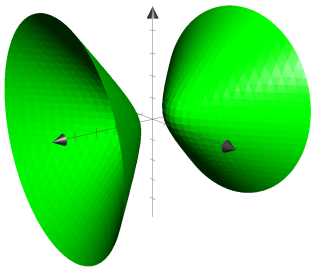


F G

projection is in same direction as $\vec{CA}$

it is the same length & direction as $\vec{DB}$

9. (4 pts) Which equation corresponds the surface drawn?



In graph, intersecting with planes of the form $x =$ constant gives circles for x big, nothing for x small of 0

Intersecting with planes of the form $z =$ constant gives hyperbolas

A. $x^2 + y^2 + z^2 = 1$ ← when $x=0$, have circle, not nothing X

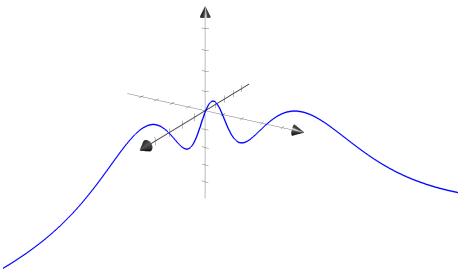
B. $x^2 - y^2 - z^2 = -1$ ← when $x=0$ have circle, not nothing X

C. $x^2 + y^2 - z^2 = -1$ ← when $x=0$ have hyperbola, not nothing X

D. $x^2 - y^2 - z^2 = 1$

E. $x - y^2 - z^2 = 1$ ← when $z=$ constant have parabola not hyperbola X

10. (4 pts) Which equation corresponds to the curve drawn?



Note x, y are unbounded when projecting onto xz or yz plane, looks vaguely like a stretched out sine or cosine curve

A. $\vec{r}(t) = < \cos(t), \cos(3t), \cos(5t) >$ ← here, x,y are bounded X

B. $\vec{r}(t) = < e^t, e^{-t}, \cos(4t) >$

C. $\vec{r}(t) = < t, 3+t, 2-t >$ ← this is a straight line

D. $\vec{r}(t) = < t, \cos(t), \sin(t) >$ ← this is a helix and y is bounded

11. (4 pts) A boat is being pulled with a force of 200 Newtons along a canal that runs due East by a rope that is at an angle of 30° North of due East. What is the work done (in Newtons) in moving the boat 30 meters?

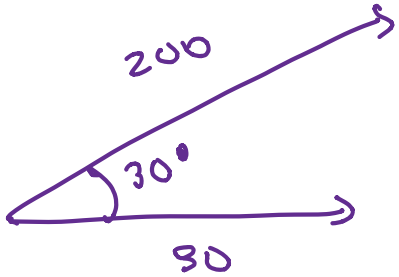
A. 1500

B. 3000

C. $3000\sqrt{3}$

D. 6000

E. $6000\sqrt{3}$



$W = \vec{F} \cdot \vec{d} = 200 \cdot 30 \cos 30^\circ = 6000 \cdot \frac{\sqrt{3}}{2}$

$= 3000\sqrt{3}$

12. (a) (6 pts) Find the point of intersection of the two curves

$$\vec{r_1}(t) = < t, 7 - t, t^2 >$$

$$\vec{r_2}(s) = < \frac{9}{s} + 1, s, 5s + 1 >$$

(b) (10 pts) Write an expression for the exact angle at which the curves intersect at that point. (If you did not complete part (a), you can write your answer for part (b) with some unknowns in it.)

(a) $t = \frac{9}{s} + 1$ $\quad 7 - t = s$ $\quad t^2 = 5s + 1$ ✓✓

$t = \frac{9}{7-t} + 1 \Rightarrow t(7-t) = 9 + 7 - t$

$\Rightarrow 7t - t^2 = 16 - t \Rightarrow t^2 - 8t + 16 = 0$

$(t-4)^2 = 0 \Rightarrow t = 4$ ✓✓ $\Rightarrow s = 3$

$(4, 3, 16)$

(b) $\vec{r_1}'(t) = \langle 1, -1, 2t \rangle$ ✓✓ $\quad \vec{r_1}'(4) = \langle 1, -1, 8 \rangle$ ✓

$\vec{r_2}'(t) = \langle -\frac{9}{s^2}, 1, 5 \rangle$ ✓✓ $\quad \vec{r_2}'(3) = \langle -1, 1, 5 \rangle$ ✓

$\theta = \cos^{-1}\left(\frac{\langle 1, -1, 8 \rangle \cdot \langle -1, 1, 5 \rangle}{\| \langle 1, -1, 8 \rangle \| \, \| \langle -1, 1, 5 \rangle \|} \right) = \cos^{-1}\left(\frac{38}{\sqrt{66}\sqrt{27}} \right)$

general formula ✓✓ or specific version

Point of intersection: $(4, 3, 16)$ ✓✓

Angle: $\cos^{-1}\left(\frac{38}{\sqrt{66}\sqrt{27}} \right)$ ✓✓

13. (12 pts) Find the equation of the plane that contains the line $x = 1 + t, y = 2 - 3t, z = 2 + 4t$ and the line $x = 2, y = -1 + 2t, z = 6 - t$.

12 pts

$\vec{n} = \langle 1, -3, 4 \rangle \times \langle 0, 2, -1 \rangle$

idea of cross product

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -3 & 4 \\ 0 & 2 & -1 \end{vmatrix} = \vec{i}(-5) - \vec{j}(-1) + \vec{k}\,2$$

$= \langle -5, 1, 2 \rangle$

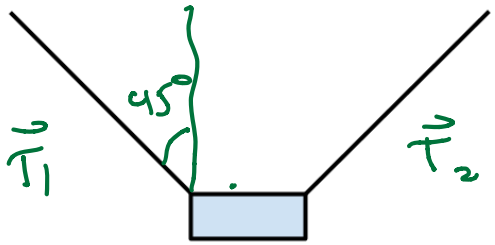
point: set $t = 0$ in 1st line: $(1, 2, 2)$

$-5(x - 1) + (y - 2) + 2(z - 2) = 0$

$-5x + y + 2z = 1$

Equation of plane: $\boxed{-5x + y + 2z = 1}$

14. (12 pts) Your friend plans to suspend a 10 pound brick by two ropes that are at angles of 45° from vertical. The ropes will break if the tension in the rope is more than 8 pounds. Will the rope break? Justify your answer numerically. You can use the fact that $\sqrt{2} \approx 1.4$.



$\|\vec{T}_1\| \cos 45° = \|\vec{T}_2\| \sin 45° \Rightarrow \|\vec{T}_1\| = \|\vec{T}_2\|$

$\|\vec{T}_1\| \sin 45° + \|T_2\| \sin 45° = 10$

$2\|\vec{T}_1\| \frac{\sqrt{2}}{2} = 10 \quad \|T_1\| = \frac{10}{\sqrt{2}}$

since $\sqrt{2} \cdot 8 \approx 1.4 \cdot 8 \simeq 11.2 > 10$

$\frac{10}{\sqrt{2}} < 8$ so rope will hold

Will the rope break? (circle one) YES or NO