Math 232 – Fall 2017 – Thomas – Test 1 **Name** ____________________

#1-5 Multiple Choice (10 each)

1. Which definite integral is the area bounded by the graphs of $y = x^2 - 6$ and $y = -x$?

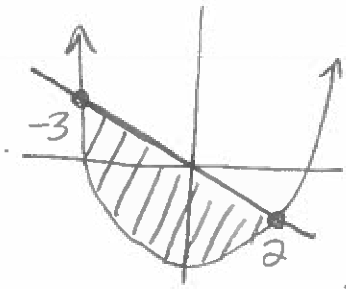
   a) $\int_{-2}^{3} (-x^2 + x - 6)\ dx$

   b) $\int_{-3}^{2} (-x^2 + x - 6)\ dx$

   c) $\int_{-2}^{3} (-x^2 - x + 6)\ dx$

   d) $\int_{-3}^{2} (-x^2 - x + 6)\ dx$

   e) $\int_{0}^{2} (-x^2 - x - 6)\ dx$



$-x-(x^2-6)$
$-x-x^2+6$

$x^2-6=-x$
$x^2+x-6=0$
$(x+3)(x-2)=0$
$x=-3 \quad x=2$

2. The base of a solid S is the region bounded by $f(x),\ x-axis,\ x=1\ and\ x=3$. Cross-sections perpendicular to the $x-axis$ are rectangles with height equal to three times the base length. Which of the following is the volume of S.

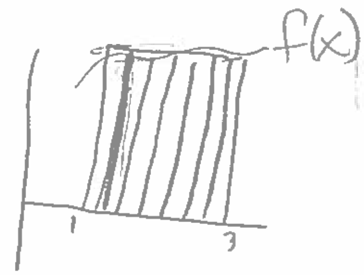
   a) $\lim_{n\to\infty} \pi \sum_{i=1}^{n} f(x_i^*)\,\Delta x$

   b) $\lim_{n\to\infty} \frac{\pi}{3} \sum_{i=1}^{n} f(x_i^*)^2\,\Delta x$

   c) $\lim_{n\to\infty} 3 \sum_{i=1}^{n} f(x_i^*)^2\,\Delta x$

   d) $\lim_{n\to\infty} 3 \sum_{i=1}^{n} f(x_i^*)\,\Delta x$

   e) $\lim_{n\to\infty} 3\pi \sum_{i=1}^{n} f(x_i^*)^2\,\Delta x$



$A = lw$
$= b\,3b$
$= 3b^2$

$\int_1^3 3b^2$

$\int_1^3 3f(x)^2\,dx$

3. Consider solving the integral $\int \frac{x^3}{\sqrt{25-x^2}}dx$. After making the appropriate trigonometric substitution, which integral must be solved to complete the solution?

a) $\int 125\sin^3\theta \; d\theta$

b) $\int \frac{\sin^3\theta}{\cos\theta} \; d\theta$

c) $\int \frac{\sin^3\theta}{25\cos\theta} \; d\theta$

d) $\int 125\tan^2\theta \; d\theta$

e) $\int 125\tan\theta\sin\theta \; d\theta$

let $x = 5\sin\theta$

$dx = 5\cos\theta \, d\theta$

$\sqrt{25-x^2}$

$\sqrt{25-25\sin^2\theta}$

$\sqrt{25(1-\sin^2\theta)}$

$\sqrt{25\cos^2\theta}$

$5\cos\theta$

$\int \frac{5^3\sin^3\theta \, \cancel{5\cos\theta} \, d\theta}{\cancel{5\cos\theta}}$

$\int 125\sin^3\theta \, d\theta$

4. A circular swimming pool has a diameter of 24 ft, the sides are 5 ft high, and the depth of the water is 4 ft. Which integral is the work done in emptying the tank from the top of the pool? (Water weighs 62.4 pounds per cubic foot.) Let y = distance from the bottom of the tank.

