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The page continued.

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Q1

10

1. (10 pts) Let  $\mathbf{F}(x, y, z) = \langle x \cos(x^2), ze^{y^2}, z^3 \rangle$ .

Does there exist a function  $f(x, y, z)$  defined on all of  $\mathbb{R}^3$  with  $\mathbf{F} = \nabla f$ ?

Circle your answer and justify (show your work).

- (a) Yes, such a function exists (and your work demonstrates why).  
 (b) No, such a function does not exist because  $\mathbf{F} \neq \mathbf{0}$ .  
 (c) No, such a function does not exist because  $\nabla \times \mathbf{F} \neq \mathbf{0}$ .  
 (d) No, such a function does not exist because  $\nabla \cdot \mathbf{F} \neq 0$ .  
 (e) Not enough information to determine.

$$\mathbf{F} = \langle x \cos(x^2), ze^{y^2}, z^3 \rangle$$

~~curl~~  
 curl  $\mathbf{F}$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ x \cos(x^2) & ze^{y^2} & z^3 \end{vmatrix}$$

$$\text{curl } \mathbf{F} = \langle -e^{y^2}, 0, 0 \rangle$$

$\neq \mathbf{0} \rightarrow$  not  
 conservative



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Problem 1 continued.

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Q2

10

2. (10 pts) Let  $\mathbf{F}(x, y, z) = \langle yz \cos(y^2), xe^y, z^3 \rangle$ .

$\mathbf{F} \rightarrow \text{curl of } \mathbf{G}$

Does there exist a vector field  $\mathbf{G}$  defined on all of  $\mathbb{R}^3$  with  $\mathbf{F} = \nabla \times \mathbf{G}$ ?

Circle your answer and justify.

(a) No, because  $\nabla \times \mathbf{F} \neq \mathbf{0}$ .

(b) No, because  $\nabla \cdot \mathbf{F} \neq 0$ .

(c) No, because  $\mathbf{F}(0, 0, 0) = \mathbf{0}$ .

(d) Yes, such a function exists (and your work demonstrates why).

(e) Not enough information to determine.

$$\text{div}(\text{curl } \mathbf{F}) = 0 \quad \checkmark$$

therefore

$$\text{if } \mathbf{G} \text{ exists } \text{div}(\mathbf{F}) = 0 \quad \rightarrow \quad \text{div of curl} = 0$$

$$\text{DNE}$$

$$\text{div } \mathbf{F} = 0 + ye^y + 3z^2 \quad \checkmark \neq 0$$



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Problem 2 continued.

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Q3

10

3. (10 pts) Let  $f(x, y, z) = x + y^3 + z^2$ . Identify the unit vector pointing in the direction in which  $f$  decreases fastest at the point  $(0, 1, 2)$ .

$D_U f$  ~~max~~ <sup>min</sup>  $\rightarrow$  opposite direction of  $\nabla f$

at min  $\nabla f \cdot U = -|\nabla f|$ ,  $\nabla f = -U$

$$\nabla f = \langle 1, 3y^2, 2z \rangle \Big|_{(0,1,2)} = \langle 1, 3, 4 \rangle$$

$$\sqrt{1+9+16} = \sqrt{26}$$

$$\nabla f = -U = \left\langle \frac{-1}{\sqrt{26}}, \frac{-3}{\sqrt{26}}, \frac{-4}{\sqrt{26}} \right\rangle$$

$$\sqrt{\frac{1}{26} + \frac{9}{26} + \frac{16}{26}} = \frac{\sqrt{26}}{26} = 1 \quad \checkmark$$



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Problem 3 continued.

