Math 232 Practice Problems for Exam 1 (some topics may not be on your midterm)

McLaughlin

Print Name____________________________________

Honor Code signature____

Clearly print UNC email address (using ONYEN)____

No calculators. Show photo ID when turning in exam.

Show all work to ensure credit. No credit if your work does not support the solution.

Due to the close proximity of your fellow students during this exam, be careful to not look at other exams. The appearance of looking around may be misunderstood and result in a zero and honor court for cheating.

(1) Set up an integral to find the area bounded by the graphs of $y = 2x^2$ and $y = 3 - 5x$. Do not integrate.

(2) Set up an integral for the length of the arc of the hyperbola $xy = 1$ from the point (1,1) to (2, 1/2).

(3) Find the value of c such that the area of the region bounded by $y = c^2 - x^2$ and the $x - axis$ on $[0, c]$ is equal to the average value of $f(x) = 3x^2$ on [0,2].

a) $\sqrt[3]{6}$

b) $\sqrt[3]{12}$

c) $\sqrt[3]{24}$

d) $\sqrt[3]{8}$

e) $\sqrt[3]{18}$

(4) Evaluate $\int_1^4 \sqrt{x}\ \ln x\, dx$

a) $-\frac{7}{2}\ln 4 - 28$

b) $16\ln 4 - \frac{28}{3}$

c) $\frac{16}{3}\ln 4 - \frac{28}{9}$

d) $-\frac{7}{2}$

e) $\frac{16}{3}\ln 4 - \frac{7}{2}$

(5) Evaluate $\int cos^{-2}x\ sin^3 x\ dx$.

a) $-\sin x - \csc x + c$

b) $\frac{cos^3 x}{3} - \sec x + c$

c) $\cos x + \frac{sin^4 x}{4} + c$

d) $\cos x + \sec x + c$

e) $\frac{\cos x}{2} - \frac{cos^5 x}{5} + c$

(6) Which definite integral is the area bounded by the graphs of $y = x^2 - 3$ and $y = 15 - x^2$?

a) $\int_{-3}^{3} (12 - 2x^2)\ dx$

b) $\int_{-15}^{15} (12 - 2x^2)\ dx$

c) $\pi \int_{-3}^{3} (18 - x^2)^2\ dx$

d) $\int_{-15}^{15} (18 - 2x^2)\ dx$

e) $\int_{-3}^{3} (18 - 2x^2)\ dx$

(7) Set up the integral to find the volume of the solid found by revolving the region bounded by $y = \sqrt{x}\,, x = 0\ and\ x = 1$ about the line $x = 1$. Do not integrate.

(8) Which definite integral is the volume of the solid found by revolving the region bounded by $y = x - 1,\ y = 3 - x$ and the $x - axis$ about the $y - axis$?

a) $\pi \int_{1}^{3} (y^2 - 4y + 10)\ dy$

b) $8\pi \int_{0}^{1} (1 - y)\ dy$

c) $\pi \int_{0}^{1} (1 - 8y^2)\ dy$

d) $\pi \int_{1}^{3} (8 - 8y)\ dy$

e) $8\pi \int_{0}^{1} (y - 1)\ dy$

(9) Find the average value of $y = x \ln x$ on $[1, 5]$.

a) $\frac{25}{8} \ln 5 - \frac{3}{2}$

b) $25 \ln 5 - \frac{3}{2}$

c) $8 \ln 4 - \frac{5}{4}$

d) $\frac{25}{8} \ln 5$

e) $\frac{8}{3} \ln 4 - \frac{5}{4}$

(10) The following integral represents the volume of a solid S. Describe S.

$\int_0^1 e^{2x}\,dx$

a) A solid obtained by rotating the region bounded by $y = e^{2x}, x = 0$ and $x = 1$ about the x – axis.

b) A solid obtained by rotating the region bounded by $y = e^{x}, x = 0$ and $x = 1$ about the x – axis.

c) The base of S is the region bounded by $y = e^{2x},\ x = 0$ and $x = 1$. Cross sections perpendicular to the x – axis are squares.

d) The base of S is the region bounded by $y = e^{x},\ x = 0$ and $x = 1$. Cross sections perpendicular to the x – axis are semicircles.

e) The base of S is the region bounded by $y = e^{x},\ x = 0$ and $x = 1$. Cross sections perpendicular to the x – axis are squares.

(11) Find A.

