

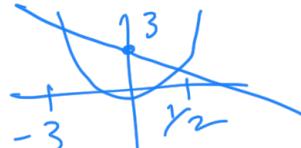
KEY

Math 232 Practice Problems for Exam 1 (some topics may not be on your midterm)

McLaughlin

- (1) Set up an integral to find the area bounded by the graphs of $y = 2x^2$ and $y = 3 - 5x$. Do not integrate.

$$\int_{-3}^{1/2} (3 - 5x - 2x^2) dx$$



$$2x^2 = 3 - 5x$$

$$2x^2 + 5x - 3 = 0$$

$$(2x - 1)(x + 3) = 0$$

$$x = \frac{1}{2}, -3$$

- (2) Set up an integral for the length of the arc of the hyperbola $xy = 1$ from the point $(1,1)$ to $(2, 1/2)$.

$$L = \int_1^2 \sqrt{1 + \frac{1}{x^4}} dx$$

$$y = \frac{1}{x}$$

$$1 \leq x \leq 2$$

$$y' = -\frac{1}{x^2}$$

- (3) Find the value of c such that the area of the region bounded by $y = c^2 - x^2$ and the $x-axis$ on $[0, c]$ is equal to the average value of $f(x) = 3x^2$ on $[0, 2]$.

a) $\sqrt[3]{6}$

b) $\sqrt[3]{12}$

c) $\sqrt[3]{24}$

d) $\sqrt[3]{8}$

e) $\sqrt[3]{18}$

$$4 = \frac{2}{3} c^3$$

$$6 = c^3$$

$$c = \sqrt[3]{6}$$

$$\begin{aligned} \text{Area} &= \int_0^c (c^2 - x^2) dx \\ &= \left[c^2 x - \frac{x^3}{3} \right]_0^c \\ &= \left(c^3 - \frac{c^3}{3} \right) = \frac{2}{3} c^3 \\ \text{ave. value} &= \frac{1}{2} \int_0^2 3x^2 dx \\ &= \frac{3}{2} \cdot \frac{x^3}{3} \Big|_0^2 = 4 \end{aligned}$$

(4) Evaluate $\int_1^4 \sqrt{x} \ln x \, dx$

a) $-\frac{7}{2} \ln 4 - 28$

b) $16 \ln 4 - \frac{28}{3}$

c) $\frac{16}{3} \ln 4 - \frac{28}{9}$

d) $-\frac{7}{2}$

e) $\frac{16}{3} \ln 4 - \frac{7}{2} = \left[\frac{16}{3} \ln 4 - \frac{2}{3} \cdot \frac{2}{3} x^{3/2} \right]_1^4$
 $= \frac{16}{3} \ln 4 - \frac{4}{9}(7) = \frac{16}{3} \ln 4 - \frac{28}{9}$

(5) Evaluate $\int \cos^{-2} x \sin^3 x \, dx$.

a) $-\sin x - \csc x + c$

b) $\frac{\cos^3 x}{3} - \sec x + c$

c) $\cos x + \frac{\sin^4 x}{4} + c$

d) $\cos x + \sec x + c$

e) $\frac{\cos x}{2} - \frac{\cos^5 x}{5} + c$

(6) Which definite integral is the area bounded by the graphs of $y = x^2 - 3$ and $y = 15 - x^2$?

a) $\int_{-3}^3 (12 - 2x^2) \, dx$

b) $\int_{-15}^{15} (12 - 2x^2) \, dx$

c) $\pi \int_{-3}^3 (18 - x^2)^2 \, dx$

d) $\int_{-15}^{15} (18 - 2x^2) \, dx$

e) $\int_{-3}^3 (18 - 2x^2) \, dx$

$$u = \ln x \quad dv = x^{1/2} \, dx$$

$$du = \frac{1}{x} \, dx \quad v = \frac{2}{3} x^{3/2}$$

$$\left[\frac{2}{3} x^{3/2} \ln x \right]_1^4 - \frac{2}{3} \int_1^4 x^{1/2} \, dx$$

$$= \left[\frac{16}{3} \ln 4 - \frac{2}{3} \cdot \frac{2}{3} x^{3/2} \right]_1^4$$

