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1. Consider the series $\sum_{n=1}^{\infty} a_n$ such that $a_n > 0$, and $S_n =$ the n-th partial sum of the series.
Complete each statement.

(6 points) (i) If $\lim_{n\to\infty}(S_n) = \frac{1}{3}$, then $\lim_{n\to\infty}(a_n) = 0$

because

$\lim_{n\to\infty}(S_n) = \frac{1}{3}$ means that the series converges to the sum $\frac{1}{3}$, so $\lim_{n\to\infty}(a_n)$ MUST $= 0$

(6 points) (ii) If $\lim_{n\to\infty}(a_n) = \frac{1}{2}$, then $\lim_{n\to\infty}(S_n) =$ does not exist

because $\lim_{n\to\infty}(a_n) \neq 0$ means the series MUST diverge.

(6 points) (iii) The series $\sum_{n=1}^{\infty}\left(\frac{|2\cos(n)+3|}{n^5+2}\right)$ *converges* *diverges* (*choose one*)

because we can use the Basic Comparison Test with $\sum_{n=1}^{\infty}\left(\frac{5}{n^5}\right)$.

$a_n = \left(\frac{|2\cos(n)+3|}{n^5+2}\right) \le \frac{5}{n^5+2} \le \frac{5}{n^5} = b_n$ $\quad\sum_{n=1}^{\infty}\left(\frac{5}{n^5}\right)$ is a convergent p-series with $p = 5 > 1$

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2. Answer each question for the series $\sum_{n=1}^{\infty}\left(\frac{6}{n(n+2)}\right)$.

(6 points) (i) Use the Limit Comparison Test to determine whether the series converges or diverges. You must show supporting work.

converges	diverges
(choose	one)

$$a_n = \frac{6}{n(n+2)} \approx \frac{6}{n^2} = b_n$$

$\sum_{n=1}^{\infty}\left(\frac{6}{n^2}\right)$ is a convergent p-series with $p = 2 > 1$

$$\lim_{n\to\infty}\left(\frac{a_n}{b_n}\right) = \lim_{n\to\infty}\left(\frac{6}{n(n+2)}\right)\left(\frac{n^2}{6}\right) \to \frac{6n^2}{6n^2} = 1 > 0 \text{ and finite}$$

(6 points) (ii) Determine the value of the partial sum S_5. You must show supporting work.

$$S_5 = \frac{6}{1\cdot 3} + \frac{6}{2\cdot 4} + \frac{6}{3\cdot 5} + \frac{6}{4\cdot 6} + \frac{6}{5\cdot 7} = 2 + \frac{3}{4} + \frac{2}{5} + \frac{1}{4} + \frac{6}{35}$$

(6 points) (iii) Determine the total sum of the series, if it exists. You must show supporting work.

$$S = \lim_{n\to\infty}(S_n) = \lim_{n\to\infty}\left(\frac{3}{1} + \frac{3}{2} - \frac{3}{n+1} - \frac{3}{n+2}\right) \to 3 + \frac{3}{2} - 0 - 0 = \boxed{3 + \frac{3}{2} = \frac{9}{2}}$$

$$\sum_{n=1}^{\infty}\left(\frac{6}{n(n+2)}\right) = \sum_{n=1}^{\infty}\left(\frac{3}{n} - \frac{3}{n+2}\right) = \left(\frac{3}{1} - \frac{3}{3}\right) + \left(\frac{3}{2} - \frac{3}{4}\right) + \left(\frac{3}{3} - \frac{3}{5}\right) + \left(\frac{3}{4} - \frac{3}{6}\right) + \ldots$$

$$\frac{6}{n(n+2)} = \frac{A}{n} + \frac{B}{n+2}$$

when $n = 0$: $6 = A(2) + B(0) \Rightarrow A = 3$

when $n = -2$: $6 = A(0) + B(-2) \Rightarrow B = -3$

$$6 = A(n+2) + B(n)$$

$$S_1 = \left(\frac{3}{1} - \frac{3}{3}\right) \qquad S_2 = \left(\frac{3}{1} - \frac{3}{3}\right) + \left(\frac{3}{2} - \frac{3}{4}\right) \qquad S_3 = \left(\frac{3}{1} - \frac{3}{3}\right) + \left(\frac{3}{2} - \frac{3}{4}\right) + \left(\frac{3}{3} - \frac{3}{5}\right) = \frac{3}{1} + \frac{3}{2} - \frac{3}{4} - \frac{3}{5}$$

$$S_4 = \frac{3}{1} + \frac{3}{2} - \frac{3}{4} - \frac{3}{5} + \left(\frac{3}{4} - \frac{3}{6}\right) = \frac{3}{1} + \frac{3}{2} - \frac{3}{5} - \frac{3}{6} \qquad S_n = \frac{3}{1} + \frac{3}{2} - \frac{3}{n+1} - \frac{3}{n+2}$$

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(6 points) 3. Choose the best answer.

