Math 110: Final Exam Version A
Fall 2018

- Calculators are NOT allowed.
- Please code true/false and multiple choice answers on a scantron. These are questions 1 - 10.
- Since you have test version A, please code the "Page Number" on the scantron as 1.
- No partial credit for multiple choice / no work needs to be shown.
- For short answer questions, you must show work for full and partial credit.
- Multiple choice and true / false problems are worth 4 points each. Free response questions are worth 6 points each.
- Sign the honor pledge below after completing the exam.
- Please put all work to be graded on the test itself.

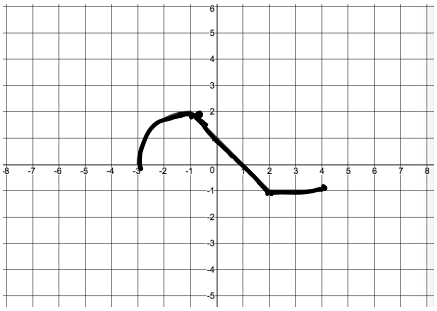
First and last name Key .....

PID .....

Honor Pledge: I have neither given nor received unauthorized help on this exam.

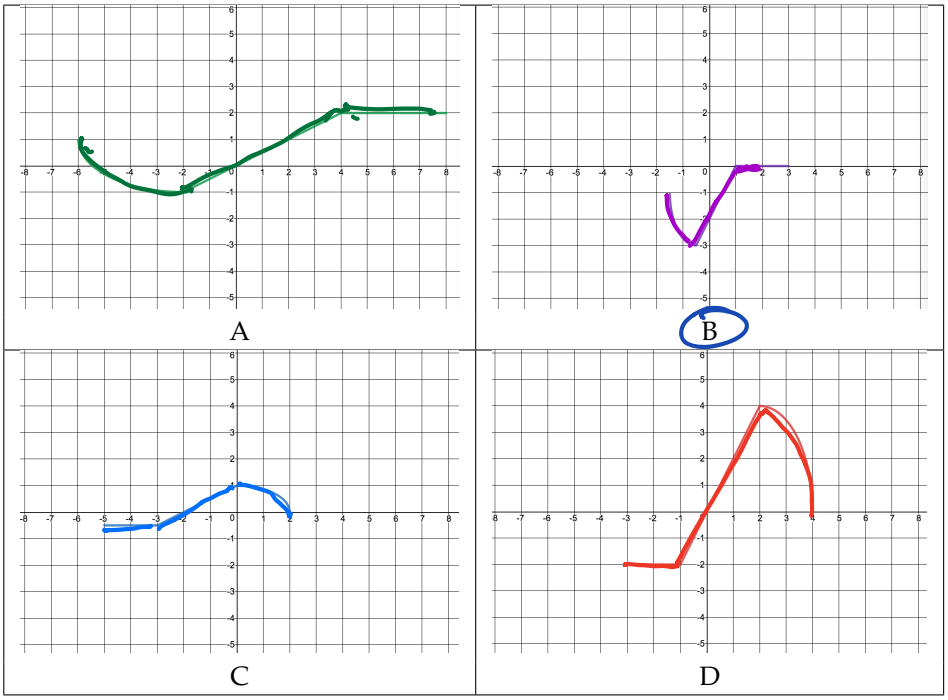
Signature: .....

1. The graph of the function $y = f(x)$ is shown below.



reflect across x-axis
shrink horiz by $\frac{1}{2}$
down by 1

Use transformations to draw the graph of $y = -f(2x) - 1$.



2. Simplify and write the answer without negative exponents.

$$\left(\frac{32x^{-7}y^{-8}}{x^{-2}y^{2}}\right)^{2/5}$$

A. $\frac{4}{x^2y^4}$

B. $\frac{1}{4x^2y^4}$

C. $\frac{32x^{14}}{y^{16}}$

D. $\frac{64}{5x^7y^4}$

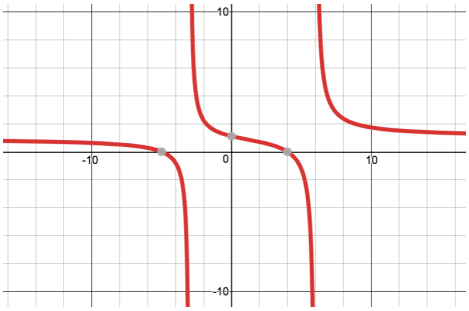
E. $\frac{64y^{12/5}}{5x^{18/5}}$

$$\left(\frac{32x^2}{x^7y^8y^2}\right)^{2/5}$$

$$= \left(\frac{32}{x^5y^{10}}\right) = \frac{32^{2/5}}{(x^5)^{2/5}(y^{10})^{2/5}}$$