$\int x \tan x\, sec^2 x\, dx = \frac{x tan^2 x}{2} - A$

a) $A = 2\int sec^2 x\, dx$

b) $A = \frac{1}{2}\int tan^2 x\, dx$

c) $A = \frac{1}{2}\int csc^3 x\, dx$

d) $A = 2\int x\cos x\ dx$

e) $A = 2\int tan^2 x\, dx$

(12) Evaluate $\int_{\pi/6}^{\pi/2} 5cos^3 x\, dx$

a) 5/8

b) 8/3

c) 5/24

d) 25/24

e) 25/8

(13) Evaluating the integral $\int \frac{2x-3}{x^3+3x}\, dx$ using partial fraction decomposition would result in the sum of integrals $\int \frac{A}{x+E}\, dx + \int \frac{Bx+C}{x^2+D}\, dx$ where constants A, B, C, D, E sum to:

a) 4

b) 7

c) 5

d) 2

e) 3

(14) Evaluate $\int tan^2(3x)\, dx$

(15) Evaluate $\int cos^3(x)\, sin^6(x)\, dx$

(16) Consider solving the integral $\int \sqrt{4-x^2}\, dx$. After making the appropriate trigonometric substitution, which integral must be solved to complete the solution?

a) $16 \int cos^2\theta\ \ d\theta$

b) $4 \int \sin\theta \cos\theta\ \ d\theta$

c) $4 \int cos^2\theta\ \ d\theta$

d) $2 \int tan^2\theta\ \ d\theta$

e) $16 \int \tan\theta \sin\theta\ \ d\theta$

True or False for problems #17, 18

(17) There exist constants A and B such that $\frac{x(x^2+4)}{x^2-4} = \frac{A}{x+2} + \frac{B}{x-2}$.

(18) Integrating the parabola $y = x^2 - 4$ from $x = -2$ to $x = 2$ is equivalent to finding the area bounded by $y = x^2$ and $y = 4$.

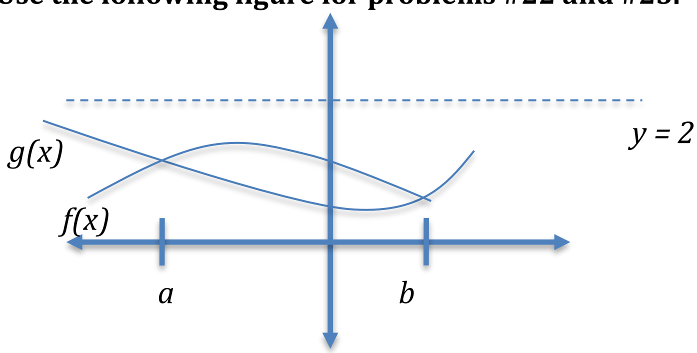
(19) Evaluate $\int x\, cos^2(x\,)\, dx$

(20) Evaluate $\int x\sqrt{16+x^2}\, dx$.

(21) Using the Partial Fraction Decomposition method, express the integral $\int \frac{x+2}{(x-3)^2} dx$ as a sum of integrals $\int \frac{A}{(x-3)} dx + \int \frac{B}{(x-3)^2} dx$. Find A and B. Show all work for credit. Do not integrate.

A=__________ B=__________

Use the following figure for problems #22 and #23.



(22) The volume of the solid generated by revolving the area bounded by $f(x)$ and $g(x)$ about the line $y = 2$ is equal to:

a) $\lim_{n\to\infty} \pi \sum_{i=1}^{n} \left[\left(2 - g(x_i^*)\right)^2 - \left(2 - f(x_i^*)\right)^2\right] \Delta x$

b) $\lim_{n\to\infty} \pi \sum_{i=1}^{n} \left[\left(g(x_i^*)\right)^2 - \left(f(x_i^*)\right)^2\right] \Delta x$

c) $\lim_{n\to\infty} \pi \sum_{i=1}^{n} \left[\left(2 + g(x_i^*)\right)^2 - \left(2 + f(x_i^*)\right)^2\right] \Delta x$

d) $\lim_{n\to\infty} \pi \sum_{i=1}^{n} \left[\left(g(x_i^*)\right)^2 - \left(2 - f(x_i^*)\right)^2\right] \Delta x$

e) $\lim_{n\to\infty} \pi \sum_{i=1}^{n} \left[\left(2 - g(x_i^*)\right)^2 - \left(f(x_i^*)\right)^2\right] \Delta x$

(23) The base of a solid S is the region bounded by $f(x)$ and $g(x)$. Cross-sections perpendicular to the $x-axis$ are squares. Which of the following is the volume of S.