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Q4 10

4. (10 pts) Evaluate  $\int_0^3 \int_{x^2}^9 x e^{-y^2} dy dx$  by integrating over the same region with the order of the iterated integrals reversed.

$$\begin{aligned} x^2 &= y \\ \sqrt{y} &= x \end{aligned}$$

$$x^2 \leq y \leq 9$$

$$0 \leq x \leq 3$$



new bounds  $\rightarrow 0 \leq x \leq \sqrt{y}$

$$0 \leq y \leq 9$$

$$\int_0^9 \int_0^{\sqrt{y}} x e^{-y^2} dx dy$$

$$\left[ \frac{1}{2} x^2 e^{-y^2} \right]_0^{\sqrt{y}} \rightarrow \frac{1}{2} y e^{-y^2}$$

$$\frac{1}{2} \int_0^9 y e^{-y^2} dy$$

$$\begin{aligned} v &= -y^2 \\ dv &= -2y \end{aligned}$$

$$\frac{1}{2} \left[ -\frac{1}{2} e^{-y^2} \right]_0^9$$

$$\int -\frac{1}{2} dv e^v = -\frac{1}{2} e^v$$

$$\frac{1}{2} \left( \left( -\frac{1}{2} e^{-81} \right) - \left( -\frac{1}{2} e^0 \right) \right)$$

$$\frac{1}{2} \left( \frac{1}{2} - \frac{1}{2} e^{-81} \right) \rightarrow \frac{1}{4} (1 - e^{-81})$$





Q5 10

5. (10 pts) Show that the vector field

$$\mathbf{F}(x, y, z) = \langle yz + y \cos(xy), xz + x \cos(xy), xy + e^z \rangle$$

is conservative and evaluate the integral  $\int_C \mathbf{F} \cdot d\mathbf{r}$  where  $C$  follows a curve along the paraboloid  $z = x^2 + y^2$  from  $(0, 0, 0)$  to  $(1, 1, 2)$ .

curl F

$i$	$-j$	$k$	
$\frac{d}{dx}$	$\frac{d}{dy}$	$\frac{d}{dz}$	
$yz + y \cos(xy)$	$xz + x \cos(xy)$	$xy + e^z$	

$$= \langle x - x, -(y - y), (z - (yz + y \cos(xy)) - (xz + x \cos(xy))) \rangle$$

$$= \langle 0, 0, 0 \rangle$$

curl F = 0 ✓ conservative

$$\int yz + y \cos xy \, dx \rightarrow xyz + \sin(xy) + f(y) + f_z$$

$$\int xz + x \cos(xy) \, dy \rightarrow xyz + \sin(xy) + f_z - e^z$$

$$\int xy + e^z \, dz \rightarrow xyz + e^z$$

Potential function

$$f = xyz + \sin(xy) + e^z + C$$

Fundamental theorem of the integrals

$$\int_C \mathbf{F} \cdot d\mathbf{r} = f(b) - f(a)$$

if  $\nabla f = \mathbf{F}$

correct answer 10

$$xyz + \sin(xy) + e^z \Big|_{(0,0,0)}^{(1,1,2)}$$

$$(2 + \sin(1) + e^2) - (0 + 0 + e^0) = 2 + \sin(1) + e^2 - 1$$

$$= \boxed{1 + \sin(1) + e^2}$$



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Problem 5 continued.

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Q6 20

6. (20 pts) Consider the function  $z = f(x, y) = 4e^{xy} + xy$ .(a) Compute the first partial derivatives of  $f$  with respect to  $x$  and  $y$ .(b) At the point  $(1, 1)$ , write the equation for the tangent plane to the surface described by the function.(c) What is the linear approximation to  $f$  at the point  $(1, 1)$ ?

a.

$$\left[ \begin{array}{l} f_x = 4ye^{xy} + y \\ f_y = 4xe^{xy} \end{array} \right]_{(1,1)} \rightarrow 4e + 1$$

$$\left[ \begin{array}{l} f_x = 4ye^{xy} + y \\ f_y = 4xe^{xy} \end{array} \right]_{(1,1)} \rightarrow 4e + 1$$

b.