$$= \frac{16}{3} \ln 4 - \frac{4}{9}(7) = \frac{16}{3} \ln 4 - \frac{28}{9}$$

$$\int \cos^{-2} x \sin^3 x \, dx$$

$$\int \cos^{-2} x (1 - \cos^2 x) \sin x \, dx$$

$$\boxed{u = \cos x}$$

$$\boxed{du = -\sin x \, dx}$$

$$\begin{aligned} & - \int u^{-2} (1-u^2) du = - \int (u^{-2} - 1) du \\ & = u + \frac{1}{u} + C \Rightarrow \cos x + \frac{1}{\cos x} \, dx \end{aligned}$$

$$= \cos x + \sec x + C$$

$$\int_{-3}^3 ((15-x^2) - (x^2-3)) \, dx \quad x^2 - 3 = 15 - x^2$$

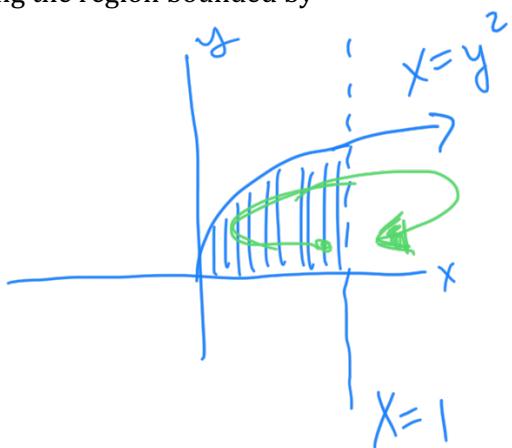
$$2x^2 = 18$$

$$x = \pm 3$$

$$= \int_{-3}^3 (18 - 2x^2) \, dx$$

- (7) Set up the integral to find the volume of the solid found by revolving the region bounded by $y = \sqrt{x}$, $x = 0$ and $x = 1$ about the line $x = 1$. Do not integrate.

$$\int_0^1 \pi (1 - y^2)^2 dy$$



- (8) Which definite integral is the volume of the solid found by revolving the region bounded by $y = x - 1$, $y = 3 - x$ and the $x - axis$ about the $y - axis$?

a) $\pi \int_1^3 (y^2 - 4y + 10) dy$

b) $8\pi \int_0^1 (1 - y) dy$

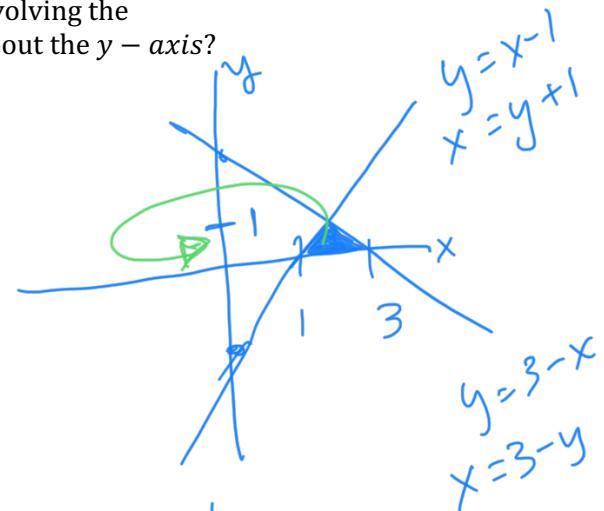
c) $\pi \int_0^1 (1 - 8y^2) dy$

d) $\pi \int_1^3 (8 - 8y) dy$

e) $8\pi \int_0^1 (y - 1) dy$

$$= \pi \int_0^1 [(9 - 6y + y^2) - (y^2 + 2y + 1)] dy$$

$$= \pi \int_0^1 (8 - 8y) dy$$



$$\pi \int_0^1 (R^2 - r^2) dy$$

- (9) Find the average value of $y = x \ln x$ on $[1, 5]$.

a) $\frac{25}{8} \ln 5 - \frac{3}{2}$

$$\frac{1}{4} \int_1^5 x \ln x dx$$

b) $25 \ln 5 - \frac{3}{2}$

c) $8 \ln 4 - \frac{5}{4}$

d) $\frac{25}{8} \ln 5$

e) $\frac{8}{3} \ln 4 - \frac{5}{4}$

$$= \frac{1}{4} \left(\frac{x^2}{2} \ln x - \frac{x^2}{4} \right) \Big|_1^5 = \frac{1}{4} \left(\frac{25}{2} \ln 5 - \frac{25}{4} + \frac{1}{4} \right) = \frac{25}{8} \ln 5 - \frac{3}{2}$$

$$\begin{aligned} u &= \ln x & du &= x dx \\ dv &= \frac{1}{x} dx & v &= x^2/2 \\ dM &= \frac{1}{x} dx & \Rightarrow & x^2 \ln x - \int_1^5 \frac{x^2}{2} dx \end{aligned}$$