The series $\frac{\pi}{1} - \frac{\pi}{\sqrt[4]{2}} + \frac{\pi}{\sqrt[4]{3}} - \frac{\pi}{\sqrt[4]{4}} + \ldots = \pi \sum_{1}^{\infty} (-1)^{n+1} \left(\frac{1}{n^{1/4}} \right)$

converges conditionally	converges absolutely	diverges
A	B	C

because **we have an alternating p-series with $p = \frac{1}{4} < 1$.**

4. Consider the series $\sum_{n=1}^{\infty} \left(\frac{5^n}{7^n - 3^n} \right)$. Determine whether the given statement is TRUE or FALSE.

(4 points) (i) $\sum_{n=1}^{\infty} \left(\frac{5^n}{7^n - 3^n} \right)$ converges by the Basic Comparison Test with $\sum_{n=1}^{\infty} \left(\frac{5^n}{7^n} \right)$

$a_n = \frac{5^n}{7^n - 3^n} > \frac{5^n}{7^n} = b_n$ $\quad \sum_{1}^{\infty} b_n = \underbrace{\sum_{1}^{\infty} \left(\frac{5}{7} \right)^n}_{\text{convergent geometric with } r = \frac{5}{7}}$

TRUE *FALSE*
(*choose one*)

because Convergence by the Basic Comparison Test requires $a_n \le b_n$, with $\sum_{n=1}^{\infty} b_n$ convergent.

But we have $a_n > b_n$, i.e., $\frac{5^n}{7^n - 3^n} > \frac{5^n}{7^n}$ since $7^n - 3^n < 7^n$.

(4 points) (ii) $\sum_{n=1}^{\infty} \left(\frac{5^n}{7^n - 3^n} \right)$ diverges by the Basic Comparison Test with $\sum_{n=1}^{\infty} \left(\frac{5^n}{7^n} \right)$.

TRUE *FALSE*
(*choose one*)

because Divergence by the Basic Comparison Test requires $a_n \ge b_n$, with $\sum_{n=1}^{\infty} b_n$ divergent.

But we have $\sum_{n=1}^{\infty} b_n = \underbrace{\sum_{1}^{\infty} \left(\frac{5}{7} \right)^n}_{\text{convergent geometric with } r = \frac{5}{7}}$

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5. Choose the best answer for each question.

(4 points) (i) The series $\sum_{n=1}^{\infty}\left(1+\frac{6}{n}\right)^{5n}$

converges conditionally	converges absolutely	diverges
A	B	C

because $\lim_{n\to\infty}(a_n)=\lim_{n\to\infty}\left(1+\frac{6}{n}\right)^{5n}\to e^{30}\neq 0$

(4 points) (ii) The series $\sum_{n=1}^{\infty}\left(\frac{(-2)^{10}}{n^{\pi}+\sqrt{n}}\right)\approx 2^{10}\cdot\sum_{n=1}^{\infty}\left(\frac{2^{10}}{n^{\pi}}\right)$

converges conditionally	converges absolutely	diverges
A	B	C

because it behaves like a p-series with $p=\pi>1$.

PART 2 --

(6 points) 1. Determine whether the given sequence converges or diverges. If it converges find the limit. You must show supporting work.

$$\left(\frac{1+p}{1}\right)^{5},\ \left(\frac{2+p}{2}\right)^{10},\ \left(\frac{3+p}{3}\right)^{15},\ \left(\frac{4+p}{4}\right)^{20},\ldots$$

We have the sequence $\left\{\left(\frac{n+\pi}{n}\right)^{5n}\right\}_{n=1}^{\infty}$ $\quad\lim_{n\to\infty}\left(\frac{n+\pi}{n}\right)^{5n}=\lim_{n\to\infty}\left(1+\frac{\pi}{n}\right)^{5n}\to e^{5\cdot\pi}$

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converges	*diverges*
with limit $= \boxed{e^{5\cdot\pi}}$	
(*choose one*)	

(6 points) 2. Use the Ratio Test to determine whether the series converges or diverges. You must show supporting work.