$$z_0 = f(1,1) = 4e + 1 = z_0$$

$$z - z_0 = f_x(x - x_0) + f_y(y - y_0)$$

$$z - (4e + 1) = (4e + 1)(x - 1) + (4e + 1)(y - 1)$$

$$z - (4e + 1) = (4e + 1)x - (4e + 1) + (4e + 1)y - (4e + 1)$$

$$z - (4e + 1) = (4e + 1)(x + y - 2)$$

$$L = ax + by + d$$

$$L = (4e + 1)x + (4e + 1)y - 2(4e + 1) + (4e + 1)$$

$$L = (4e + 1)x + (4e + 1)y - (4e + 1)$$

$$L = (4e + 1)(x + y - 1)$$

(if needs to be simplified)

c.



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Problem 6 continued.

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$$r = \langle \cos\theta, \sin\theta, 0 \rangle$$

$$r_u = \langle \cos\theta, \sin\theta, 1 \rangle$$

$$r_v = \langle -\sin\theta, \cos\theta, 0 \rangle$$

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Q7 20

7. (20 pts) Consider the cone  $z^2 = x^2 + y^2$  between  $z = 0$  and  $z = 1$ .  
 (a) Find the surface area of this cone.  
 (b) Find the volume of the region above this cone and inside the sphere of radius  $\sqrt{2}$  centered at the origin that encloses the cone.

a.  $SA = \iint |r_u \times r_v| dA$  or  $\iint \sqrt{1 + \frac{d^2z}{dx^2} + \frac{d^2z}{dy^2}} dA$

$z = \sqrt{x^2 + y^2}$  ( $x^2 + y^2 = z^2$ )

$F_x = \frac{x}{\sqrt{x^2 + y^2}}$   $r = \langle x, y, \sqrt{x^2 + y^2} \rangle$   $\frac{1}{\sqrt{x^2 + y^2}}$

$F_y = \frac{y}{\sqrt{x^2 + y^2}}$   $\sqrt{1 + \frac{x^2}{x^2 + y^2} + \frac{y^2}{x^2 + y^2}} = \sqrt{1 + \frac{x^2 + y^2}{x^2 + y^2}} = \sqrt{2}$

b.  $\iint \sqrt{2} dA \rightarrow \sqrt{2} \int_0^{2\pi} \int_0^1 r dr d\theta$

$\frac{1}{2} r^2 \Big|_0^1 = \frac{1}{2}$

$\frac{\sqrt{2}}{2} \int_0^{2\pi} d\theta = \sqrt{2} \pi$

$\boxed{\sqrt{2} \pi}$

$x^2 + y^2 + z^2 = 2$

$\int_0^{2\pi} \int_0^{\frac{\pi}{4}} \int_0^{\sqrt{2} \cos\phi} \rho^2 \sin\phi d\rho d\phi d\theta$

$\left[ \frac{1}{3} \rho^3 \right]_0^{\sqrt{2} \cos\phi} = \frac{2\sqrt{2}}{3} \int_0^{\frac{\pi}{4}} \cos^3\phi d\phi d\theta$

$\frac{2\sqrt{2}}{3} \left[ -\cos\phi \right]_0^{\frac{\pi}{4}} \rightarrow \frac{2\sqrt{2}}{3} \left( -\frac{\sqrt{2}}{2} - (-1) \right)$

$\frac{2\sqrt{2}}{3} \left( 1 - \frac{\sqrt{2}}{2} \right) \int_0^{2\pi} d\theta = \left( \frac{2\sqrt{2}}{3} - \frac{2(\sqrt{2})}{6} \right) 2\pi = \frac{4\pi}{3} (\sqrt{2} - 1)$



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Problem 7 continued.



