Math 232 – Fall 2017 – Thomas – Test 1   Name _____

I, _____ , have neither given nor received unauthorized aid on this test.

- *Show all work! Work may include explanations in phrases/sentences. Use proper mathematical notation and make complete mathematical statements.*

- *If you need more room to write, get blank paper from me. Do not use your own paper.*

- *Only Scientific Calculators are allowed. **NO** Graphing Calculators. Test is designed to be completed without a calculator.*

- ***Exact solutions only.***

- *Multiple choice and True/False will be graded correct or incorrect, free response will be graded based on partial credit (NO WORK NO CREDIT)*

#1-5 Multiple Choice (10 points each)

1. Which definite integral is the area bounded by the graphs of $y = x^2 - 6$ and $y = -x$?

   a) $\int_{-2}^{3} (-x^2 + x - 6)\ dx$

   b) $\int_{-3}^{2} (-x^2 + x - 6)\ dx$

   c) $\int_{-2}^{3} (-x^2 - x + 6)\ dx$

   d) $\int_{-3}^{2} (-x^2 - x + 6)\ dx$

   e) $\int_{0}^{2} (-x^2 - x - 6)\ dx$

2. The base of a solid S is the region bounded by $f(x)$, $x - axis$, $x = 1\ and\ x = 3$. Cross-sections perpendicular to the $x - axis$ are rectangles with height equal to three times the base length. Which of the following is the volume of S.

   a) $\lim\limits_{n \to \infty} \pi \sum_{i=1}^{n} f(x_i^*)\ \Delta x$

   b) $\lim\limits_{n \to \infty} \frac{\pi}{3} \sum_{i=1}^{n} f(x_i^*)^2\ \Delta x$

   c) $\lim\limits_{n \to \infty} 3 \sum_{i=1}^{n} f(x_i^*)^2\ \Delta x$

   d) $\lim\limits_{n \to \infty} 3 \sum_{i=1}^{n} f(x_i^*)\ \Delta x$

   e) $\lim\limits_{n \to \infty} 3\pi \sum_{i=1}^{n} f(x_i^*)^2\ \Delta x$

3. Consider solving the integral $\int \frac{x^3}{\sqrt{25-x^2}} dx$. After making the appropriate trigonometric substitution, which integral must be solved to complete the solution?

a) $\int 125 \sin^3 \theta \ d\theta$

b) $\int \frac{\sin^3 \theta}{\cos \theta} \ d\theta$

c) $\int \frac{\sin^3 \theta}{25 \cos \theta} \ d\theta$

d) $\int 125 \tan^2 \theta \ d\theta$

e) $\int 125 \tan \theta \sin \theta \ d\theta$

4. A circular swimming pool has a diameter of 24 ft, the sides are 5 ft high, and the depth of the water is 4 ft. Which integral is the work done in emptying the tank from the top of the pool? (Water weighs 62.4 pounds per cubic foot.) Let $y =$ distance from the bottom of the tank.

a) $\int_0^1 62.4(12)^2 \pi (5-y)dy$

b) $\int_0^4 62.4(12)^2 \pi (5-y)dy$

c) $\int_0^4 62.4(24)^2 \pi (5-y)dy$

d) $\int_0^1 62.4(14)^2 \pi (5-y)dy$

e) $\int_0^4 62.4(12)^2 (5-y)dy$

5. Consider the integral $\int \frac{x-1}{x^2+x} \, dx$. The partial fraction decomposition would be:

a) $\int \frac{1}{x} + \frac{-2}{x+1} \, dx$

b) $\int \frac{-2}{x} + \frac{1}{x+1} \, dx$

c) $\int \frac{2}{x} + \frac{-1}{x+1} \, dx$

d) $\int \frac{1}{x} + \frac{2}{x+1} \, dx$

e) $\int \frac{-1}{x} + \frac{2}{x+1} \, dx$

#6 True/False (14 points total)

6. A solid is formed by revolving the region bounded by $y = \sqrt{x}$ and $y = x$ about the $x - axis$.

   a. The shape of the cross section is a washer.   **TRUE**   **FALSE**

   b. To find the volume, the following integral is used: **TRUE**   **FALSE**

   $$V = \pi \int_0^1 (x - x^2) \, dx$$

#7-9 Free Response/Partial Credit (12 points each)

7. Evaluate $\int \sin^3 x \cos^6 x \ dx$.

8. Find the average value of $f(x) = \sin 4x$ on the interval of $[-\pi, \pi]$.

9. Evaluate $\int_1^\infty x \ln x \, dx$. State if the integral converges or diverges.

    (Hint: Factor out the variable before you take the limit)