(10) The following integral represents the volume of a solid S . Describe S .

$$\int_0^1 e^{2x} dx = \int_0^1 (e^x)^2 dx$$

- a) A solid obtained by rotating the region bounded by $y = e^{2x}$, $x = 0$ and $x = 1$ about the x -axis.
- b) A solid obtained by rotating the region bounded by $y = e^x$, $x = 0$ and $x = 1$ about the x -axis.
- c) The base of S is the region bounded by $y = e^{2x}$, $x = 0$ and $x = 1$. Cross sections perpendicular to the x -axis are squares.
- d) The base of S is the region bounded by $y = e^x$, $x = 0$ and $x = 1$. Cross sections perpendicular to the x -axis are semicircles.
- e) The base of S is the region bounded by $y = e^x$, $x = 0$ and $x = 1$. Cross sections perpendicular to the x -axis are squares.

(11) Find A .

$$\int x \tan x \sec^2 x dx = \frac{x \tan^2 x}{2} - A$$

By parts :

a) $A = 2 \int \sec^2 x dx$

$$u = x \quad dv = \tan x \sec^2 x dx$$

b) $A = \frac{1}{2} \int \tan^2 x dx$

$$du = dx \quad v = \frac{\tan^2 x}{2} dx$$

c) $A = \frac{1}{2} \int \csc^3 x dx$

$$\frac{x \tan^2 x}{2} - \int \frac{\tan^2 x}{2} dx$$

d) $A = 2 \int x \cos x dx$

using
u-sub :
 $u = \tan x$

e) $A = 2 \int \tan^2 x dx$

(12) Evaluate $\int_{\pi/6}^{\pi/2} 5 \cos^3 x dx$

a) $5/8$

b) $8/3$

c) $5/24$

d) $25/24$

e) $25/8$

change limits of integration

$$\int \cos^3 x dx = \int \cos^2 x \cos x dx$$

$$= \int (1 - \sin^2 x) \cos x dx$$

$$\Rightarrow \int_{\pi/2}^{\pi} (1 - u^2) du = \left(u - \frac{u^3}{3} \right) \Big|_{\pi/2}^{\pi}$$

$$u = \sin x \\ du = \cos x dx$$

$$= \left(1 - \frac{1}{3} \right) - \left(\frac{1}{2} - \frac{1}{24} \right) = \frac{5}{24}$$

(13) Evaluating the integral $\int \frac{2x-3}{x^3+3x} dx$ using partial fraction decomposition would

result in the sum of integrals $\int \frac{A}{x+E} dx + \int \frac{Bx+C}{x^2+D} dx$ where constants A, B, C, D, E sum to:

a) 4

b) 7

c) 5

d) 2

e) 3

$$\frac{2x+3}{x(x^2+3)} = \frac{A}{x} + \frac{Bx+C}{x^2+3}$$

$$2x+3 = A(x^2+3) + (Bx+C)x$$

$$X=0 \Rightarrow 3 = 3A, A=1$$

Equate linear coeff. $\Rightarrow 2 = C$

Equate quadratic coeff. $\Rightarrow D=1+B$

$$\begin{cases} A=1 \\ C=2 \\ B=-1 \\ D=3 \\ E=0 \end{cases}$$

(14) Evaluate $\int \tan^2(3x) dx$

$$\int (\sec^2(3x) - 1) dx$$

$$= \frac{1}{3} \tan(3x) - x + C$$

(15) Evaluate $\int \cos^3(x) \sin^6(x) dx$

$$\int \cos^3 x \sin^6 x \cos x dx$$

$$\int (1 - \sin^2 x) \sin^6 x \cos x dx$$

$$\int (1 - u^2) u^6 du = \int (u^6 - u^8) du$$

$$= \frac{u^7}{7} - \frac{u^9}{9} + C \Rightarrow \frac{\sin^7 x}{7} - \frac{18 \sin^9 x}{9} + C$$

$$u = \sin x$$

$$du = \cos x dx$$

- (16) Consider solving the integral $\int \sqrt{4 - x^2} dx$. After making the appropriate trigonometric substitution, which integral must be solved to complete the solution?