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$$x^4 - x^3$$

Q8 20

8. (20 pts) Find and correctly classify all of the local minima, local maxima, and saddles of  $f(x, y) = x^2 + y^2 - x^2y$ .

$$f_x = 2x - 2xy = 0 \implies 2x(1-y) = 0$$

$$f_y = 2y - x^2 = 0$$

$$2y = x^2$$

$$2(1) - x^2 = 0$$

$$x = \sqrt{2}$$

(0,0)

( $\sqrt{2}, 1$ )

$$x^2 = 0, 2y = 0$$

$$y = 0$$

( $-\sqrt{2}, 1$ )

$$2x - 2xy = 2y - x^2$$

~~$x = \sqrt{2}$~~

$$2x + x^2 = 2y + 2xy$$

$$2x(1+x) = 2y(1+x)$$

$$x = -1, y = 0$$

$$2(-1) - 2(-1)$$

$$2\sqrt{2} - 2\sqrt{2}(-1) = 4\sqrt{2}$$

$$-2\sqrt{2} - 2(-\sqrt{2}) = 0$$

$$D = F_{xx}F_{yy} - F_{xy}^2$$

$$F_{xx} = 2 - 2y$$

$$D > 0 \rightarrow F_{xx} < 0 \rightarrow \text{max}$$

$$\hookrightarrow F_{xx} > 0 \rightarrow \text{min}$$

$$F_{yy} = 2$$

$$D < 0 \rightarrow \text{saddle}$$

$$F_{xy} = -2x$$

$$(0,0) \rightarrow D = (2-0)(2) - (0)^2 = 4$$

correct answer 20

$$(\sqrt{2}, 1) \rightarrow D = (0)(2) - (-2(\sqrt{2}))^2 < 0$$

$\rightarrow$  saddle point

$$(-\sqrt{2}, 1) \rightarrow D = (0)(2) - (0) < 0 \rightarrow \text{saddle point}$$

(0,0)  $\rightarrow$  local min.

( $\pm\sqrt{2}, 1$ ) saddle points

points



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Problem 8 continued.



$$\begin{matrix} i & -j \\ dx & dy \\ P & Q \end{matrix} \quad \begin{matrix} -\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \\ P \\ Q \end{matrix}$$

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Q9 20

9. (20 pts) Evaluate  $\int_C [x^1 dx + (x + \cos y) dy]$  with  $C$  the boundary of the rectangular region defined by  $-2 \leq x \leq 2$  and  $-3 \leq y \leq 2$ , oriented clockwise.

⊖ orientation

Green's Theorem

$$\iint (\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}) dA$$

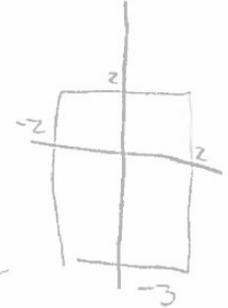
$$(-) \iint (1 - 0) dA$$

$$- \int_{-3}^2 \int_{-2}^2 1 dx dy$$

$$x \Big|_{-2}^2 (2 - (-3)) = 4$$

$$-4 \int_{-3}^2 dy \Rightarrow y \Big|_{-3}^2 = 5$$

**-20**



$C_1 = \langle -2, y \rangle$   
 $r'(t) = \langle -1, 0 \rangle$

$C_2 = \langle x, 2 \rangle$   
 $r'(t) = \langle 1, 0 \rangle$

$C_3 = \langle -2, y \rangle$   
 $r'(t) = \langle 0, 1 \rangle$

$C_4 = \langle x, -3 \rangle$   
 $r'(t) = \langle 1, 0 \rangle$

$C_1 = \langle -2, 3 \rangle$   
 $r'(t) = \langle 0, 1 \rangle$



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Problem 9 continued.