$$= \frac{\left(\sqrt[5]{32}\right)^2}{x^{5\cdot 2/5}y^{10\cdot 2/5}} = \frac{2^2}{x^2y^4} = \frac{4}{x^2y^4}$$

3. Find the equation of the graph.



V.A. at $x=-3$ $x=6$

x-intercepts at $x=-5$ $x=4$

A. $y = \frac{(x-4)}{(x+3)(x-6)}$

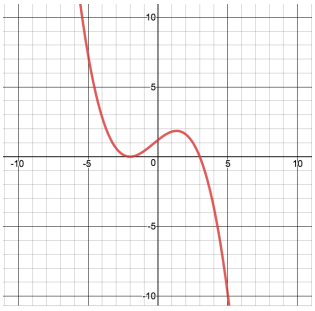
B. $y = \frac{(x-4)}{(x-1)(x-2)}$

C. $y = \frac{(x-4)(x+5)}{(x-1)(x-2)}$

D. $y = \frac{(x+3)(x-6)}{(x+5)(x-4)}$

E. $y = \frac{(x+5)(x-4)}{(x+3)(x-6)}$

4. Find the equation of the graph:



Zero of degree 2 at $x=-2$

Zero of degree 1 at $x=3$

negative leading coeff

$y = \frac{1}{10}(x+2)^2(x-3)$

A. $y = -\frac{1}{10}(x+2)(x-3)$

B. $y = \frac{1}{10}(x+2)(x-3)$

C. $y = -\frac{1}{10}(x+2)^2(x-3)$

D. $y = \frac{1}{10}(x+2)(x-3)^2$

E. $y = -\frac{1}{10}(x+2)^2(x-3)^2$

5. Find the equation of a line that is perpendicular to the line $3x+4y=7$ and goes through the point $(6,5)$.

A. $3x-4y=-2$

B. $4x-3y=7$

C. $4x-3y=9$

D. $4x+3y=7$

E. $4x+3y=39$

$4y = -3x+7$

$y = -\frac{3}{4}x + \frac{7}{4}$

$m_2 = \frac{4}{3}$

$y = \frac{4}{3}x + b$

$5 = \frac{4}{3}\cdot 6 + b \Rightarrow 5 = 8 + b \Rightarrow b = -3$

$y = \frac{4}{3}x - 3$

$3y = 4x - 9 \Rightarrow 4x - 3y = 9$

6. For the functions $f(x) = x^2 + x$ and $g(x) = \dfrac{1}{x+4}$, find an expression for $f \circ g(x)$.

A. $\dfrac{1}{x^2+x+4}$

B. $\dfrac{1}{(x+4)^2+x}$

C. $\dfrac{x+5}{(x+4)^2}$

D. $\dfrac{x^2+x}{x+4}$

E. $\dfrac{x^3+4x^2+1}{x+4}$

$f(g(x)) = f\left(\frac{1}{x+4}\right)$

$= \left(\frac{1}{x+4}\right)^2 + \frac{1}{x+4}$

$= \frac{1}{(x+4)^2} + \frac{1}{x+4} \cdot \frac{x+4}{x+4}$

$= \frac{1+x+4}{(x+4)^2} = \frac{x+5}{(x+4)^2}$

7. Solve the inequality. Write your answer in interval notation.

$2|3-2x|+1 > 5$

A. $(0, \infty)$

B. $(-\frac{1}{2}, \frac{1}{2})$

C. $(\frac{1}{2}, \frac{5}{2})$

D. $(-\infty, \frac{1}{2}) \cup (3, \infty)$

E. $(-\infty, \frac{1}{2}) \cup (\frac{5}{2}, \infty)$

$2|3-2x| > 4$

$|3-2x| > 2$

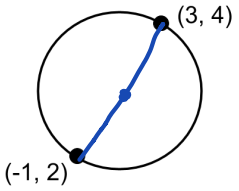
$3-2x$: -2 and 3

$3-2x < -2$ or $3-2x > 2$

$-2x < -5$ or $-2x > -1$

$x > \frac{5}{2}$ or $x < \frac{1}{2}$

8. The two points $(3,4)$ and $(-1,2)$ lie on a circle, on opposite sides of a diameter. Find the equation of the circle.



