Math 232 Exam 2

Key

Print Name____

Honor Code signature____

Clearly print UNC email address (using ONYEN)____

No calculators. Show photo ID when turning in exam.
Problem worth is next to each problem.
Show all work to ensure credit. No credit if your work does not support the solution.

Due to the close proximity of your fellow students during this exam, be careful to not look at other exams. The appearance of looking around may be misunderstood and result in a zero and honor court for cheating.

2**1)** Every monotonic sequence is convergent.

a) True
b) False

needs to be bounded too

2**2)** An infinite series $\sum_{n=1}^{\infty} a_n$ is convergent if $\lim_{n\to\infty} a_n = 0$.

a) True
b) False

ex) $\sum \frac{1}{n}$

2**3)** An infinite sequence $\{a_n\}$ is convergent *only if* $\lim_{n\to\infty} a_n = 0$.

a) True
b) False

ex) $\left\{1+\frac{1}{n}\right\} \to 1$ as $n \to \infty$

2**4)** The improper integral $\int_1^2 (2-x)^p\,dx$ is convergent for $p = -3$.

a) True
b) False

$\int_1^2 \frac{1}{(2-x)^2}\,dx$ diverges, shifted $p = 2 > 1$

2**5)** A series $\sum_{n=1}^{\infty} a_n$ is convergent if $\lim_{n\to\infty} S_n$ converges.
(S_n = *nth partial sum*)

a) True
b) False

For #6-7, determine if the infinite sequence is convergent or divergent. If convergent, to what value? You must state a theorem or test to support your solution.

5**6)** $\left\{\frac{2-\pi}{\sqrt[3]{x}}\right\}$

typo, should be n

$$\lim_{n\to\infty} \frac{2-\pi}{n^{1/3}} = 0$$

5**7)** $\left\{\frac{(n-4)(n+2)}{(n-1)(n+6)}\right\}$

$$\lim_{n\to\infty} a_n = 1$$

by def. of convergence of an infinite sequence

For #8-9, determine if the infinite series is convergent or divergent. If convergent, find the sum. You must state a theorem or test to support your solution.

5**8)** $\sum_{n=2}^{\infty} \frac{3^{n+1}}{5^{n+1}}$

$$\sum_{n=2}^{\infty} \frac{3}{5}\left(\frac{3}{5}\right)^n$$

geometric, $r = 3/5 \Rightarrow$ conv.

$$a = (3/5)^3 = \frac{27}{125}$$

$$\frac{\frac{27}{125}}{1-3/5} = \frac{5}{2}\left(\frac{27}{125}\right) = \frac{27}{50}$$

5**9)** $\sum_{n=2}^{\infty} \frac{(n-2)(n+2)}{(n-1)(n+8)}$

$$\lim_{n\to\infty} a_n = 1 \neq 0$$

diverges by Divergence Test

7**10)** Which of the series are Conditionally Convergent? Circle letter of solution(s). You may have more than one answer. Briefly support your choice(s).

A) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

$\sum \frac{1}{n}$ div harmonic

$\sum \frac{(-1)^n}{n}$ conv. by AST

B) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

$\sum \frac{1}{\sqrt{n}}$ div. $p = \frac{1}{2}$

$\sum \frac{(-1)^n}{\sqrt{n}}$ conv. by AST

C) $\sum_{n=1}^{\infty} \frac{(-1)^n}{2^n}$

$\leftarrow$ Absol. conv. since $\sum \frac{1}{2^n}$ conv. geom, $r = \frac{1}{2}$

8**11)** Using the Remainder Estimate for Alternating Series, find an upper bound for the error using S_4 as an approximation to S.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^3} = \underbrace{1 - \frac{1}{8} + \frac{1}{27} - \frac{1}{64}}_{S_4} + \underbrace{\frac{1}{125}}_{b_5} - + \cdots$$

$$b_5 = \frac{1}{125}$$

8**12)** Determine the convergence or divergence of $\int_3^\infty \frac{1}{\sqrt{x^2-1}}\,dx$ using the Comparison Theorem.