a) $16 \int \cos^2 \theta d\theta$

b) $4 \int \sin \theta \cos \theta d\theta$

c) $4 \int \cos^2 \theta d\theta$

d) $2 \int \tan^2 \theta d\theta$

e) $16 \int \tan \theta \sin \theta d\theta$

$$x = 2 \sin \theta \\ dx = 2 \cos \theta d\theta$$

$$\int \sqrt{4 - 4 \sin^2 \theta} (2 \cos \theta) d\theta$$

$$= \int 2 \sqrt{1 - \sin^2 \theta} (2 \cos \theta) d\theta = 4 \int \cos^2 \theta d\theta$$

$$= 4 \int \frac{1}{2} (1 + \cos 2\theta) d\theta = 2 \left(\theta + \frac{1}{2} \sin 2\theta \right) + C$$



$$\sin \theta = \frac{x}{2} \Rightarrow \theta = \sin^{-1} \left(\frac{x}{2} \right)$$

$$\text{Use } \sin 2\theta = 2 \sin \theta \cos \theta$$

$$\Rightarrow \sin 2\theta = 2 \left(\frac{x}{2} \right) \left(\frac{\sqrt{4-x^2}}{2} \right) \Rightarrow$$

$$= 2 \left[\sin^{-1} \left(\frac{x}{2} \right) + \right]$$

$$\left. \left(\frac{x}{2} \right) \left(\frac{\sqrt{4-x^2}}{2} \right) \right] + C$$

~~-2 to 2~~ integral negative

True or False for problems #17, 18

F (17) There exist constants A and B such that $\frac{x(x^2+4)}{x^2-4} = \frac{A}{x+2} + \frac{B}{x-2}$.

~~Degree of numerator > degree denominator~~

F (18) Integrating the parabola $y = x^2 - 4$ from $x = -2$ to $x = 2$ is equivalent to finding the area bounded by $y = x^2$ and $y = 4$.

(19) Evaluate $\int x \cos^2(x) dx$

$$\int x \left(\frac{1}{2} (1 + \cos 2x) \right) dx$$

$$= \frac{1}{2} \int (x + x \cos 2x) dx$$

$$\begin{aligned} u &= x & dv &= \cos 2x dx \\ du &= dx & v &= \frac{1}{2} \sin 2x \end{aligned}$$

$$\Rightarrow \frac{x}{2} \sin(2x) - \frac{1}{2} \int \sin 2x dx$$

$$\left. -\frac{1}{2} \cos 2x \right|$$

$$= \frac{1}{2} \left(\frac{x^2}{2} + \frac{x}{2} \sin 2x + \frac{1}{4} \cos 2x \right) + C$$

(20) Evaluate $\int x \sqrt{16+x^2} dx$.

$$x = 4 \tan \theta \\ dx = 4 \sec^2 \theta d\theta$$

$$\begin{aligned} & \int 4 \tan \theta \sqrt{16 + 16 \tan^2 \theta} (4 \sec^2 \theta) d\theta \\ &= 64 \int \tan \theta (\sec \theta) \sec^2 \theta d\theta = 64 \int \sec^3 \theta \tan \theta d\theta \\ &= 64 \int \sec^2 \theta \sec \theta \tan \theta d\theta \\ &\quad u = \sec \theta \\ &\quad du = \sec \theta \tan \theta d\theta \\ \Rightarrow & 64 \int u^2 du = \frac{64}{3} u^3 + C \Rightarrow \frac{64}{3} \sec^3 \theta d\theta \\ \text{Diagram: } & \tan \theta = \frac{x}{4} \Rightarrow \sec \theta = \frac{\sqrt{x^2+16}}{4} \Rightarrow \frac{64}{3} \left(\frac{\sqrt{x^2+16}}{4} \right)^3 + C \end{aligned}$$

(21) Using the Partial Fraction Decomposition method, express the integral $\int \frac{x+2}{(x-3)^2} dx$ as a sum of integrals $\int \frac{A}{(x-3)} dx + \int \frac{B}{(x-3)^2} dx$. Find A and B. Show all work for credit. Do not integrate.

