

**McCombs Math 232 Test 2 Part 1 Fall 2019 Key**  
**Text Sections 11.1–11.4**

(10 points) 1. Choose the Taylor polynomial of order 3 centered at  $a = 1$  for the function

$$f(x) = \frac{1}{(1+2x)^2}$$

You must show work that supports your choice.

$p_3(x) = \frac{1}{9} - \frac{4}{27}(x-1) + \frac{8}{27}(x-1)^2 - \frac{192}{243}(x-1)^3$ <p>A</p>	$p_3(x) = \frac{1}{9} - \frac{4}{27}(x+1) + \frac{8}{27}(x+1)^2 - \frac{32}{243}(x+1)^3$ <p>B</p>
$p_3(x) = \frac{1}{9} - \frac{4}{27}(x+1) + \frac{8}{27}(x+1)^2 - \frac{192}{243}(x+1)^3$ <p>C</p>	$D. p_3(x) = \frac{1}{9} - \frac{4}{27}(x-1) + \frac{4}{27}(x-1)^2 - \frac{32}{243}(x-1)^3$ <p>D</p>

$$f(x) = \frac{1}{(1+2x)^2} = (1+2x)^{-2} \Rightarrow f(1) = \frac{1}{9} \Rightarrow c_0 = \frac{1}{9}$$

$$f'(x) = -2(1+2x)^{-3}(2) \Rightarrow f'(1) = -\frac{4}{27} \Rightarrow c_1 = -\frac{4}{27}$$

$$f''(x) = 6(1+2x)^{-4}(4) \Rightarrow f''(1) = \frac{24}{81} \Rightarrow c_2 = \left(\frac{24}{81}\right)\left(\frac{1}{2!}\right) = \frac{4}{27}$$

$$f'''(x) = -24(1+2x)^{-5}(8) \Rightarrow f'''(1) = -\frac{192}{243} \Rightarrow c_3 = \left(-\frac{192}{243}\right)\left(\frac{1}{6}\right) = -\frac{32}{243}$$

(10 points) 2. The Taylor series centered at  $a = 7$  for the function  $y = f(x)$  is given by

$$f(x) = 1 + 6(x-7)^3 + 10(x-7)^6 + 28(x-7)^9 + 5(x-7)^{12} + \dots$$

<p>(i) Determine the exact value of <math>f''(7)</math>.</p> <p>You must show work that supports your answer.</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> <math>f''(7) = 0</math> </div>	<p>(ii) Determine the exact value of <math>f^{(9)}(7)</math>.</p> <p>You must show work that supports your answer.</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> <math>f^{(9)}(7) = 28(9!)</math> </div>
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(10 points) 3. Use the Absolute Ratio Test to determine the interval of convergence for the given power series. Make sure you check the endpoints, if any. You must show work that supports your answer.

$$\sum_{n=1}^{\infty} \frac{(7x-2)^n}{\sqrt[5]{n}}$$

convergence interval:  $\frac{1}{7} \leq x < \frac{3}{7}$

$$\lim_{n \rightarrow \infty} \left( \frac{|7x-2|^{n+1}}{\sqrt[5]{n+1}} \right) \left( \frac{\sqrt[5]{n+1}}{|7x-2|^n} \right) = \lim_{n \rightarrow \infty} \left( \frac{|7x-2|^{n+1}}{|7x-2|^n} \right) \left( \frac{\sqrt[5]{n}}{\sqrt[5]{n+1}} \right) \rightarrow |7x-2|(1)$$

For convergence, we need  $|7x-2| < 1$

$$-1 < 7x-2 < 1$$

$$\frac{1}{7} < x < \frac{3}{7}$$

Check endpoints

when  $x = \frac{1}{7}$ :  $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[5]{n}}$  which converges as an alternating p-series,  $p = 1/5$

when  $x = \frac{3}{7}$ :  $\sum_{n=1}^{\infty} \frac{(1)^n}{\sqrt[5]{n}}$  which diverges as a p-series,  $p = 1/5$

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- (10 points) 4. Use a known series to construct the first five nonzero terms of the Taylor series centered at  $a = 0$  for the given function and determine the convergence interval. You must show work that supports your answer.

