Algebra Project Ideas
Analysis Project Ideas

**Fall 2019.ANA1**

**Mentor:** Hunter Dinkins

**Text:** *Resources for the Study of Real Analysis* by Robert Brabenec

**Additional Materials:** May occasionally need to reference basic analysis concepts and definitions (limits, derivatives, integrals, etc.) depending on the background of the student.

**Project:** The story of the development of Calculus and Analysis is fascinating. Newton and Leibniz first discovered Calculus in the mid 16th century; however it was not until the 19th century that mathematicians saw the need for greater rigor and precision in the subject, leading to the formulation of limits, derivatives, and integrals as we know them today. The clean and precise treatments of Calculus and Analysis in typical undergraduate courses, though efficient, can leave the student with a cloudy understanding of the need for certain technical notions. In this project, we will enrich our understanding of and appreciation for Calculus and Analysis by taking a problem based approach with emphasis on the historical context. From this, students will gain an understanding of the historical development of Analysis, and will also engage with some of the key problems that led to the subject taking the form that it has today. In contrast to typical courses, the explicit main priority of this project is not to gain technical fluency with some mathematical concepts, but is instead to help us love, appreciate, and enjoy mathematics more.

**Suggested Prerequisites:** MATH 231, MATH 232, MATH 381. The project will be adapted based on whether or not the student has taken MATH 521.

**Keywords:** calculus, analysis, history of calculus, Euler, Newton, Cauchy, infinite series, Bernoulli family, limits

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**Fall 2019.ANA2**

**Mentor:** Collin Kofroth

**Text:** *Partial Differential Equations I: Basic Theory* by Michael Taylor

**Project:** One of the most fundamental techniques in the theory of linear partial differential equations is encompassed by the field of Fourier analysis. Fourier series allows one to approximate a function (in the right sense) by sums of trigonometric functions, and the Fourier transform is an integral transform that fluidly transfers a function from time to frequency space. In this project, we will study the basics of both. Time permitting, we will discuss Fourier series (either on the unit circle $S^1$ or on the n-dimensional torus $T^n$), the Fourier transform, Schwartz functions, tempered distributions, and application to evolution equations.

**Suggested Prerequisites:** MATH 381 and MATH 383 are required; MATH 521 is recommended

**Keywords:** PDE, differential equations, Fourier
Geometry/Topology Project Ideas

Fall 2019.TG1

Mentor: Yiyan Shou

Text: *Topology from the Differentiable Viewpoint* by John Milnor

Additional Materials: Analysis II, Tao

Project: Differential topology is a field of mathematics concerned with topological spaces that carry additional analytic structure. If you have taken a multivariable calculus course, you will have already encountered such spaces: curves and surfaces in $\mathbb{R}^3$. In a typical multivariable calculus course, we extend the familiar analytic notions of differentiation and integration to curves and surfaces by means of a parametrization, an identification of a curve to $\mathbb{R}$ and a surface to $\mathbb{R}^2$. In differential topology, we consider more general spaces admitting parametrizations that allow us to transfer onto them the analytic structure of $\mathbb{R}^n$. The resulting analytic structure on the space is deeply intertwined with the underlying topology. A central theme in differential topology is that analytic properties force topological features to appear and vice versa. This project is a broad survey of these connections between analysis and topology.

The main text, *Topology from the Differentiable Viewpoint*, explores a varied selection of classical topics in differential topology. The text is mostly self contained, but a foundation in multivariable analysis (MATH 522) would be extremely beneficial. The project can, however, be approached with nothing more than a multivariable calculus (MATH 233) background. In this case, we will supplement the reading with selections from Tao’s Analysis II. Despite the nomenclature, knowledge of topology is helpful but not essential. The project will culminate in a final presentation on a central idea or result to one topic of the main text of your choosing.

Suggested Prerequisites: MATH 233

Keywords: topology, analysis, manifold, differential topology

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Fall 2019.TG2

Mentor: Collin Kofroth

Text: *An Introduction to Manifolds* by Loring Tu

Project: A term that many undergraduates will hear thrown around frequently without clear understanding is the term “manifold.” While the general, intuitive idea is clear after briefly perusing Wikipedia, the precise mathematical description of manifold can prove somewhat esoteric for many students for years. This leads to the question asked by many undergraduates: “What exactly is a manifold?” This is the question that we aim to answer. We will start by discussing the concept of a manifold from both an intuitive and precise viewpoint. From there, we will select from the variety of objects fundamentally studied on manifolds (functions, vector fields, pushforwards, pullbacks, exterior derivative, Lie derivatives, differential forms, etc.) and provide a brief overview, depending on the background of the student. An example schedule, for a student with the listed prerequisites, could be something like studying manifolds, functions between manifolds, differential forms on manifolds, and integration of differential forms. A student lacking some of the prerequisites might pursue something less advanced and more tractable, with prerequisites filled in as needed.

Suggested Prerequisites: MATH 381 is required, MATH 521 is highly recommended, MATH 547/577 and/or MATH 550 would be useful

Keywords: topology, manifolds

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Fall 2019.TG3

Mentor: Russell Arnold

Text: *From Geometric Calculus to Clifford Algebra* by David Hestenes

Additional Material: *Geometric Algebra* by Eric Chisholm

Project: This reading course will lead to a new way of understanding the unifying themes behind basic geometric operations (reflection, projection, rotation), Grassmann algebras and complex numbers. We will learn about the so called geometric product, sometimes called the Clifford product, of multivectors (and what a multivector is). The algebra we will study has found favor among some in the physics community for, among other things, aiding in the geometric interpretation of the Dirac equation.

Suggested Prerequisites: MATH 233 and nothing else

Keywords: geometric algebra, Clifford Algebras
Mentor: Bradley Hicks

Text: *Mathematics of Social Choice* by Christoph Börgers

Project: The subject of focus for this project will be a mathematical understanding of voting. While many democratic countries, states, cities, towns, etc. all use voting as a way to construct their government, often voting looks different depending on the laws, rules, or the reason for voting. When we think of voting we often imagine the scenario of two candidates competing for a seat to represent us, but careful examination of your local ballot will show a variety of voting methods with different rules. Our particular interest will be understanding these situations and more importantly exploring ranked-choice voting where voters cast ballots ranking candidates and methods for producing a winner from this information. In terms of policy, we need a mathematical understanding of these methods and how to compare, interpret, and justify the outcomes. The goal of the project will be an understanding of Arrow’s Impossibility Theorem, what it actually means, and to give the beginning math major an interesting avenue into thinking like a mathematician about ideas in social science, economics, and politics. The intrepid student may wish to do some coding and simulations with the methods we discuss, or leap into related subjects like weighted voting.

Suggested Prerequisites: This project is best for a student who has completed Math 381 for experience writing proofs and exposure to sets, functions, and basic combinatorics. The ideal student will have some interest in or knowledge of political policy, economics, or social sciences, as I do not deep knowledge of these fields.

Keywords: voting, social choice, ranked-choice voting, Arrow’s Impossibility Theorem, political science, economics, social science