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Q10 20

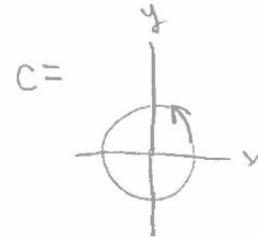
10. (20 pts) Use Stokes' Theorem to evaluate  $\oint_C \mathbf{F} \cdot d\mathbf{r}$  for  $\mathbf{F}(x, y, z) = 2z\mathbf{i} + 3x\mathbf{j} + 4y\mathbf{k}$  where  $C$  is the circle of radius 1 in the  $xy$ -plane centered at the origin and oriented counterclockwise when viewed from above the  $xy$ -plane. (Do not evaluate the line integral; you must evaluate the integral obtained via Stokes' Theorem.)

$$\text{Stokes Theorem: } \oint_C \mathbf{F} \cdot d\mathbf{r} = \iint_S \text{curl } \mathbf{F} \cdot d\mathbf{s}$$

$$\mathbf{F} = \langle 2z, 3x, 4y \rangle$$

$$\hookrightarrow \iint_S \text{curl } \mathbf{F} \cdot \mathbf{n} \, ds$$

$$\begin{array}{c} \text{curl } \mathbf{F} \\ \left| \begin{array}{ccc} \mathbf{i} & -\mathbf{j} & \mathbf{k} \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ 2z & 3x & 4y \end{array} \right| = \langle 4, -(0-2), 3 \rangle \\ = \langle 4, 2, 3 \rangle \end{array}$$



$$\mathbf{n} = \langle 0, 0, 1 \rangle$$

$$\begin{aligned} \iint_{\text{Circle } r=1} \text{curl } \mathbf{F} \cdot \mathbf{n} \, ds &= \iint_{\text{Circle}} \langle 4, 2, 3 \rangle \cdot \langle 0, 0, 1 \rangle \, ds \\ &= \iint_{\text{Circle}} 0 + 0 + 3 \, ds \\ &= \iint_{\text{Circle}} 3 \, ds \end{aligned}$$

$$\int_0^{2\pi} \int_0^1 3r \, dr \, d\theta$$

$$\left[ \frac{3}{2} r^2 \right]_0^1 = \frac{3}{2} \rightarrow \frac{3}{2} \int_0^{2\pi} d\theta = \frac{3(2\pi)}{2} = 3\pi$$



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Problem 10 continued.

$$r = \langle \cos t, \sin t, 0 \rangle$$

$$\langle 2x, 3y, 4z \rangle = \langle -2\sin t, 2\cos t, 0 \rangle$$

$$\int_0^{2\pi} 0 + 3\cos^2 t$$

$$\frac{3}{2} \int_0^{2\pi} 1 - \cos 2t$$

$$\left[ \frac{3}{2}t - \frac{1}{2}\sin 2t \right]_0^{2\pi} = \frac{3}{2}(2\pi) - 0 - 0 = 3\pi \checkmark$$



Q11 18

11. (20 pts) Consider the surface integral  $\iint_S \mathbf{F} \cdot d\mathbf{S}$  with  $\mathbf{F}(x, y, z) = 2xy\mathbf{i} + xe^z\mathbf{j} + z^3\mathbf{k}$  where  $S$  is the surface of the cylinder  $y^2 + z^2 = 4$  with  $0 \leq x \leq 2$ . (a) Parametrize this surface and write down (but do not evaluate) the iterated integrals for the surface integral. (b) Let  $S'$  be the closed surface with outward-facing normals obtained by taking the union of the surface  $S$  with the planes  $x = 0$  and  $x = 2$ . Use the Divergence Theorem to evaluate the integral  $\iint_{S'} \mathbf{F} \cdot d\mathbf{S}$ .

$\iint_S \mathbf{F}(r(u,v)) \cdot (\mathbf{r}_u \times \mathbf{r}_v)$

a.  $S = r(u,v) = \langle u, 2\cos v, 2\sin v \rangle$

$0 \leq u \leq 2, 0 \leq v \leq 2\pi$

$\mathbf{r}_u = \langle 1, 0, 0 \rangle$

$\mathbf{r}_v = \langle 0, -2\sin v, 2\cos v \rangle$

$\mathbf{F}(r(u,v)) = \langle 4u\cos v, ue^{2\sin v}, 8\sin^3 v \rangle$



$\mathbf{r}_u \times \mathbf{r}_v$

i	0j	k
1	0	0
0	-2sin v	2cos v

$\iint_S \mathbf{F}(r(u,v)) \cdot (\mathbf{r}_u \times \mathbf{r}_v) = \langle 4u\cos v, ue^{2\sin v}, 8\sin^3 v \rangle \cdot \langle 0, -2\cos v, -2\sin v \rangle$