$$f(x) = \frac{x^2}{3+6x}$$

$$\text{convergence interval: } -\frac{1}{2} < x < \frac{1}{2}$$

$$f(x) = \left(\frac{x^2}{3}\right) \left(1 - 2x + 4x^2 - 8x^3 + 16x^4 + \dots\right) = \frac{x^2}{3} - \frac{2x^3}{3} + \frac{4x^4}{3} - \frac{8x^5}{3} + \frac{16x^6}{3} + \dots$$

$$f(x) = \frac{x^2}{3+6x} = \left(\frac{x^2}{3}\right) \left(\frac{1}{1+2x}\right)$$

$$\frac{1}{1+2x} = \frac{1}{1-(-2x)} = 1 - 2x + 4x^2 - 8x^3 + 16x^4 + \dots \quad \text{valid for } |-2x| < 1 \Rightarrow -\frac{1}{2} < x < \frac{1}{2}$$

$$f(x) = \left(\frac{x^2}{3}\right) \left(1 - 2x + 4x^2 - 8x^3 + 16x^4 + \dots\right) = \frac{x^2}{3} - \frac{2x^3}{3} + \frac{4x^4}{3} - \frac{8x^5}{3} + \frac{16x^6}{3} + \dots$$

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(10 points) 5. (i) Use the first five non-zero terms of a known series to approximate the value of the integral. You must show work that supports your answer.

$$\int_0^1 \ln(1+x^4) dx$$

$$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n+1} \binom{x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} + \dots,$$

$$\ln(1+x^4) = \sum_{n=1}^{\infty} (-1)^{n+1} \binom{x^{4n}}{n} = x^4 - \frac{x^8}{2} + \frac{x^{12}}{3} - \frac{x^{16}}{4} + \frac{x^{20}}{5} + \dots,$$

$$\int_0^1 \ln(1+x^4) dx = \int_0^1 \left( x^4 - \frac{x^8}{2} + \frac{x^{12}}{3} - \frac{x^{16}}{4} + \frac{x^{20}}{5} + \dots \right) dx \quad \text{valid for } -1 < x \leq 1$$

$$\approx \left[ \frac{x^5}{5} - \frac{x^9}{18} + \frac{x^{13}}{39} - \frac{x^{17}}{68} + \frac{x^{21}}{105} \right]_0^1 = \boxed{\frac{1}{5} - \frac{1}{18} + \frac{1}{39} - \frac{1}{68} + \frac{1}{105}}$$

**BONUS**

(2 points) (ii) The value obtained in part (i) has an error less than or equal to

$$\boxed{\frac{1}{6 \cdot 25} = \frac{1}{150}}$$

Explain your answer.

Since we have a convergent alternating series the partial sum of the first five terms  $a_1 - a_2 + a_3 - a_4 + a_5$  has an error that is less than or equal to  $|a_6|$ .

$$\int_0^1 \left( x^4 - \frac{x^8}{2} + \frac{x^{12}}{3} - \frac{x^{16}}{4} + \frac{x^{20}}{5} - \frac{x^{24}}{6} + \dots \right) dx = \underbrace{\frac{1}{5} - \frac{1}{2 \cdot 9} + \frac{1}{3 \cdot 13} - \frac{1}{4 \cdot 17} + \frac{1}{5 \cdot 21}}_{S_5} - \underbrace{\frac{1}{6 \cdot 25}}_{a_6}$$

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Part 2-----

1. Find the exact sum of the series. You must show work that supports your answer.

$$(i) \quad 3 - \frac{3}{5} + \frac{(3/25)}{2!} - \frac{(3/125)}{3!} + \frac{(3/625)}{4!} - \dots$$

$$3 \left( 1 - \frac{1}{5} + \frac{(-1/5)^2}{2!} - \frac{(-1/5)^3}{3!} + \frac{(-1/5)^4}{4!} - \dots \right) = \boxed{3e^{-1/5}}$$

Taylor Series centered at  $a=0$   
for  $e^x$  when  $x = -1/5$

$$(ii) \quad 0.5\rho - \frac{(0.5\rho)^3}{3!} + \frac{(0.5\rho)^5}{5!} - \frac{(0.5\rho)^7}{7!} + \frac{(0.5\rho)^9}{9!} - \dots$$

$$0.5\pi - \frac{(0.5\pi)^3}{3!} + \frac{(0.5\pi)^5}{5!} - \frac{(0.5\pi)^7}{7!} + \frac{(0.5\pi)^9}{9!} - \dots = \boxed{\sin(-0.5\rho) = -1}$$

