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$$\cos(x)=\sum_{0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!}=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\cdots,\text{ valid for all } x$$

$$\sin(x)=\sum_{0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!}=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\cdots,\text{ valid for all } x$$

$$\ln(1+x)=\sum_{1}^{\infty}(-1)^{n+1}\left(\frac{x^n}{n}\right)=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\ldots,\text{ valid for } -1<x\le 1$$

(10 points) 1. Choose the Taylor polynomial of order 3 centered at a = 1 for the function

$$f(x)=\frac{1}{(1+2x)^2}$$

You must show work that supports your choice.

$$p_3(x)=\frac{1}{9}-\frac{4}{27}(x+1)+\frac{4}{27}(x+1)^2-\frac{32}{243}(x+1)^3$$

A

$$p_3(x)=\frac{1}{9}-\frac{4}{27}(x-1)+\frac{8}{27}(x-1)^2-\frac{192}{243}(x-1)^3$$

B

$$p_3(x)=\frac{1}{9}-\frac{4}{27}(x+1)+\frac{8}{27}(x+1)^2-\frac{192}{243}(x+1)^3$$

C

D. $p_3(x)=\frac{1}{9}-\frac{4}{27}(x-1)+\frac{4}{27}(x-1)^2-\frac{32}{243}(x-1)^3$

D

(10 points) 2. The Taylor series centered at a = 7 for the function $y=f(x)$ is given by

$$f(x)=1+6(x-7)^3+10(x-7)^6+28(x-7)^9+5(x-7)^{12}+\ldots.$$

(i) Determine the exact value of $f''(7)$.

You must show work that supports your answer.

$f''(7)=$

(ii) Determine the exact value of $f^{(9)}(7)$.

You must show work that supports your answer.

$f^{(9)}(7)=$

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(10 points) 3. Use the Absolute Ratio Test to determine the interval of convergence for the given power series. Make sure you check the endpoints, if any. You must show work that supports your answer.

$$\sum_{n=1}^{\infty} \frac{(7x-2)^n}{\sqrt[5]{n}}$$

convergence interval:

(10 points) 4. Use a known series to construct the first five non-zero terms of the Taylor series centered at $a = 0$ for the given function and determine the convergence interval. You must show work that supports your answer.

$$f(x) = \frac{x^2}{3+6x}$$

convergence interval:

$f(x) =$

(10 points) 5. (i) Use the first five non-zero terms of a known series to approximate the value of the integral. You must show work that supports your answer.

$$\int_0^1 \ln\left(1+x^4\right)\, dx$$

(ii) The value obtained in part (i) has an error less than or equal to

Explain your answer.

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Part 2---

$$\cos(x) = \sum_{0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots, \text{ valid for all } x$$

$$\sin(x) = \sum_{0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots, \text{ valid for all } x$$

$$\ln(1+x) = \sum_{1}^{\infty} (-1)^{n+1} \left(\frac{x^n}{n}\right) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots, \text{ valid for } -1 < x \le 1$$

1. Find the exact sum of the series. You must show work that supports your answer.

(i) $$3 - \frac{3}{5} + \frac{(3/25)}{2!} - \frac{(3/125)}{3!} + \frac{(3/625)}{4!} \cdots$$

(ii) $$0.5\pi - \frac{(0.5\pi)^3}{3!} + \frac{(0.5\pi)^5}{5!} - \frac{(0.5\pi)^7}{7!} + \frac{(0.5\pi)^9}{9!} + \ldots$$

2. Use Integration by Parts to evaluate the given integral.
You must show work that supports your answer.

$$\int \sin(\ln(x))\, dx$$

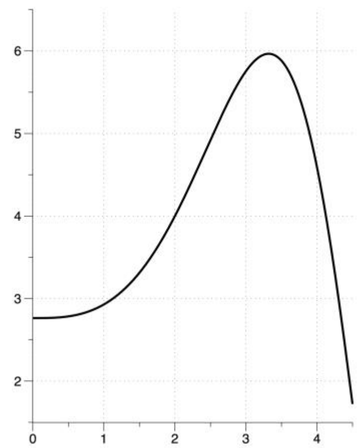
3. Use Integration by Parts to evaluate the given integral.
You must show work that supports your answer.

$$\int 12x \cdot \cos(4x)\, dx$$

4. Note that $\frac{d}{dx}\left(\sin^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}}$. Use Integration by Parts to evaluate the given integral. You must show work that supports your answer.

$$\int \sin^{-1}(x)\, dx$$

5. The graph of the function $y = f(x)$ is shown below.



Determine whether the statement is true or false.
You must explain your answers.

(i) The 3rd degree Taylor polynomial for f, centered at a = 2, is given by

$$p_3(x) = 4 - 1.67(x-2) + 0.51(x-2)^2 - 0.22(x-2)^3$$

TRUE FALSE

because

(ii) The 3rd degree Taylor polynomial for f, centered at a = 2, is given by

$$p_3(x) = 4 + 1.67(x-2) - 0.51(x-2)^2 - 0.22(x-2)^3$$

TRUE FALSE

because

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