

## Key- Math 231 Final Exam Review

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1. Find the equation of the line tangent to the curve  $x^2 + 4yx + y^2 = 13$  at the point  $(2, 1)$ .

$$y - 1 = (-4/5)(x - 2)$$

2. Find the slope of the normal line to  $y = (2 + x)e^{-x}$  at the point  $(0, 2)$ .

1

3. Find  $\frac{dy}{dx}$  if  $\cos(xy) - y^3 = 4x^2y$ .

$$(y \sin xy + 8xy) / (-x \sin xy - 3y^2 - 4x^2)$$

4. Find the exact area bounded by  $y = x(x^2 - 4)$ , the  $x$ -axis,  $x = -2$ , and  $x = 2$ .

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5. Find the exact area bounded by  $f(x) = \begin{cases} -x - 1 & , -3 \leq x \leq 0 \\ -\sqrt{1 - x^2} & , 0 \leq x \leq 1 \end{cases}$  and the  $x$ -axis.

$$5/2 + \pi/4$$

6. Find  $\lim_{x \rightarrow 0} \frac{\tan \pi x}{\ln(1 + x)}$ .

$\pi$

7. Find  $\lim_{x \rightarrow \infty} x^3 e^{-x}$

0

8. Find  $y'$  if  $y = (\tan x)^{\cos x}$ .

$$y' = y(-\sin x \ln(\tan x) + \csc x) = (\tan x)^{\cos x} (-\sin x \ln(\tan x) + \csc x)$$

9. Find  $y'$  if  $y = (\cot x)^{\sqrt{x^5 + 1}}$ .

$$y' = (\cot x)^{\sqrt{x^5 + 1}} \left( \frac{5x^4}{2\sqrt{x^5 + 1}} \ln(\cot x) - \sqrt{x^5 + 1} / \cos x \sin x \right)$$

10. Given  $g(x) = \int_{\sqrt{x}}^1 \sin(t^4) dt$ , find  $g'(x)$ .

$$g'(x) = -\sin(x^2) \frac{1}{2\sqrt{x}}$$

11. A particle moves along a line with velocity function  $v(t) = t^2 - t$ , where  $v$  is measured in meters per second. Find (a) the displacement and (b) the distance traveled by the particle during the time interval  $[0,5]$ .

175/6

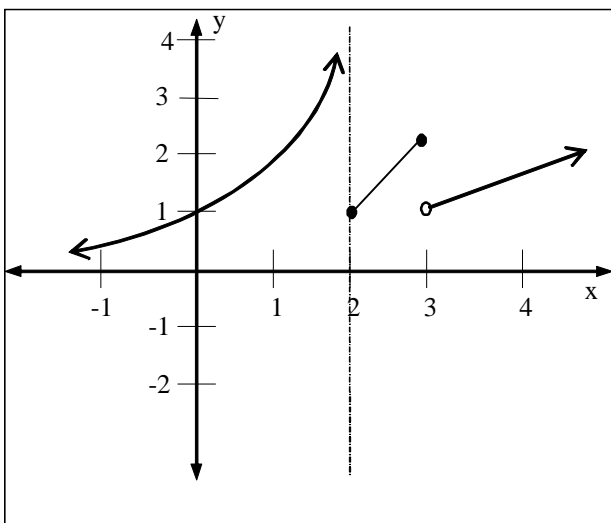
12. Find  $g'(x)$  if  $g(x) = 6^{\sqrt{\csc x}}$ .

$$-6^{\sqrt{\csc x}} \ln 6 \left(\frac{1}{2}\right) (\csc x)^{-1/2} \csc x \cot x$$

13. For a function  $g(x)$ , given that  $g'(x) = \frac{-4x+3}{x^3}$  and  $g''(x) = \frac{8x-9}{x^4}$ , find all intervals where  $g(x)$  is simultaneously decreasing AND concave down.

$(-\infty, 0), (3/4, 9/8)$

14. Consider the function  $g(x)$  whose graph is shown below. Find each of the following. If the limit does not exist, describe the outcome as  $-\infty$ ,  $\infty$ , or LDNE.