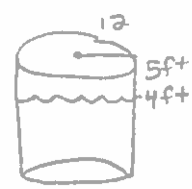
a) $\int_0^1 62.4(12)^2\pi(5-y)dy$

b) $\int_0^4 62.4(12)^2\pi(5-y)dy$

c) $\int_0^4 62.4(24)^2\pi(5-y)dy$

d) $\int_0^1 62.4(14)^2\pi(5-y)dy$

e) $\int_0^4 62.4(12)^2(5-y)dy$



V= density · Area

$A = \pi r^2$

$A = \pi(12)^2$

$V = \int_0^4 62.4\,\pi(12)^2(5-y)dy$

5. Consider the integral $\int \frac{x-1}{x^2+x}\,dx$. The partial fraction decomposition would be:

   a) $\int \frac{1}{x} + \frac{-2}{x+1} dx$

   b) $\int \frac{-2}{x} + \frac{1}{x+1} dx$

   c) $\int \frac{2}{x} + \frac{-1}{x+1} dx$

   d) $\int \frac{1}{x} + \frac{2}{x+1} dx$

   e) $\int \frac{-1}{x} + \frac{2}{x+1} dx$

$\int \frac{X-1}{X(X+1)} dX = \int \frac{-1}{X} + \frac{2}{X+1} dX$

$\frac{X-1}{X(X+1)} = \frac{A}{X} + \frac{B}{X+1}$

$X-1 = A(X+1) + B(X)$

$X-1 = AX + A + BX$

$1 = A + B \rightarrow 1 = -1 + B$

$-1 = A \qquad +1 \quad +1$

$2 = B$

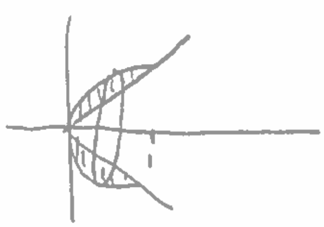
#6 True/False (14 points)

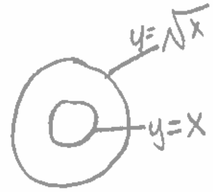
6. A solid is formed by revolving the region bounded by $y = \sqrt{x}$ and $y = x$ about the $x-axis$.

   a. The shape of the cross section is a washer. **TRUE** **FALSE**

   b. To find the volume, the following integral is used: **TRUE** **FALSE**

   $V = \pi \int_0^1 (x - x^2) dx$





$A = \pi r_o^2 - \pi r_i^2$

$A = \pi(\sqrt{x})^2 - \pi(x)^2$

$A = \pi(X - X^2)$

$V = \int_0^1 \pi (X - X^2) dX$

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#7-9 Free Response/Partial Credit (12 each)

7. Evaluate $\int \sin^3 x \cos^6 x \; dx$.

$\int \sin x \sin^2 x \cos^6 x \, dx$

$\int \sin x (1-\cos^2 x) \cos^6 x \, dx$

let $u = \cos x$

$du = -\sin dx$

$\boxed{-\frac{\cos^7 x}{7} + \frac{\cos^9 x}{9} + C}$

$-\int (1-u^2) u^6 \, du$

$-\int u^6 - u^8 \, du$

$-\frac{u^7}{7} + \frac{u^9}{9} + C$

8. Find the average value of $f(x) = \sin 4x$ on the interval of $[-\pi, \pi]$.

$\text{avg value} = \frac{1}{b-a} \int_a^b f(x) dx$

$= \frac{1}{\pi - -\pi} \int_{-\pi}^{\pi} \sin 4x$

$= \frac{1}{2\pi} \left[\frac{-\cos 4x}{4} \right]_{-\pi}^{\pi}$

$= \frac{1}{8\pi} \left[-\cos(4\pi) + \cos(-4\pi) \right]$

$= \frac{1}{8\pi}(-1 + 1) = \frac{1}{8\pi}(0) = \boxed{0}$

9. Evaluate $\int_1^\infty x \ln x \, dx$. State if the integral converges or diverges.

Hint: Factor b/f take limit

no discontinuity $[1, \infty)$

$$\lim_{t \to \infty} \int_1^t x \ln x \, dx = \lim_{t \to \infty} \frac{x^2}{2} \ln x - \int_1^t \frac{x^2}{2} \frac{1}{x} dx =$$

$u = \ln x \quad dv = x\,dx$

$du = \frac{1}{x} dx \quad v = \frac{x^2}{2} dx$

$$\lim_{t \to \infty} \frac{x^2}{2} \ln x - \int_1^t \frac{x}{2} dx = \lim_{t \to \infty} \frac{x^2}{2} \ln x - \frac{x^2}{4} \Big]_1^t$$

$$= \lim_{t \to \infty} \frac{t^2}{2} \ln t - \frac{t^2}{4} - \frac{1}{4} \ln(1) + \frac{1^2}{4}$$

$$= \lim_{t \to \infty} \frac{t^2}{2} \ln t - \frac{t^2}{4} + \frac{1}{4}$$

$$= t^2\left(\frac{1}{2} \ln t - \frac{1}{4}\right) + \frac{1}{4}$$

$$= \infty(\infty)$$

$\boxed{\text{diverges}}$