Math 232: Test 2A
Spring 2016
Instructor: Linda Green

- Calculators are allowed.
- For short answer questions, you must show work for full and partial credit.
- No partial credit for multiple choice / no work needs to be shown.
- Give exact values instead of decimal approximations unless otherwise specified.
- Sign the honor pledge below after completing the exam.

First and last name ..... key .....

PID .....

UNC Email .....

Honor Pledge: I have neither given nor received unauthorized help on this exam.

Signature: .....

True or False. (2 points each) Recall that true means always true, and false means sometimes or always false.

1. True or False: If $\sum_{n=1}^{\infty} a_n$ converges then $\sum_{n=1}^{\infty} (a_n + 1)$ converges. $\lim_{n\to\infty}(a_n+1) = \lim_{n\to\infty} a_n + 1 = 0 + 1 = 1$ so diverges by Div Test

2. True or False: If $\sum_{n=1}^{\infty} a_n$ converges, where $a_n > 0$ for all n, then $\sum_{n=1}^{\infty} (-1)^n a_n$ converges.
$\sum |(-1)^n a_n| = \sum a_n$ so $\sum (-1)^n a_n$ conv. abs.

3. True or False: a_n and b_n are both positive and $\lim_{n\to\infty} \frac{a_n}{b_n} = 0$ then $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ both converge or both diverge. Example: $a_n = 0$, $b_n = 1/n$

4. True or False: Suppose $a_n = f(n)$ for a continuous, positive, decreasing function $f(x)$, and $\int_1^{\infty} f(x)\,dx = 2$. Then $\sum_{n=1}^{\infty} a_n = 2$ also . Both converge, but not nec. to exact same value

5. True or False: If $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = -0.8$, then the series $\sum_{n=1}^{\infty} a_n$ converges. $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 0.8$ so conv. abs by Ratio Test

6. True or False: If $\{a_n\}_{n=1}^{\infty}$ is decreasing, and each a_n is positive, then $\{a_n\}_{n=1}^{\infty}$ converges.
monotonic & bdd $\Rightarrow$ sequence conv.

7. True or False: If $\{a_n\}_{n=1}^{\infty}$ is decreasing, and each a_n is positive, then $\sum_{n=1}^{\infty} a_n$ converges.
For example, $a_n = \frac{1}{n}$ or $a_n = \frac{n-1}{n}$

8. True or False: If $\sum_{n=1}^{\infty} |a_n|$ converges then $\sum_{n=1}^{\infty} \frac{a_n}{n}$ converges.

$\sum \left|\frac{a_n}{n}\right|$ conv. by comp to $\sum |a_n|$

since $\left|\frac{a_n}{n}\right| < |a_n|$.

$\therefore \sum \frac{a_n}{n}$ conv. abs.

9. (5 pts) Find the limit of the SEQUENCE, if it converges: $\left\{\frac{3+2n^2}{5-3n^2}\right\}_{n=1}^{\infty}$

A. Converges to $-\frac{2}{3}$.

B. Converges to $\frac{3}{5}$.

C. Diverges to ∞.

D. Diverges to $-\infty$.

E. Diverges (but not to ∞ or $-\infty$).

$\lim_{n\to\infty} \frac{3+2n^2}{5-3n^2} = -\frac{2}{3}$

10. (5 pts) Find the limit of the SEQUENCE, if it converges: $\{2+(-1)^n\}_{n=1}^{\infty}$

A. Converges to 1.

B. Converges to 2.

C. Converges to 3.

D. Diverges to ∞.

E. Diverges (but not to ∞).

1, 3, 1, 3, ... oscillates

11. (5 pts) Which series is absolutely convergent?

A. $\sum_{n=1}^{\infty} \frac{\cos(n)}{n!}$ ← yes, b/c $\sum \left|\frac{\cos(n)}{n!}\right|$ conv. by comp to $\sum \frac{1}{n!}$ ← ratio test

B. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ ← no b/c $\sum \frac{1}{n}$ diverges by p-test

C. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ ← no b/c $\sum \left|\frac{1}{\sqrt{n}}\right| = \sum \frac{1}{\sqrt{n}}$, diverges by p-test

D. None of these series are absolutely convergent.

12. (5 pts) Find all the values of r for which the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^r}$ converges.