A. $(x-1)^2 + (y-3)^2 = 5$

B. $(x+1)^2 + (y+3)^2 = 20$

C. $(x-1)^2 - (y-3)^2 = 25$

D. $(x-1)^2 + (y-3)^2 = \sqrt{5}$

E. $(x+2)^2 + (y+1)^2 = \sqrt{20}$

center at midpoint $\left(\frac{3-1}{2}, \frac{4+2}{2}\right) = (1, 3)$

radius: $\sqrt{(3-1)^2+(4-3)^2}$

$= \sqrt{2^2+1^2} = \sqrt{5}$

$(x-1)^2 + (y-3)^2 = 5$

9. A model rocket is launched and its height in meters at time t seconds is given by the equation $h(t) = 6t - 3t^2$. Find the time(s) at which the height of the rocket is 2 meters.

A. $t = -2$

B. $t = 1$

C. $t = 2$

D. $t = 1, 2$

E. $t = 1 + \frac{\sqrt{3}}{3}, 1 - \frac{\sqrt{3}}{3}$

omitted due to typo

$2 = 6t - 3t^2$

$3t^2 - 6t + 2 = 0$

$t = \dfrac{6 \pm \sqrt{36 - 4(3)(2)}}{2(3)}$

$= 1 \pm \frac{2\sqrt{3}}{6}$

$= 1 \pm \frac{\sqrt{3}}{3}$

$t = \dfrac{6}{6} \pm \dfrac{\sqrt{12}}{6}$

10. Use log properties to write the following expression as a single log. You may assume that all variables are positive.

$$\frac{1}{2}\log(x+5) - 8\log(x) + 2\log(y)$$

A. $\log\left(\dfrac{(x+5)^{1/2}}{x^8y^2}\right)$

B. $\log\left(\dfrac{(x+5)^{1/2}y^2}{x^8}\right)$

C. $\log\left(\dfrac{(x+5)y}{8x}\right)$

D. . $\log\left(\frac{1}{2}(x+5) - 8x + 2y\right)$

E. $\log\left((x+5)^{1/2} - x^8 + y^2\right)$

$\log(x+5)^{1/2} - \log x^8 + \log y^2$

$\log \dfrac{(x+5)^{1/2}}{x^8} + \log y^2$

$\log\left[\dfrac{(x+5)^{1/2}}{x^8} \cdot y^2\right]$

$\log \dfrac{(x+5)^{1/2}\, y^2}{x^8}$

4 pts per part
12 pts total

Each part:
2 pts answer
2 pts explanation

11. TRUE or FALSE and justify your answer.

(a) $\log(x-y) = \dfrac{\log(x)}{\log(y)}$. (circle one) TRUE or FALSE

Explanation / Counterexample

for example, if $x = 10$, $y = 10$

left side is $\log(0)$ DNE

right side is $\frac{1}{1} = 1$

(b) The equation $4|x+5| < -2$ has no solutions. (circle one) TRUE or FALSE

Explanation / Counterexample

$|x+5| < -\frac{1}{2}$

can't have negative abs value

so no solutions

(c) $\sqrt{a^2+25} = a+5$. (circle one) TRUE or FALSE

Explanation / Counterexample

$a = 1$

$\sqrt{1^2+25} = \sqrt{26} \neq 1+5 = 6$

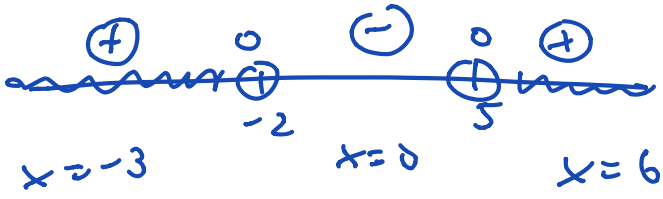
Since $26 \neq 6^2 = 36$

12. Find the domain of the function $f(x) = 2\sqrt{x^2 - 3x - 10}$. Write your answer in interval notation.

$x^2 - 3x - 10 \geq 0$ 2 pts inequality (can get full credit without writing this down if other work is correct)

$(x-5)(x+2) \geq 0$

2 pts $x = 5$, $x = -2$ (1 pt each)



Answer: $(-\infty, -2] \cup [5, \infty)$ 2 pts answer -1 for end pt errors

13. The population of spiders on an island (in millions) is growing according to the equation $y = 30e^{0.05t}$. If this growth rate continues, find the amount of time it will take the population to double. Write your answer in a form that you could type into your calculator.