- a)  $\lim_{x \rightarrow 3^+} g(x) = \underline{\text{pos\_inf}}$
- b)  $\lim_{x \rightarrow 2^-} g(x) = \underline{\text{pos\_inf}}$
- c)  $\lim_{x \rightarrow 0} g(x) = \underline{0}$
- d)  $\lim_{x \rightarrow -\infty} g(x) = \underline{-\infty}$

15. Compute the EXACT value of each limit, if it exists. If the limit does not exist, describe the outcome as  $-\infty$ ,  $\infty$ , or LDNE.

a)  $\lim_{x \rightarrow 8^-} \frac{12x-1}{x^2-4x-32}$   
 $-\infty$

b)  $\lim_{x \rightarrow \infty} \frac{\sqrt{7}x^5 - 5x^2}{x^2 - 6x^5}$   
 $(-\sqrt{7})/6$

c)  $\lim_{x \rightarrow 3^+} f(x)$  if  $f(x) = \begin{cases} 8x^2 - 1, & x > 3 \\ 5x - 2, & x \leq 3 \end{cases}$   
 71

d)  $\lim_{x \rightarrow 3^-} \frac{3x^2 - 7x - 6}{|x - 3|}$   
 -11

e)  $\lim_{x \rightarrow 16} \frac{4 - \sqrt{x}}{16x - x^2}$   
 1/128

f)  $\lim_{x \rightarrow 2^-} \frac{x^2 - x - 2}{(x - 2)^3}$   
 inf

g)  $\lim_{x \rightarrow -\infty} \frac{3x^6 - x^7}{\sqrt{-x} - 5x^8}$   
 0

h)  $\lim_{x \rightarrow \infty} \frac{\sqrt{7x^8} - x}{\sqrt{x} + 5x^4}$   
 (sqrt7)/5

16. a) Find the  $x$ -coordinate(s) of all point(s), if any, on the graph of the curve  $y = \frac{x^2 + x}{2 - x}$  at which the tangent line is parallel to the line  $y + x = 0$ .

none

b) Find the equation of the normal line to the curve  $y = \frac{3x^2 + 2x}{5x - 2}$  at  $x = 1$ .

$y - 5/3 = 9(x - 1)$

17. Use the **limit definition** of the derivative to find  $f'(x)$  for the function  $f(x) = \sqrt{x} + 3$ .

18. Use the **limit definition** of the derivative to find  $f'(x)$  for the function  $f(x) = \frac{1}{x + 2}$ .

19. a) Express the definite integral  $\int_1^2 e^{x^2} dx$  as the limit of a Riemann sum.

b) Evaluate the Riemann sum for  $f(x) = e^{x^2}$ ,  $[1, 2]$  with 4 subintervals taking the sample points to be the left endpoints.

$$a) \int_1^2 e^{x^2} dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n e^{(x_i^*)^2} \Delta x$$

$$b) \left( e + e^{25/16} + e^{9/4} + e^{49/16} \right) \frac{1}{4} \approx 9.58942$$

20. Find the EXACT value of  $c$  so that the function  $f$  given below is continuous at  $x = 9$ .

$$f(x) = \begin{cases} \frac{|x-9|}{(9-x)(9+x)} & \text{for } x < 9 \\ 3x - c & \text{for } x \geq 9 \end{cases}$$

$$c = 27 - \frac{1}{18} = \frac{485}{18}$$

21. State the three points to be satisfied for a function  $f(x)$  to be continuous at  $x = a$ .

1.  $f(a)$  exists \_\_\_\_\_

2. limit as  $x$  approaches  $a$  exists \_\_\_\_\_

3. above two are equal \_\_\_\_\_

Use the above definition to determine whether  $f(x) = \frac{x^3 - 1}{x^2 - 3x + 2}$  is continuous or discontinuous at