Taylor Series centered at  $a=0$   
for  $\sin(x)$  when  $x = -0.5\pi$

2. Use Integration by Parts to evaluate the given integral.

$$\int \ln(\sin(x)) dx$$

$$u = \sin(\ln(x)) \quad dv = dx$$

$$du = \cos(\ln(x)) \left(\frac{1}{x}\right) dx \quad v = x$$

$$\int \ln(\sin(x)) dx = x \sin(\ln(x)) - \int x \cos(\ln(x)) \left(\frac{1}{x}\right) dx \quad u = \cos(\ln(x)) \quad dv = dx$$

$$= x \sin(\ln(x)) - \int \cos(\ln(x)) dx \quad du = -\sin(\ln(x)) \left(\frac{1}{x}\right) dx \quad v = x$$

$$\int \ln(\sin(x)) dx = x \sin(\ln(x)) - \left( x \cos(\ln(x)) + \int x \sin(\ln(x)) \left(\frac{1}{x}\right) dx \right)$$

$$\int \ln(\sin(x)) dx = x \sin(\ln(x)) - x \cos(\ln(x)) - \int \sin(\ln(x)) dx$$

$$2 \int \ln(\sin(x)) dx = x \sin(\ln(x)) - x \cos(\ln(x))$$

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$$\int \ln(\sin(x)) dx = \frac{x \sin(\ln(x)) - x \cos(\ln(x))}{2} + C$$

3. Use Integration by Parts to evaluate the given integral.

$$\int 12x \cos(4x) dx \quad \begin{array}{ll} u = 12x & dv = \cos(4x) dx \\ du = 12 dx & v = \frac{1}{4} \sin(4x) \end{array}$$

$$\int 12x \cos(4x) dx = 3x \sin(4x) - \frac{12}{4} \int \sin(4x) dx$$

$$= 3x \sin(4x) + \frac{3}{4} \cos(4x) + C$$

4. Note that  $\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$ . Use Integration by Parts to evaluate the integral.

$$\int \sin^{-1}(x) dx \quad \begin{array}{ll} u = \sin^{-1}(x) & dv = dx \\ du = \frac{1}{\sqrt{1-x^2}} dx & v = x \end{array}$$

$$\int \sin^{-1}(x) dx = x \sin^{-1}(x) - \int \frac{x}{\sqrt{1-x^2}} dx$$

$$p = 1 - x^2$$

$$dp = -2x dx \quad \Rightarrow \quad -\frac{1}{2} dp = x dx$$

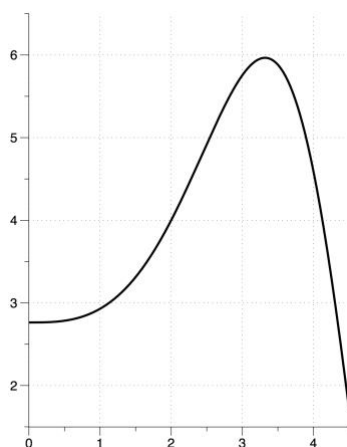
$$\int \sin^{-1}(x) dx = x \sin^{-1}(x) + \frac{1}{2} \int p^{-1/2} dp$$

$$= x \sin^{-1}(x) + \frac{1}{2} (2p^{1/2})$$

$$= x \sin^{-1}(x) + (1-x^2)^{1/2} + C$$

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5. The graph of the function  $y = f(x)$  is shown below.



Determine whether the statement is true or false.  
You must explain your answers.

(i) The 3rd degree Taylor polynomial for  $f$ , centered at  $a = 2$ , is given by

$$p_3(x) = 4 - 1.67(x-2) + 0.51(x-2)^2 - 0.22(x-2)^3$$

TRUE FALSE

because the coefficient on the  $(x-2)$  term tells us that  $f'(2) = -1.67 < 0$ , which means the curve should be decreasing at  $x = 2$ . However, this curve is increasing at  $x = 2$ .

(ii) The 3rd degree Taylor polynomial for  $f$ , centered at  $a = 2$ , is given by

$$p_3(x) = 4 + 1.67(x-2) - 0.51(x-2)^2 - 0.22(x-2)^3$$

TRUE FALSE

because the coefficient on the  $(x-2)^2$  term tells us that  $f''(2) = (-0.51)(2!) < 0$ , which means the curve should be concave down at  $x = 2$ . However, this curve is concave up at  $x = 2$ .

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