$y = 30e^{0.05t}$

$60 = 30e^{0.05t}$ 2 pts plug in 60 for y

$2 = e^{0.05t}$ 1 pt

$\ln 2 = \ln e^{0.05t}$ 1 pt ln of each side

$\ln 2 = 0.05t$ 1 pt

$t = \frac{\ln 2}{0.05}$ 1 pt answer

Answer: $\frac{\ln 2}{0.05}$

-1 if get $\frac{1}{2(6+x)}$ due to not distributing negative sign, but all else correct

14. Simpify:

$$\frac{\frac{1}{6+x}-\frac{1}{6}}{\frac{x}{3}}$$

2 pts for multiplying by reciprocal

2 pts for adding fractions

$$\frac{\frac{1}{6+x}\cdot\frac{6}{6}-\frac{1}{6}\cdot\frac{6+x}{6+x}}{\frac{x}{3}} = \frac{\frac{6}{(6+x)6}-\frac{6+x}{(6+x)6}}{\frac{x}{3}}$$

$$= \frac{\frac{6-(6+x)}{(6+x)(6)}}{\frac{x}{3}} = \frac{\frac{-x}{6(6+x)}}{\frac{x}{3}} = \frac{-x}{6(6+x)}\cdot\frac{3}{x}$$

$$= \frac{-1}{2(6+x)}$$

Answer: $\frac{-1}{2(6+x)}$ 2 pts correct final answer

15. Solve for y: $y-4=\sqrt{31-6y}$

$(y-4)^2=(\sqrt{31-6y})^2$ ✓✓

$y^2-8y+16=31-6y$

$y^2-2y-15=0$ ✓✓

$(y-5)(y+3)=0$

$y=5 \quad y=-3$ ✓✓

check: $y=5$

$5-4 \stackrel{?}{=} \sqrt{31-30}$ ✓

$y=-3$

$-3-4 \stackrel{?}{=} \sqrt{31-6(-3)}$

$-7 \stackrel{?}{=} \sqrt{49}$ ✗

extraneous

Answer: $y=5$

-1 if did not exclude extraneous solution