a)  $x = 0$  *cont.*

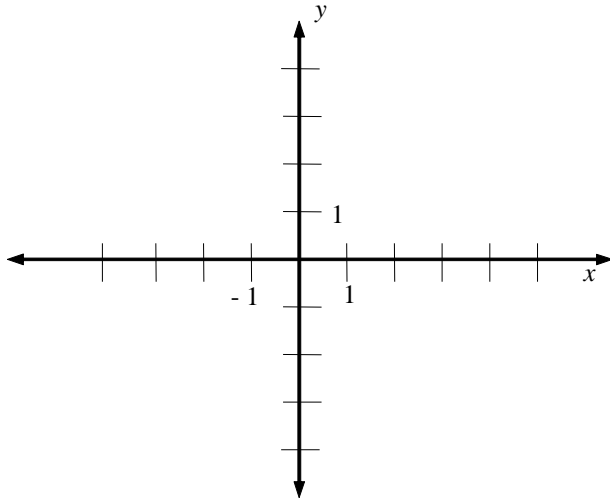
b)  $x = 1$  *hole*

c)  $x = 2$  *inf.*

d) If the function is discontinuous at  $x = 0, 1$  and/or  $2$ , describe the type of discontinuity.

22. Sketch the graph of a function  $f(x)$  that satisfies the following conditions.

- $f(0) = 3$ ,  $f(-2) = 2$ ,  $f(x)$  has exactly one point of discontinuity
- $f'(0) = 0$
- $f'(x) < 0$  on  $(0, 4)$ ,  $f'(x) > 0$  on  $(-\infty, 0) \cup (4, \infty)$
- $f''(x) < 0$  on  $(-2, 4) \cup (4, \infty)$ ,  $f''(x) > 0$  on  $(-\infty, -2)$
- $\lim_{x \rightarrow \infty} f(x) = \infty$ ,  $\lim_{x \rightarrow 4} f(x) = -\infty$



23. Given the function  $f(x) = \sqrt[5]{x}$ ,
- Find the linear approximation,  $L(x)$ , to the function at  $a = 32$ .
  - Use your result from part (a) to approximate  $\sqrt[5]{33}$ . Answer to **4** decimal places or exact value.

a)  $L(x) = 2 + \frac{1}{80}(x - 32)$

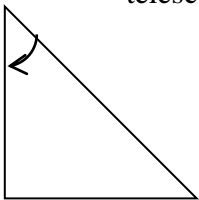
b)  $L(33) = 2 + \frac{1}{80} = \frac{161}{80} \approx 2.0125$

24. Given the function  $f(x) = \sin(x - 1)$ ,
- Find the linear approximation,  $L(x)$ , to the function at  $a = 1$ .
  - Use your result from part (a) to approximate  $\sin(-0.1)$ .

a)  $L(x) = x - 1$

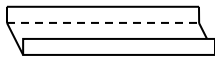
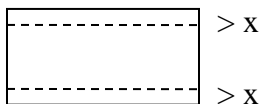
b)  $L(0.9) = -0.1$

25. A woman standing on a cliff is watching a motorboat through a telescope as the boat approaches the shoreline directly below her. If the telescope is 300 feet above the water level and if the boat is approaching at 5 feet per second, at what rate is the angle,  $A$ , of the telescope changing when the boat is 150 feet from the shore? (radian mode)



$$\frac{dA}{dt} = \frac{1}{75} \text{ rad/sec} \approx 0.01333\bar{3} \text{ rad/sec}$$

26. A long rectangular sheet of metal, 9 inches wide, is to be turned up at both sides to make a horizontal gutter with vertical sides. How many inches should be turned up at each side for maximum carrying capacity?



2.25 in

27. Using calculus, determine the absolute minimum and maximum for the function

$$f(x) = \frac{-x^8}{8} + x^6 - 5 \text{ on the interval } [0, 1].$$

*absol min of -5, absol max of -33/8*

28. Using calculus, determine all local minimum and maximum for the function

$$f(x) = e^x(x^2 - x) \text{ . (change problem-find } x \text{ values only)}$$

*max at  $-1/2 - (\text{sqrt}5)/2$ , min at  $-1/2 + (\text{sqrt}5)/2$*

29. Suppose a function  $g(x)$  has the following derivative:  $g'(x) = x^4(x^2 + 2)(3x + 7)(x - 5)^3$ .

On what interval(s) is  $g(x)$  increasing?

