Math 232: Test 3A
Spring 2016
Instructor: Linda Green

- Calculators are allowed.
- For short answer questions, you must show work for full and partial credit.
- No partial credit for multiple choice / no work needs to be shown.
- Since you have test version A, please code your scantron sequence number as 111111 (all 1's).
- Give exact values instead of decimal approximations unless otherwise specified.
- Sign the honor pledge below after completing the exam.

First and last name ....

PID ....

UNC Email ....

Honor Pledge: I have neither given nor received unauthorized help on this exam.

Signature: ....

True or False. (2 points each) Recall that true means always true, and false means sometimes or always false. n represents non-negative integers and the coefficients c_n are real numbers that could be positive, negative, or 0.

1. True or False: If $f(x) = \sum_{n=0}^{\infty} 3x^n$, then $f^{(n)}(0) = 3$ for all $n \geq 0$.

   A. True

   B. False

2. True or False: Suppose the series $\sum_{n=0}^{\infty} a_n x^n$ and the series $\sum_{n=0}^{\infty} c_n x^n$ converge to the same values for all x. Then $a_n = c_n$ for all n.

   A. True

   B. False

3. True or False: If the power series $\sum_{n=0}^{\infty} c_n x^n$ converges for all x-values, then the power series $\sum_{n=1}^{\infty} n \cdot c_n x^{n-1}$ also converges for all x-values.

   A. True

   B. False

4. True or False: If the radius of convergence for $f(x) = \sum_{n=0}^{\infty} c_n x^n$ is 9, then the radius of convergence of $f(x) = \sum_{n=0}^{\infty} c_n x^{2n}$ is 3.

   A. True

   B. False

5. True or False: If $f(x) = \sum_{n=0}^{\infty} c_n x^n$ is the Taylor series for $f(x) = 3x^2 - 5x + 6$, then $c_n = 0$ for all $n > 2$.

   A. True

   B. False

6. True or False: If $f(x) = \sum_{n=0}^{\infty} c_n 4^n$ converges, then $f(x) = \sum_{n=0}^{\infty} c_n(-4)^n$ also converges.

   A. True
   B. False

7. True or False: If $f(x) = \sum_{n=0}^{\infty} c_n(-5)^n$ converges, then $f(x) = \sum_{n=0}^{\infty} c_n 3^n$ also converges.

   A. True
   B. False

8. True or False: If $f(x) = \sum_{n=0}^{\infty} \frac{x^n}{(n+1)!2^n}$ and $g(x) = \sum_{n=1}^{\infty} \frac{x^{n-1}}{n!2^{n-1}}$, then $f(x) = g(x)$ for all values of x.

   A. True
   B. False

9. (5 pts) Find the radius of convergence of $\sum_{n=1}^{\infty} \frac{(-2)^n(x-3)^n}{n^3}$

   A. $\frac{1}{2}$
   B. 1
   C. 2
   D. 3
   E. $\frac{7}{2}$

10. (5 pts) The series $\sum_{n=1}^{\infty} \frac{(-1)^n(x-7)^n}{4^n\sqrt{n}}$ has a radius of convergence of 4. What is its interval of convergence?

    A. $(3, 11)$
    B. $(3, 11]$
    C. $[3, 11)$
    D. $[3, 11]$

11. (5 pts) The sum of the series $\sqrt{3} - \frac{3\sqrt{3}}{3} + \frac{3^2\sqrt{3}}{5} - \frac{3^3\sqrt{3}}{7} + \cdots$ is equal to which of the following expressions?

A. $\sqrt{3}\arctan(3)$

B. $\arctan(\sqrt{3})$

C. $\sqrt{3}\sin(3)$

D. $\sin(\sqrt{3})$

E. $\ln(1+\sqrt{3})$

12. (10 pts) Use Taylor series to find the limit:

$$\lim_{x \to 0} \frac{xe^x - x}{\ln(1 + 5x^2)}$$

13. (14 pts) In order to approximate $e^{x/2}$ on the interval $[0.2, 1.4]$, Anne wants to use a degree 3 Taylor polynomial centered at 0.8, but Wendy wants to use a degree 6 Taylor polynomial centered at $x = 0$, instead. Whose estimate will be more accurate? Use remainder estimates to give a quantitative justification for your answer.

Whose estimate will be more accurate? Anne Wendy

14. (10 pts)

(a) Find the Maclaurin series for $x^5 \cos(5x^3)$.

(b) Find the Maclaurin series for $\int x^5 \cos(5x^3)\, dx$.

15. (10 pts) Find the Taylor polynomial of degree 3 for the function $f(x) = \sqrt{3-x}$ centered around $a = 2$.

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \cdots \qquad R = 1$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \qquad R = \infty$$

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots \qquad R = \infty$$

$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots \qquad R = \infty$$

$$\tan^{-1} x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots \qquad R = 1$$

$$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots \qquad R = 1$$

$$(1+x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n = 1 + kx + \frac{k(k-1)}{2!} x^2 + \frac{k(k-1)(k-2)}{3!} x^3 + \cdots \qquad R = 1$$