Behaves like $\frac{1}{\sqrt{x^2}} = \frac{1}{x}$ for large x. Guess diverges.

$$0 \le \underbrace{\frac{1}{x}}_{\text{diverges},\ p=1} \le \frac{1}{\sqrt{x^2}} \le \frac{1}{\sqrt{x^2-1}}$$

Therefore $\int_3^\infty \frac{1}{\sqrt{x^2-1}}\,dx$ diverges

8**13)** Evaluate the improper integral or show that it is divergent. Use correct notation for all steps. $\int_2^\infty \frac{1}{x\,\ln x}\,dx$

$$\lim_{t\to\infty}\int_2^t \frac{1}{x\ln x}\,dx = \lim_{t\to\infty} \ln|\ln x|\Big]_2^t = \lim_{t\to\infty}\left(\ln|\ln t| - \ln|\ln 2|\right) = \infty$$

diverges

$$\int \frac{1}{x\ln x}\,dx \Rightarrow \begin{array}{l} u=\ln x \\ du = \frac{1}{x}\,dx \end{array} \Rightarrow \int \frac{1}{u}\,du = \ln|u| + C$$

$$\Rightarrow \ln|\ln x| + C$$

14) Determine the convergence or divergence of the series using the Comparison Test. $\sum_{n=1}^\infty \frac{\cos^2 n}{4^n+5}$

Behaves like $\frac{1}{4^n}$ for large n. Numerator bounded.

$$0 \le \frac{\cos^2 n}{4^n+5} \le \frac{1}{4^n}$$

converges, geometric $r=\frac{1}{4}$

8**15)** Using the Ratio Test, determine the interval of convergence of the series $\sum_{n=1}^\infty \frac{(x-3)^n}{n2^n}$

State all convergence tests used to form the solution.

$$\lim_{n\to\infty} \frac{|(x-3)^{n+1}|\, n\, 2^n}{|(x-3)^n|\,(n+1)\,2^{n+1}} = \lim_{n\to\infty} \frac{|x-3|}{2}\left(\frac{n}{n+1}\right)$$

$$= \frac{|x-3|}{2} < 1$$

must be <1 for conv.

$$-1 < \frac{x-3}{2} < 1$$

$$1 < x < 5$$

Check endpoints (Ratio Test limit = 1, inconclusive)

$x=5 \Rightarrow \sum \frac{2^n}{n2^n} = \sum \frac{1}{n}$ diverges, harmonic

$x=1 \Rightarrow \sum \frac{(-1)^n 2^n}{n2^n} = \sum \frac{(-1)^n}{n}$ converges, alter. harmonic

$$\boxed{\Rightarrow [1,5)}$$

7**16)** Which of the series converges by the Alternating Series Test? Circle letter of solution(s). You may have more than one answer. Briefly support solution below each one.

A) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$

$b_n = \frac{1}{n!}$

decreases and $\lim_{n\to\infty} \frac{1}{n!} = 0$

B) $\sum_{n=1}^{\infty} (-1)^{n+1} \left(\frac{n+3}{n+4}\right)$

$b_n = \frac{n+3}{n+4}$

$\lim_{n\to\infty} b_n = 1 \neq 0$

C) $\sum_{n=1}^{\infty} (-1)^n \left(\frac{5}{4}\right)^n$

diverges

geom, $r = \frac{5}{4} > 1$

8**17)** Determine the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{n^2}{n^3 - 4n + 5}$ using the Limit Comparison Test.

behaves like $\frac{n^2}{n^3} = \frac{1}{n}$ for large n. Guess diverges

let $b_n = \frac{1}{n}$

$$\lim_{n\to\infty} \frac{\frac{n^2}{n^3-4n+5}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n^3}{n^3-4n+5} = 1$$

both diverge

8**18)** Using the Remainder Estimate for the Integral Test, find an upper bound for the error using S_4 as an approximation to S.

$$\sum_{n=1}^{\infty} \frac{1}{n^2}$$

$$\int_4^{\infty} \frac{1}{x^2}\,dx = \lim_{t\to\infty} \int_4^t x^{-2}\,dx$$

$$= \lim_{t\to\infty} \left(-\frac{1}{x}\right)\Big|_4^t = \lim_{t\to\infty} \left(-\frac{1}{t} + \frac{1}{4}\right) = \frac{1}{4}$$