$$A = -1 \quad B = 5$$

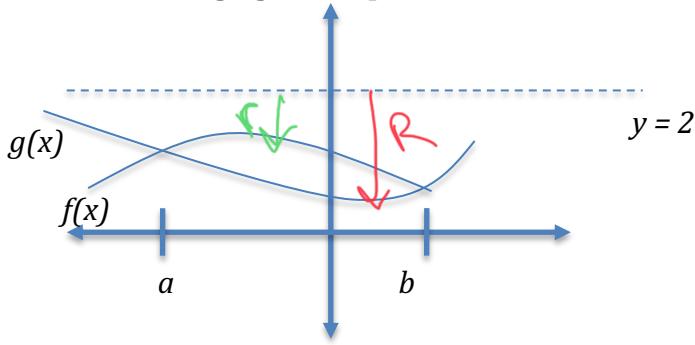
$$\frac{x+2}{(x-3)^2} = \frac{A}{(x-3)} + \frac{B}{(x-3)^2}$$

$$x+2 = A(x-3) + B$$

$$x=3 \Rightarrow B=5$$

$$\text{Equate constants} \quad 2 = 3A + 5 \\ A = -1$$

Use the following figure for problems #22 and #23.



- (22) The volume of the solid generated by revolving the area bounded by $f(x)$ and $g(x)$ about the line $y = 2$ is equal to:

- a) $\lim_{n \rightarrow \infty} \pi \sum_{i=1}^n [(2 - g(x_i^*))^2 - (2 - f(x_i^*))^2] \Delta x$
- b) $\lim_{n \rightarrow \infty} \pi \sum_{i=1}^n [(g(x_i^*))^2 - (f(x_i^*))^2] \Delta x$
- c) $\lim_{n \rightarrow \infty} \pi \sum_{i=1}^n [(2 + g(x_i^*))^2 - (2 + f(x_i^*))^2] \Delta x$
- d) $\lim_{n \rightarrow \infty} \pi \sum_{i=1}^n [(g(x_i^*))^2 - (2 - f(x_i^*))^2] \Delta x$
- e) $\lim_{n \rightarrow \infty} \pi \sum_{i=1}^n [(2 - g(x_i^*))^2 - (f(x_i^*))^2] \Delta x$