a) $\lim_{n\to\infty} \pi \sum_{i=1}^{n} [f(x_i^*)]^2 \, \Delta x$

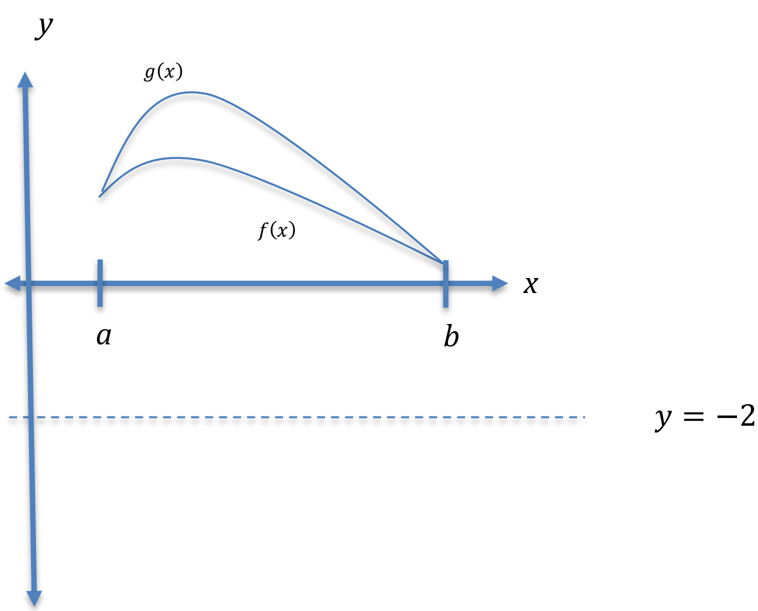
b) $\lim_{n\to\infty} \sum_{i=1}^{n} [f(x_i^*) - g(x_i^*)\,]\Delta x$

c) $\lim_{n\to\infty} 2\pi \sum_{i=1}^{n} [f(x_i^*)^2 - g(x_i^*)^2] \, \Delta x$

d) $\lim_{n\to\infty} \sum_{i=1}^{n} [f(x_i^*) - g(x_i^*)]^2 \, \Delta x$

e) $\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*) \, g(x_i^*) \, \Delta x$

Use the following figure for problems #24 and #25.



(24) The area bounded by the graphs of $y = f(x)$ *and* $y = g(x)$ is equal to:

a) $\lim_{n\to\infty} \sum_{i=1}^{n} \pi\left(g(x_i^*) \ - f(x_i^*)\right)^2 \Delta x$

b) $\lim_{n\to\infty} \sum_{i=1}^{n} (f(x_i^*) - g(x_i^*))\Delta x$

c) $\lim_{n\to\infty} \sum_{i=1}^{n} \left(g(x_i^*) \ - f(x_i^*)\right) \Delta x$

d) $\lim_{n\to\infty} \sum_{i=1}^{n} \pi(g(x_i^*)^2 - f(x_i^*)^2) \, \Delta x$

e) $\lim_{n\to\infty} \sum_{i=1}^{n} \pi\left(f(x_i^*) - g(x_i^*)\right)^2 \Delta x$

(25) The volume of the solid generated by revolving the area bounded by the graphs of $y = f(x)\ and\ y = g(x)$ about the line $y = -2$ is equal to:

a) $\lim\limits_{n\to\infty} \pi \sum_{i=1}^{n} (g(x_i^*) - f(x_i^*) + 2)^2\ \Delta x$

b) $\lim\limits_{n\to\infty} \pi \sum_{i=1}^{n} [(g(x_i^*) - 2)^2 - (f(x_i^*) - 2)^2]\ \Delta x$

c) $\lim\limits_{n\to\infty} \sum_{i=1}^{n} (g(x_i^*) - f(x_i^*) - 4)\ \Delta x$

d) $\lim\limits_{n\to\infty} \pi \sum_{i=1}^{n} [(g(x_i^*) + 2)^2 - (f(x_i^*) + 2)^2]\ \Delta x$

e) $\lim\limits_{n\to\infty} \pi \sum_{i=1}^{n} [(f(x_i^*) + 2)^2 - (g(x_i^*) + 2)^2]\ \Delta x$

(26) A tank in the shape of a right circular cone is full of water. If the height of the tank is 12 feet and the radius of the top is 3 feet, which integral is the work done in pumping the water to a height 10 feet above the top of the tank. (Water weighs 62.4 pounds per cubic foot.) Let $\delta = 62.4$

a) $\frac{\delta\pi}{16}\int_0^{22}(22x^2 - x^3)\ dx$

b) $\frac{\delta\pi}{16}\int_0^{12}(12x^2 - x^3)\ dx$

c) $\delta\pi \int_0^{22}(x^3 - 12)\ dx$

d) $10\delta\pi \int_0^{12}(12x^2 - x^3)\ dx$

e) $\frac{\delta\pi}{16}\int_0^{12}(22x^2 - x^3)\ dx$