*$(-\text{inf}, -7/3), (5, \text{inf})$*

30. Use Newton's method to find all roots of the given equation to 6 decimal places.

$$\cos x = \sqrt{x}$$

*0.641714*

31. An object was dropped off a building and hit the ground with a speed of 73 feet per second.

What is the height of the building? (use  $a(t) = -32 \frac{ft}{s^2}$ )

*5329/64, which is approximately equal to 83.265625*

32. Let  $s(t) = t^3 - 12t + 3$ ,  $t \geq 0$  be a position function for an object that moves along a horizontal line. Chart the motion of the object. When is the object speeding up? When is the object moving right?

*the object is speeding up when  $t > 2$ . The object is also moving right when  $t > 2$ .*

33. Evaluate the Riemann sum for  $f(x) = x \ln(1 + x^2)$ ,  $[0,3]$  with 4 subintervals taking the sample points to be the right endpoints.

6.45375357

- a) What integral does the Riemann sum approximate?  
 b) Express the definite integral in part a) as the limit of a Riemann sum.

34. Evaluate each integral.

a)  $\int_1^2 \frac{(x+2)(x-3)}{x^3} dx$        $\ln 2 - 11/4$

b)  $\int_0^1 \frac{6}{x^2 + 1} dx$        $3\pi/2$

c)  $\int (e^{\cos x} \cdot \sin x + \cos 9x) dx$        $-e^{\cos x} + 1/9 \sin 9x + c$

d)  $\int (\csc^2 4x + \frac{\sin^{-1} x}{\sqrt{1-x^2}}) dx$        $-1/4 \cot 4x + (\arcsin x)^2/2 + c$

e)  $\int \frac{(\ln x)^4}{x} dx$        $((\ln x)^5)/5 + c$

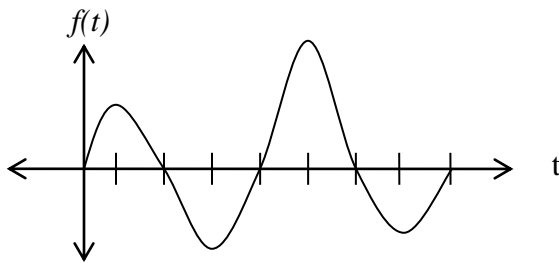
f)  $\int_0^1 3x(5+x^2)^8 dx$        $(1/6)(6^9 - 5^9)$

35. Find the interval on which  $y = \int_0^x \frac{1}{1+t+t^2} dt$  is concave up.

*The function is concave up when  $x < -1/2$ .*

36. Let  $g(x) = \int_0^x f(t) dt$  where  $f$  is the function whose graph is shown.

At what values of  $x$  do the local maximum and minimum values of  $g$  occur?



Local max at  $x=2,6$

Local min at  $x=4$

37. Find all values of  $c$  in the given interval that satisfy the Mean Value Theorem for :

$$f(x) = x^3 + x - 1, [0,2]$$

$\pm 2/\sqrt{3}$

38. Let  $f(2) = 8$ ,  $f'(x) \leq 6$  for  $2 \leq x \leq 5$ . How large can  $f(5)$  possibly be?

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39. Graph  $f(x) = \frac{x^2}{x^2 - 9}$ . Label all asymptotes, local max/mins, inflection points, if they exist.

40. The diameter of a sphere is measured to be 6 inches with a possible error of 0.05 inches. Use differentials to estimate the maximum error in the calculated volume.

$36\pi(0.025)$

**This review may not cover every topic on the final exam. It is essential to reread the text, go over your instructor's class notes, review all assigned HW then find similar problems in the chapter reviews for more drill. Begin your final exam review at least 2 weeks prior to the final.**