16. Solve for y. Your answer should be in terms of x.

$$x = \frac{2-y}{3y+1}$$

$(3y+1)x = \frac{2-y}{3y+1} \cdot (3y+1)$

$(3y+1)x = 2-y$ ✓✓

$3yx + x = 2-y$ ✓

$3yx + y = 2-x$ ✓

$y(3x+1) = 2-x$ ✓

$y = \frac{2-x}{3x+1}$

Answer: $\frac{2-x}{3x+1}$ ✓

17. Find the values of the expressions. Write DNE if the value is undefined.

1 pt each
no partial credit
1 pt for free
Since only 5 parts

(a) $\log_2 \frac{1}{8} =$ -3

$2^{-3} = \frac{1}{8}$

(b) $\log_9 3 =$ $\frac{1}{2}$

$9^{1/2} = 3$

(c) $\log_4 0 =$ DNE

$4^{\square} = 0$ has no solution

(d) $\ln e^2 =$ 2

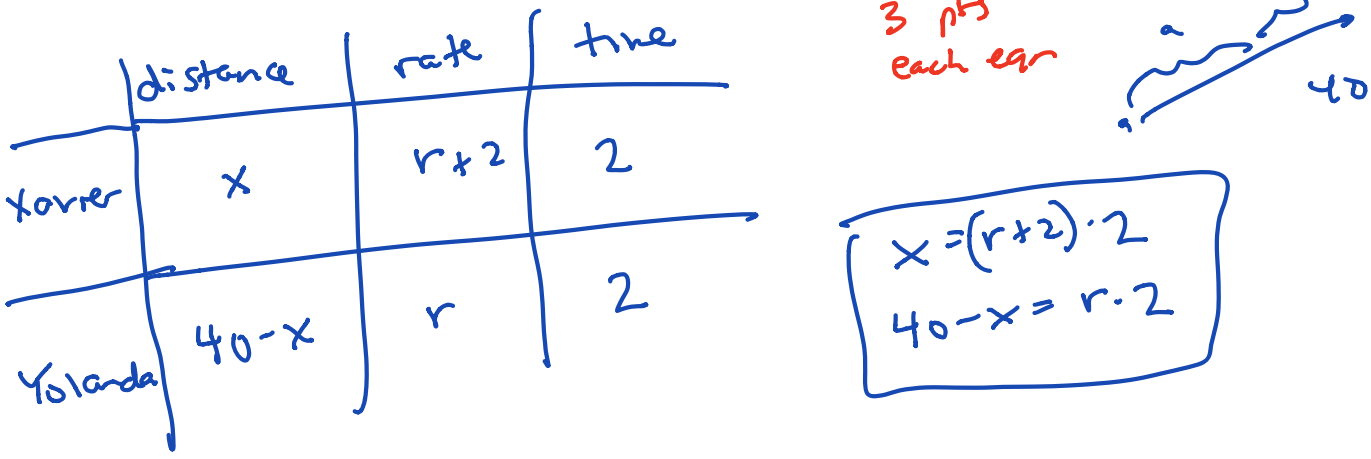
$\log_e e^2 = 2$

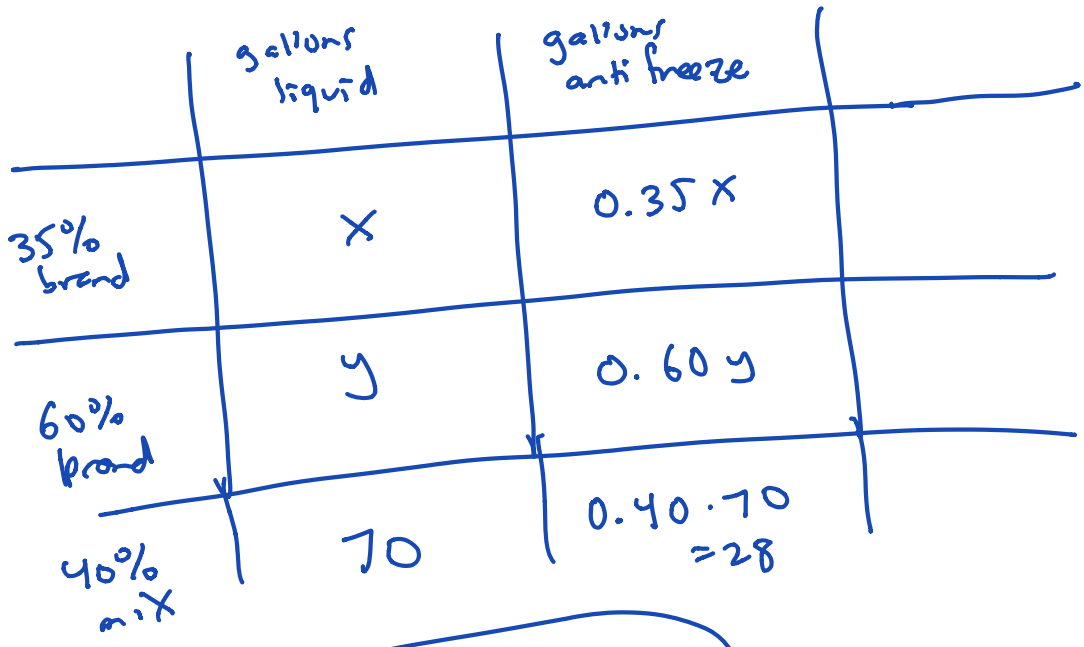
(e) $5^{\log_5 10} =$ 10

18. Choose ONE of the problems below and SET UP a system of two equations in two unknowns that you could use to solve it. You DO NOT need to finish solving the problem. Please circle the problem you choose.

    (a) Xavier and Yolanda leave at the same time and bicycle towards each other from towns 40 miles apart. Yolanda bikes 2 miles per hour faster than Xavier. They meet somewhere in between after 2 hours of biking. How fast do they each bike?