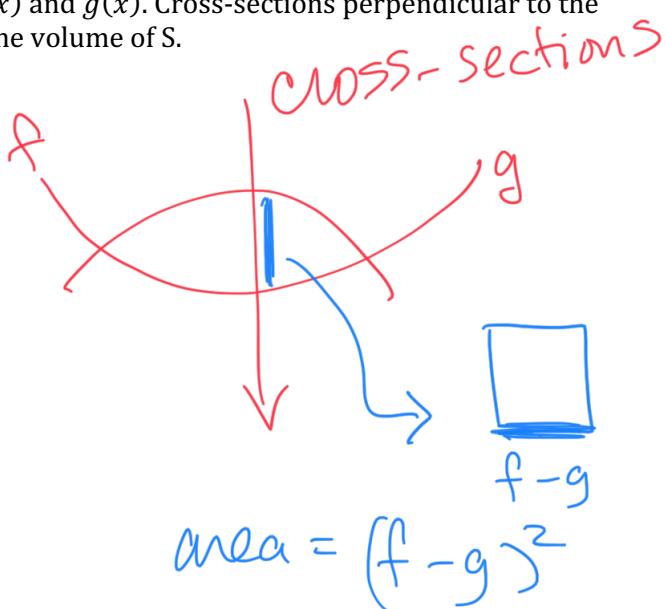
$$\pi \int_a^b (R^2 - r^2) dr$$

$$R = 2 - g(x)$$

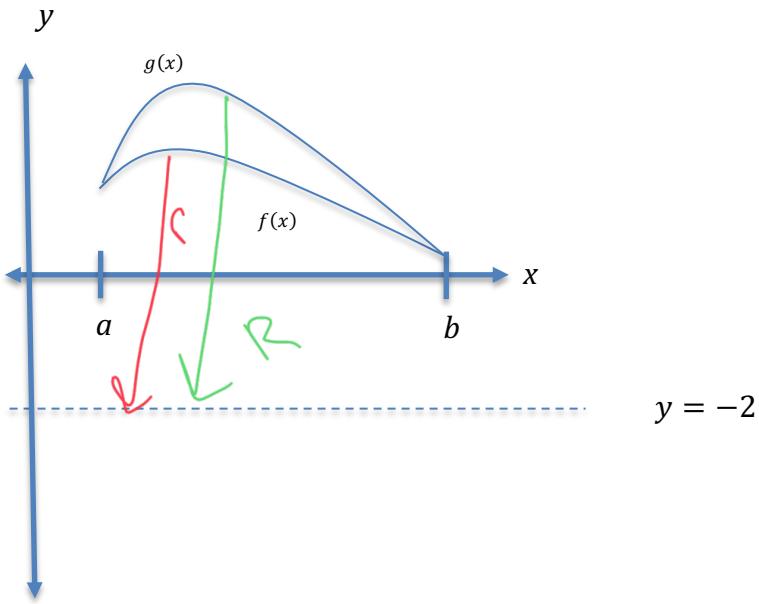
$$r = 2 - f(x)$$

- (23) The base of a solid S is the region bounded by $f(x)$ and $g(x)$. Cross-sections perpendicular to the x -axis are squares. Which of the following is the volume of S.

- a) $\lim_{n \rightarrow \infty} \pi \sum_{i=1}^n [f(x_i^*)]^2 \Delta x$
- b) $\lim_{n \rightarrow \infty} \sum_{i=1}^n [f(x_i^*) - g(x_i^*)] \Delta x$
- c) $\lim_{n \rightarrow \infty} 2\pi \sum_{i=1}^n [f(x_i^*)^2 - g(x_i^*)^2] \Delta x$
- d) $\lim_{n \rightarrow \infty} \sum_{i=1}^n [f(x_i^*) - g(x_i^*)]^2 \Delta x$
- e) $\lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i^*) g(x_i^*) \Delta x$



Use the following figure for problems #24 and #25.



(24) The area bounded by the graphs of $y = f(x)$ and $y = g(x)$ is equal to:

a) $\lim_{n \rightarrow \infty} \sum_{i=1}^n \pi(g(x_i^*) - f(x_i^*))^2 \Delta x$

b) $\lim_{n \rightarrow \infty} \sum_{i=1}^n (f(x_i^*) - g(x_i^*)) \Delta x$

c) $\lim_{n \rightarrow \infty} \sum_{i=1}^n (g(x_i^*) - f(x_i^*)) \Delta x$

d) $\lim_{n \rightarrow \infty} \sum_{i=1}^n \pi(g(x_i^*)^2 - f(x_i^*)^2) \Delta x$

e) $\lim_{n \rightarrow \infty} \sum_{i=1}^n \pi(f(x_i^*) - g(x_i^*))^2 \Delta x$

$$\int_a^b (g(x) - f(x)) dx$$

(25) The volume of the solid generated by revolving the area bounded by the graphs of $y = f(x)$ and $y = g(x)$ about the line $y = -2$ is equal to:

a) $\lim_{n \rightarrow \infty} \pi \sum_{i=1}^n (g(x_i^*) - f(x_i^*) + 2)^2 \Delta x$

b) $\lim_{n \rightarrow \infty} \pi \sum_{i=1}^n [(g(x_i^*) - 2)^2 - (f(x_i^*) - 2)^2] \Delta x$

c) $\lim_{n \rightarrow \infty} \sum_{i=1}^n (g(x_i^*) - f(x_i^*) - 4) \Delta x$

d) $\lim_{n \rightarrow \infty} \pi \sum_{i=1}^n [(g(x_i^*) + 2)^2 - (f(x_i^*) + 2)^2] \Delta x$

e) $\lim_{n \rightarrow \infty} \pi \sum_{i=1}^n [(f(x_i^*) + 2)^2 - (g(x_i^*) + 2)^2] \Delta x$

$$\pi \int_a^b (R^2 - r^2) dx$$

$$R = g + 2$$

$$r = f + 2$$

- (26) A tank in the shape of a right circular cone is full of water. If the height of the tank is 12 feet and the radius of the top is 3 feet, which integral is the work done in pumping the water to a height 10 feet above the top of the tank. (Water weighs 62.4 pounds per cubic foot.) Let $\delta = 62.4$