A. $r > 0$

B. $r > 1$

C. It always converges, for all values of r.

D. It never converges, for any value of r.

for $r > 0$, n^r increases to ∞ so $\frac{1}{n^r}$ decreases and $\lim_{n\to\infty} \frac{1}{n^r} = 0$

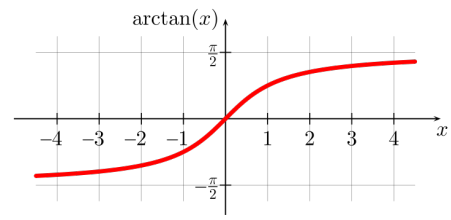
for $r < 0$ $\lim_{n\to\infty} \frac{1}{n^r} \neq 0$

13. (10 pts) Find the sum of the series.

$$\sum_{n=1}^{\infty}(\arctan(n) - \arctan(n+1))$$

$S_1 = \arctan(1) - \arctan(2)$

$S_2 = \arctan(1) - \cancel{\arctan(2)} + \cancel{\arctan(2)} - \arctan(3)$



$S_3 = \arctan(1) - \cancel{\arctan(2)} + \cancel{\arctan(2)} - \cancel{\arctan(3)} + \cancel{\arctan(3)} - \arctan(4)$

$= \arctan(1) - \arctan(4)$

$S_n = \arctan(1) - \arctan(n+1)$

$S_\infty = \lim_{n\to\infty} S_n = \lim_{n\to\infty} \arctan(1) - \arctan(n+1)$

$= \arctan(1) - \pi/2$

$= \pi/4 - \pi/2 = -\pi/4$

(−1) if fail to evaluate arctan(1)

−1 if decimal answer instead of exact answer if supporting work

2 pts for decimal answer without supporting work

Sum: $-\pi/4$

14. (10 pts) Find the sum of the series.

$$\sum_{n=2}^{\infty} \frac{2^n + (-1)^n}{3^n}$$

$$\sum_{n=2}^{\infty} \frac{2^n + (-1)^n}{3^n} = \sum_{n=2}^{\infty} \frac{2^n}{3^n} + \sum_{n=2}^{\infty} \frac{(-1)^n}{3^n}$$

$a = \left(\frac{2}{3}\right)^2$ ✓✓

$r = \frac{2}{3}$ ✓

(-1) if start at $n=1$ or $n=0$

$$\frac{a}{1-r} = \frac{\frac{4}{9}}{1-\frac{2}{3}} \checkmark$$

$$= \frac{\frac{4}{9}}{\frac{1}{3}}$$

$$= \frac{4}{9} \cdot \frac{3}{1} = \frac{4}{3}$$

$a = \left(\frac{-1}{3}\right)^2$ ✓✓

$r = -\frac{1}{3}$ ✓

(-1) if start at $n=1$ or $n=0$

$$\frac{a}{1-r} = \frac{\frac{1}{9}}{1+\frac{1}{3}} \checkmark$$

$$= \frac{\frac{1}{9}}{\frac{4}{3}}$$

$$= \frac{1}{9} \cdot \frac{3}{4} = \frac{1}{12}$$

$$\frac{4}{3} + \frac{1}{4} = \frac{16}{12} + \frac{1}{12} = \frac{17}{12}$$

Sum: 17/12 ✓✓

2 pts for $\frac{a}{1-r}$ formula even if other parts are missing

can give 2 pts for splitting sum if nothing else is correct

15. (10 pts) It is a fact that $\int e^{-\sqrt{y}}\,dy = -2e^{-\sqrt{y}}(\sqrt{y}+1)$. Use this fact to find $\int_0^\infty e^{-\sqrt{y}}\,dy$.

$$\int_0^\infty e^{-\sqrt{y}}\,dy = \lim_{t\to\infty}\int_0^t e^{-\sqrt{y}}\,dy$$ limit idea

$$= \lim_{t\to\infty} -2e^{-\sqrt{y}}(\sqrt{y}+1)\Big|_0^t$$

$$= \lim_{t\to\infty} -2e^{-\sqrt{t}}(\sqrt{t}+1) + 2e^{-\sqrt{0}}(\sqrt{0}+1)$$

plugging in

$$= \lim_{t\to\infty} -\frac{2(\sqrt{t}+1)}{e^{\sqrt{t}}} + 2$$

$$= 2 - 2\lim_{t\to\infty}\frac{\sqrt{t}+1}{e^{\sqrt{t}}} \qquad \frac{\infty}{\infty}$$

$$= 2 - 2\lim_{t\to\infty}\frac{\cancel{\frac{1}{2}t^{-1/2}}}{e^{\sqrt{t}}\,\cancel{\frac{1}{2}t^{-1/2}}}$$

$$= 2 - 0 = 2$$

CONVERGES to: 2 or DIVERGES

16. (10 pts) Choose ONE of the following two questions to answer, using remainder methods from class.