$\int_0^{2\pi} \int_0^2 (-2u\cos v e^{2\sin v} - 16\sin^4 v) du dv$

$= \langle 0, -(2\cos v), -2\sin v \rangle$

$d\mathbf{S}$  should be in reverse direction in order to point outwards

✓ b.  $\iiint_{S'} \mathbf{F} \cdot d\mathbf{S} = \iiint_{S'} \text{div} \mathbf{F} \, dV$      $\text{div} \mathbf{F} = 2y + 0 + 3z^2$

→ cylindrical coordinates  $x = x, y = r\cos\theta, z = r\sin\theta$

$\int_0^{2\pi} \int_0^2 \int_0^2 (2r\cos\theta + 3r^3\sin^2\theta) r \, dx \, dr \, d\theta$

$\left[ x(2r\cos\theta + 3r^3\sin^2\theta) \right]_0^2 = 2(2r^2\cos\theta + 3r^3\sin^2\theta)$

$2 \int_0^{2\pi} \int_0^2 \left[ \frac{2}{2} r^3 \cos\theta + \frac{3}{4} r^4 \sin^2\theta \right]_0^2 \, dr \, d\theta \rightarrow \frac{16}{3} \cos\theta + \frac{16(3)}{4} \sin^2\theta$



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Problem 11 continued.

$$2 \int_0^{2\pi} \frac{16}{3} \cos \theta + 12 \sin^2 \theta \, d\theta$$

$$\frac{16}{3} \sin \theta + \left( 12 \left( \frac{1}{2} \right) (1 - \cos 2\theta) \right)$$

$$6\theta - \frac{1}{2}(6) \sin 2\theta$$

$$2 \left[ \frac{16}{3} \sin \theta + 6\theta - 3 \sin 2\theta \right]_0^{2\pi}$$

$$2 \left( (0 + 12\pi - 0) - (0) \right) = \boxed{24\pi}$$

$$\int_0^{2\pi} \int_0^2 (-2v \cos v\theta - 16 \sin^4 v) \, dv \, d\theta$$

$$\int_0^{2\pi} (-4 \cos v\theta - 2(16 \sin^4 v)) \, dv \, d\theta$$

$$\left[ -2 \sin v\theta - 2e^{2 \sin v\theta} \right]_0^2 \Big|_0^{2\pi}$$

$$x=0 \rightarrow -2xy \rightarrow 0 \int_0^{2\pi} \int_0^2 4y^2 \sin^4 \theta \, dy \, d\theta$$

$$x=2 \int_0^{2\pi} \int_0^2 4y \, dy \, d\theta$$

$$\frac{1}{4} \int_0^{2\pi} (1 - \cos 2v)(1 - \cos 2v) \, dv \, d\theta$$

$$\frac{1}{4} \int_0^{2\pi} (1 - 2 \cos 2v + \cos^2(2v)) \, dv \, d\theta$$

$$\frac{1}{4} \int_0^{2\pi} \left( 1 - 2 \cos 2v + \frac{1}{2}(1 + \cos 4v) \right) \, dv \, d\theta$$

$$\frac{1}{4} \int_0^{2\pi} \left( \frac{3}{2} - 2 \cos 2v + \frac{1}{2} \cos 4v \right) \, dv \, d\theta$$

$$\frac{1}{4} \left( \frac{3}{2} \theta - \sin 2v + \frac{1}{8} \sin 4v \right) \Big|_0^{2\pi}$$

$$\frac{1}{4} \left( \frac{3\pi}{2} + \frac{\pi}{4} \right) = \frac{3\pi}{4} = 24\pi$$