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Math 233, Final Exam, Spring 2018

May 3rd, 2018

Last name

First name

UNC E-MAIL (ONYEN)

- Closed book, closed notes, no calculators.
- Show photo ID when turning in exam.
- Partial credit is important—try all problems.
- Take integrals as far as you can analytically, leaving them as iterated or definite integrals if you must.
- You must show full analytical work to receive full credit, even on the multiple choice problems.
- By putting your name on your paper, you implicitly pledge your adherence to the honor code.

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Title page continued.

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- 1. (10 pts) Let $\mathbf{F}(x, y, z) = \langle x \cos(x^2), z e^{y^2}, z^3 \rangle$. Does there exist a function f(x, y, z) defined on all of \mathbb{R}^3 with $\mathbf{F} = \nabla f$?
 - Circle your answer and justify (show your work).
 - (a) Yes, such a function exists (and your work demonstrates why).
 - (b) No, such a function does not exist because $\mathbf{F} \neq \mathbf{0}$.
 - (c) No, such a function does not exist because $\nabla \times \mathbf{F} \neq \mathbf{0}$.
 - (d) No, such a function does not exist because $\nabla \cdot \mathbf{F} \neq 0$.
 - (e) Not enough information to determine.



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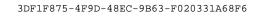
Problem 1 continued.

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- 2. (10 pts) Let $\mathbf{F}(x, y, z) = \langle yz \cos(y^2), xe^y, z^3 \rangle$. Does there exist a vector field **G** defined on all of \mathbb{R}^3 with $\mathbf{F} = \nabla \times \mathbf{G}$? Circle your answer and justify.
 - (a) No, because $\nabla \times \mathbf{F} \neq \mathbf{0}$.
 - (b) No, because $\nabla \cdot \mathbf{F} \neq 0$.
 - (c) No, because F(0, 0, 0) = 0.
 - (d) Yes, such a function exists (and your work demonstrates why).
 - (e) Not enough information to determine.





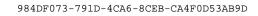
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Problem 2 continued.

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3. (10 pts) Let $f(x, y, z) = x + y^3 + z^2$. Identify the unit vector pointing in the direction in which f decreases fastest at the point (0, 1, 2).



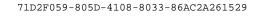


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Problem 3 continued.



4. (10 pts) Evaluate $\int_0^3 \int_{x^2}^9 x e^{-y^2} dy dx$ by integrating over the same region with the order of the iterated integrals reversed.





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Problem 4 continued.

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5. (10 pts) Show that the vector field

$$\mathbf{F}(x, y, z) = \langle yz + y\cos(xy), xz + x\cos(xy), xy + e^z \rangle$$

is conservative and evaluate the integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C follows a curve along the paraboloid $z = x^2 + y^2$ from (0, 0, 0) to (1, 1, 2).



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Problem 5 continued.

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6. (20 pts) Consider the function $z = f(x, y) = 4e^{xy} + xy$.

(a) Compute the first partial derivatives of f with respect to x and y. (b) At the point (1, 1), write the equation for the tangent plane to the

surface described by the function.

(c) What is the linear approximation to f at the point (1, 1)?



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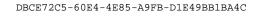
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Problem 6 continued.

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- 7. (20 pts) Consider the cone z² = x² + y² between z = 0 and z = 1.
 (a) Find the surface area of this cone.
 - (b) Find the volume of the region above this cone and inside the sphere of radius $\sqrt{2}$ centered at the origin that encloses the cone.



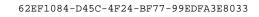


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Problem 7 continued.



8. (20 pts) Find and correctly classify all of the local minima, local maxima, and saddles of $f(x, y) = x^2 + y^2 - x^2y$.





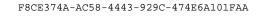
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Problem 8 continued.

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9. (20 pts) Evaluate $\int_C x^4 dx + (x + \cos y) dy$ with C the boundary of the rectangular region defined by $-2 \le x \le 2$ and $-3 \le y \le 2$, oriented *clockwise*.





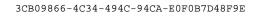
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Problem 9 continued.

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10. (20 pts) Use Stokes' Theorem to evaluate $\oint_C \mathbf{F} \cdot d\mathbf{r}$ for $\mathbf{F}(x, y, z) = 2z\mathbf{i}+3x\mathbf{j}+4y\mathbf{k}$ where C is the circle of radius 1 in the xy-plane centered at the origin and oriented counterclockwise when viewed from above the xy-plane. (Do not evaluate the line integral; you must evaluate the integral obtained via Stokes' Theorem.)





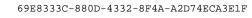
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Problem 10 continued.

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11. (20 pts) Consider the surface integral ∬_S F ⋅ dS with F(x, y, z) = 2xyi + xe^zj + z³k where S is the surface of the cylinder y² + z² = 4 with 0 ≤ x ≤ 2. (a) Parametrize this surface and write down (but do not evaluate) the iterated integrals for the surface integral.
(b) Let S' be the closed surface with outward-facing normals obtained by taking the union of the surface S with the planes x = 0 and x = 2. Use the Divergence Theorem to evaluate the integral ∰_{S'} F ⋅ dS.





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Problem 11 continued.