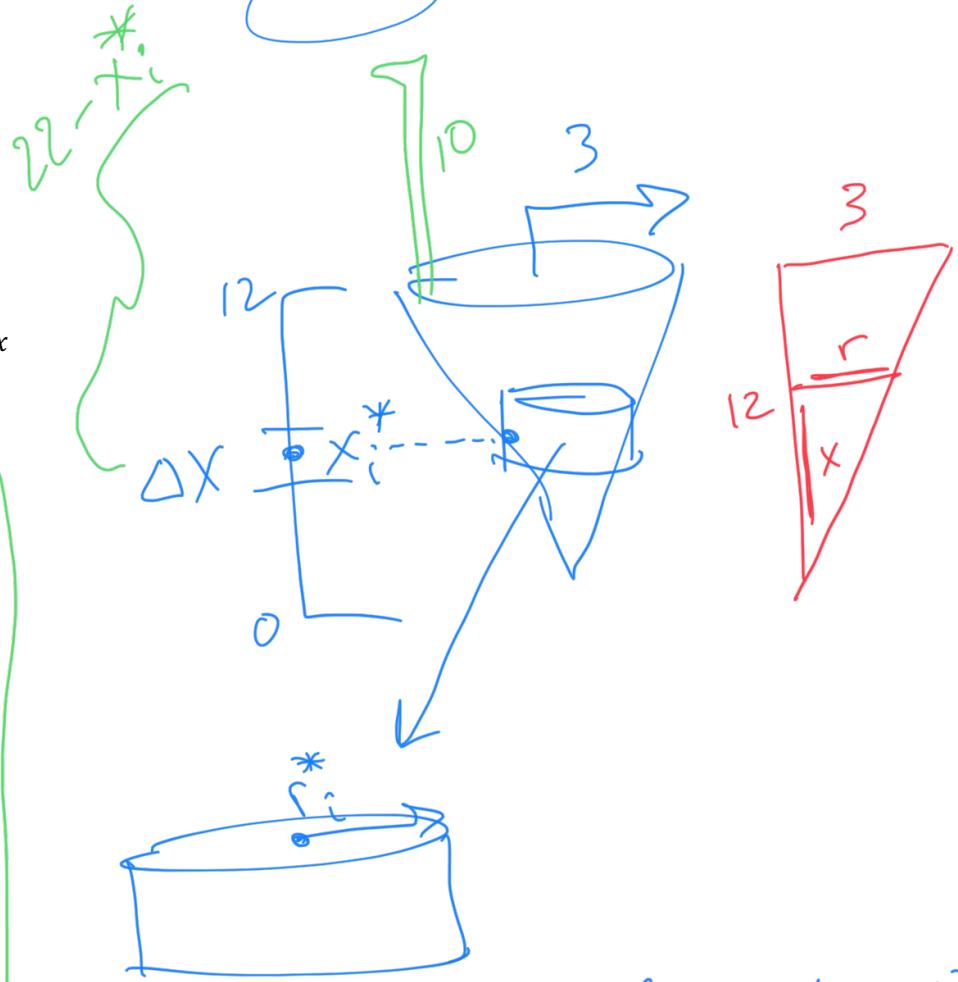
a) $\frac{\delta\pi}{16} \int_0^{22} (22x^2 - x^3) dx$

b) $\frac{\delta\pi}{16} \int_0^{12} (12x^2 - x^3) dx$

c) $\delta\pi \int_0^{22} (x^3 - 12) dx$

d) $10\delta\pi \int_0^{12} (12x^2 - x^3) dx$

e) $\frac{\delta\pi}{16} \int_0^{12} (22x^2 - x^3) dx$



$$W_i = F_i \cdot D_i$$

$$W_i = \left(\frac{\pi x_i^*}{16} \Delta x \right) S \cdot (22 - x_i^*)$$

$$\int_0^{12} \frac{S\pi}{16} (22x^2 - x^3) dx$$

$$F_i = \text{weight} = \text{volume}_i (62.4)$$

$$\text{volume}_i = \pi r_i^{*2} \Delta x$$

$$\frac{r_i^*}{x_i^*} = \frac{3}{12} = \frac{1}{4}$$

using similar triangles

$$r_i^* = \frac{x_i^*}{4} \Rightarrow \text{volume}_i = \pi \left(\frac{x_i^*}{4}\right)^2 \Delta x$$