(a) Suppose you use a partial sum to approximate $\sum_{n=1}^{\infty} \frac{1}{n^3}$. How many terms are needed to guarantee that the approximation will be accurate to within 0.01?

$$R_n < \int_n^\infty \frac{1}{x^3}\,dx = \lim_{t\to\infty} \int_n^t x^{-3}\,dx$$

$$= \lim_{t\to\infty} \frac{x^{-2}}{-2}\Big|_n^t = \lim_{t\to\infty} \frac{-t^{-2}}{2} + \frac{n^{-2}}{2}$$

$$= \lim_{t\to\infty} \cancel{\frac{-1}{2t^2}}^{\,0} + \frac{1}{2n^2} = \frac{1}{2n^2}$$

want $\frac{1}{2n^2} \le 0.01 \Rightarrow 100 \le 2n^2 \Rightarrow n^2 \ge 50$

$\Rightarrow n \ge 7.1$

$n \ge 8$

Answer: 8

(b) Suppose you use a partial sum to approximate $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3}$. How many terms are needed to guarantee that the approximation will be accurate to within 0.01?

$|R_n| \le b_{n+1} = \frac{1}{(n+1)^3}$ for n+1, not n

need $\frac{1}{(n+1)^3} \le 0.01 \Rightarrow 100 \le (n+1)^3$

$\Rightarrow n+1 \ge \sqrt[3]{100} \Rightarrow n \ge \sqrt[3]{100} - 1$

≈ 3.64

$n \ge 4$

Answer: 4

For each series, circle CONVERGES or DIVERGES, circle the correct justification, and fill in the blanks. If more than one justification applies, just circle one justification that represents the first step in your argument. You DO NOT have to complete the problem or show work.

17. (6 pts) $\sum_{n=1}^{\infty} \frac{2}{1+3^{-n}}$ CONVERGES DIVERGES

A. Divergence Test, where limit of terms is $\boxed{2}$

B. Comparison Test (ordinary or limit), comparing series with $\sum b_n$ where $b_n =$ $\boxed{}$

C. Integral Test, using function $f(x) =$ $\boxed{}$

D. Alternating Series Test

E. Ratio Test, where the limit of ratio is $\boxed{}$

18. (6 pts) $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln(n)}$ CONVERGES DIVERGES

A. Divergence Test, where limit of terms is $\boxed{}$

B. Comparison Test (ordinary or limit), comparing series with $\sum b_n$ where $b_n =$ $\boxed{}$

C. Integral Test, using function $f(x) =$ $\boxed{}$

D. Alternating Series Test

E. Ratio Test, where the limit of ratio is $\boxed{}$

19. (6 pts) $\sum_{n=1}^{\infty} \frac{1}{n+2^n}$ CONVERGES DIVERGES

A. Divergence Test, where limit of terms is [ ]

B. Comparison Test (ordinary or limit), comparing series with $\sum b_n$ where $b_n =$ [$\frac{1}{2^n}$]

$\lim_{n\to\infty} \frac{\frac{1}{n+1+2^{n+1}}}{\frac{1}{n+2^n}} \ldots = \frac{1}{2}$ using L'H

C. Integral Test, using function $f(x) =$ [ ]

OR D. Alternating Series Test

E. Ratio Test, where the limit of ratio is [$\frac{1}{2}$]

20. (6 pts) $\sum_{n=2}^{\infty} \frac{1}{n\ln(n)}$ CONVERGES DIVERGES

A. Divergence Test, where limit of terms is [ ]

B. Comparison Test (ordinary or limit), comparing series with $\sum b_n$ where $b_n =$ [ ]

C. Integral Test, using function $f(x) =$ [$\frac{1}{x\ln x}$]

D. Alternating Series Test

E. Ratio Test, where the limit of ratio is [ ]

give credit for $f(x) = \ln(\ln(x))$ since it was unclear what $f(x)$ meant.