$$\sum_{n=1}^{\infty}\left(\frac{3^n}{5^{n+1}(n+4)}\right)$$

converges	*diverges*
(*choose one*)	

because $\boxed{\lim_{n\to\infty}\left(\frac{a_{n+1}}{a_n}\right)=\frac{3}{5}<1}$

$$\lim_{n\to\infty}\left(\frac{a_{n+1}}{a_n}\right)=\lim_{n\to\infty}\left(\frac{3^{n+1}}{5^{n+2}(n+5)}\right)\left(\frac{5^{n+1}(n+4)}{3^n}\right)$$

$$=\lim_{n\to\infty}\left(\frac{5^{n+1}}{5^{n+2}}\right)\left(\frac{3^{n+1}}{3^n}\right)\left(\frac{n+4}{n+5}\right)\to\left(\frac{1}{5}\right)(3)(1)=\frac{3}{5}<1$$

(6 points) 3. Determine whether the series is absolutely convergent, conditionally convergent, or divergent. You must show supporting work.

$$\sum_{n=0}^{\infty}(-1)^n\left(\frac{\sqrt{n^6+9n+5}}{n^4+1}\right)\approx\sum_{n=0}^{\infty}(-1)^n\left(\frac{\sqrt{n^6}}{n^4}\right)\approx\sum_{n=0}^{\infty}(-1)^n\left(\frac{1}{n}\right)$$

absolutely convergent	*conditionally convergent*	*divergent*
	(*choose one*)	

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because **it acts like the alternating harmonic series**

4. Consider the series $\sum_{n=1}^{\infty}\left(\frac{x^n}{10^n}\right)$.

(6 points) (i) Determine the sum of the series when $x = 5$.
You must show supporting work.

$$\sum_{n=1}^{\infty}\left(\frac{5^n}{10^n}\right)=\sum_{n=1}^{\infty}\left(\frac{5}{10}\right)^n \quad \text{geometric series with } r=\frac{1}{2} \text{ and } a=\frac{1}{2}$$

$$sum=\frac{a}{1-r}=\frac{\frac{1}{2}}{1-\frac{1}{2}}=1$$

(6 points) (ii) Determine all values of x for which the series converges.
You must show supporting work.

$$\sum_{n=1}^{\infty}\left(\frac{x^n}{10^n}\right)=\sum_{n=1}^{\infty}\left(\frac{x}{10}\right)^n \quad \text{geometric series with } r=\frac{x}{10}$$

The series converges provided that $|r|<1$.

$$\left|\frac{x}{10}\right|<1 \Rightarrow -1<\frac{x}{10}<1 \Rightarrow \boxed{-10<x<10}$$

5. Consider the series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ such that $a_n > 0$ and $b_n > 0$.

(6 points) (i) If the series $\sum_{n=1}^{\infty} a_n$ diverges, then the sequence $\left\{a_n\right\}_1^{\infty}$ MUST diverge also.

TRUE	*FALSE*
(choose one)	

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because for example, $\sum_{1}^{\infty}\left(\frac{1}{n}\right)$ diverges, but $\left\{\frac{1}{n}\right\}_1^{\infty}$ converges to 0.

(6 points) (ii) If the sequence $\left\{a_n\right\}_1^{\infty}$ converges, then the series $\sum_{n=1}^{\infty} a_n$ MUST converge also.

TRUE *FALSE*
(*choose one*)

because for example, $\left\{\frac{1}{n}\right\}_1^{\infty}$ converges to 0, but $\sum_{1}^{\infty}\left(\frac{1}{n}\right)$ diverges.

BONUS

(4 points) Give SPECIFIC examples of series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$

so that $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ both diverge, but $\sum_{n=1}^{\infty}\left(a_n \cdot b_n\right)$ converges.

$$\sum_{n=1}^{\infty} a_n = \underbrace{\sum_{n=1}^{\infty} \boxed{\frac{1}{n}}}_{\substack{\text{divergent} \\ \text{harmonic series}}} \qquad \sum_{n=1}^{\infty} b_n = \underbrace{\sum_{n=1}^{\infty} \boxed{\frac{1}{n^{1/2}}}}_{\substack{\text{divergent p-series} \\ p=\frac{1}{2}<1}}$$

$$\sum_{n=1}^{\infty}\left(a_n \cdot b_n\right) = \sum_{n=1}^{\infty}\left(\frac{1}{n} \cdot \frac{1}{n^{1/2}}\right) = \underbrace{\sum_{n=1}^{\infty}\left(\frac{1}{n^{3/2}}\right)}_{\substack{\text{convergent p-series} \\ p=\frac{3}{2}>1}}$$