    (b) A chemical company makes two brands of antifreeze. The first brand is 35% pure antifreeze, and the second brand is 60% pure antifreeze. In order to obtain 70 gallons of a mixture that contains 40% pure antifreeze, how many gallons of each brand of antifreeze must be used?





19. The amount of coffee beans in a coffee shop decreases at a constant rate. On November 4 there were 105 pounds of coffee beans in the shop. On November 11 there were 70 pounds of coffee beans.

(a) Write an equation to express the pounds of coffee beans C in terms of time t since the end of October.

t	C
4	105
11	70

$m = \frac{70-105}{11-4} = \frac{-35}{70} = -5$

$C = -5t + b$

$105 = -5 \cdot 4 + b \Rightarrow b = 105 + 20 = 125$

2 pts slope
-1 pt if wrong sign
2 pts intercept

-1 if write t in terms of C

Answer: $C = -5t + 125$

(b) If this rate continues, and no more coffee beans are brought in, when will the shop run out of coffee beans?

$0 = -5t + 125 \Rightarrow 125 = 5t \Rightarrow t = 25$

1 pt set $C = 0$

1 pt answer

Answer: 25 days (Nov. 25)

20. A wedding dress was purchased for \$1200. Suppose that its value decreases by a fixed percent each year, and two years after purchase, the value is \$900. Write an equation to express the value V in terms of the time in years t since purchase. Your equation should only have two variables: V and t.

$y = 1200(1+r)^t$

$900 = 1200(1+r)^2 \Rightarrow \frac{900}{1200} = (1+r)^2$

$\Rightarrow \frac{3}{4} = (1+r)^2 \Rightarrow 1+r = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2} \Rightarrow r = \frac{\sqrt{3}}{2} - 1$

$y = 1200\left(1 + \frac{\sqrt{3}}{2} - 1\right)^t \Rightarrow y = 1200\left(\frac{\sqrt{3}}{2}\right)^t$

Answer: $y = 1200\left(\frac{\sqrt{3}}{2}\right)^t$ or $y = 1200\, e^{(\ln 3/4)/2\, t}$

OR:

2 pts exp equation
1 pt plugging in numbers

$y = 1200\, e^{rt}$

$900 = 1200\, e^{r \cdot 2} \Rightarrow \frac{900}{1200} = e^{r \cdot 2}$

$\Rightarrow \frac{3}{4} = e^{2r} \Rightarrow \ln\frac{3}{4} = \ln e^{2r} \Rightarrow 2r = \ln\frac{3}{4} \Rightarrow r = \frac{\ln\frac{3}{4}}{2}$

2 pts r

$y = 1200\, e^{\ln 3/4/2\, t}$

1 pt final answer

21. Find the x and y intercepts of the graph of $y = \log_3(x^2 + 1) - 2$

x-intercept: $0 = \log_3(x^2+1) - 2 \Rightarrow 2 = \log_3(x^2+1)$

$\Rightarrow 3^2 = x^2+1 \Rightarrow 8 = x^2 \Rightarrow x = \pm\sqrt{8} = \pm 2\sqrt{2}$

y-intercept: $x = 0 \quad y = \log_3(0+1) - 2 = 0 - 2 = -2$

1 pt plug in $x=0$
2 pts solve for y -1 if leave answer as $\log_3 1$

1 pt plug in $y=0$
2 pts solve for x
+1 for $x = \sqrt{8}$ only

-1 if switch x & y intercepts

x-intercept(s): $(\sqrt{8}, 0)$ $(-8, 0)$

y-intercept(s): $(0, -2)$

22. Solve the system of equations:

$$x^2 + 3y = 4$$
$$2x - y = 4$$

$y = 2x - 4 \Rightarrow x^2 + 3(2x-4) = 4$. 2 pts getting to fn of one variable

$\Rightarrow x^2 + 6x - 12 = 4$

$\Rightarrow x^2 + 6x - 16 = 0$

$\Rightarrow (x+8)(x-2) = 0 \Rightarrow x = -8, \; x = 2$

$x = -8 \Rightarrow y = 2(-8) - 4 \Rightarrow y = -20$

$x = 2 \Rightarrow y = 2(2) - 4 \Rightarrow y = 0$

Answer: $(-8, -20)$ and $(2, 0)$

4 pts answer (1 pt each #)
-1 if (x,y) pairs mismatched