MATH 152 FINAL EXAMINATION

Spring Semester 2014

NAME: ______________________________________ RAW SCORE: ________________

Maximum raw score possible is 80.

INSTRUCTOR: ________________________ SECTION NUMBER: ________

MAKE and MODEL of CALCULATOR USED: ___________________________________

- Answers are to be given in exact form (e.g., ln(3) and not 1.099, $\sqrt{2}$ and not 1.41, $\pi/4$ and not 0.79, 1/3 and not 0.333333, etc.) unless decimal approximations are requested. <u>Terminating</u> decimal numbers may be written in either decimal or fractional form. For example, $\frac{29}{20} = 1.45$; either form is acceptable. **No credit** will be given for a decimal approximation where an 'exact-form' answer is expected.

- With the exception of symbolic manipulators such as the TI 89-95, calculators (graphing or scientific) are permitted. The use of stored formulas is prohibited.

- You must use calculus when instructed to do so and for all maximization and minimization problems. **No credit** will be given for solutions found by trial and error.

- Show all work on these pages. If you need more room for your work, use the back of a page, and make a note to that effect. Place all answers in the spaces provided.

- **Except for problems otherwise labeled, <u>no credit</u> will be given for correct answers without adequate supporting work. You should also assume that, except for problems otherwise labeled, notation and clarity of writing "count" -- <u>regardless of the format of the problem</u> .**

- There are 11 pages (including this one) and 18 problems on the exam. Except as otherwise noted, each question is worth 4 points. Verify **now** that no problems are missing. If your exam is incomplete, **immediately** obtain a complete copy from your instructor. If you turn in an incomplete exam, the missing problems will receive grades of zero.

- Abide by the Honor Code and, having done so, sign the honor pledge when you finish the exam.

I pledge that I have neither given nor received any unauthorized assistance on this exam.

____

(signature)

1. (6 pts) **Work not graded.**

a. Use the following information about a function *f* and its derivatives to sketch a graph of *f*. Be sure that you clearly indicate

i. any asymptotic behavior as x approaches $-\infty$ or ∞

ii. where the function *f* is increasing and where the function *f* is decreasing

iii. where *f* has local extreme values

iv. where *f* is concave up and where *f* is concave down

v. where *f* has points of inflection

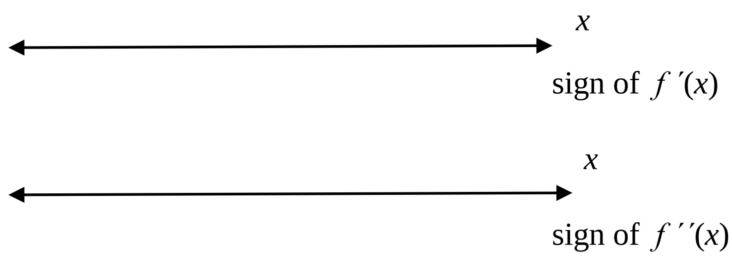
<u>Descriptions.</u>

* $f(1) = 0$; for all other values of x, $f(x) > 0$

* $\lim_{x \to -\infty} f(x) = \infty$ and $\lim_{x \to \infty} f(x) = 0$

* $f'(1) = f'(6) = 0$;
for x in the interval $(1, 6)$, $f'(x) > 0$;
for x in the interval $(-\infty, 1) \cup (6, \infty)$, $f'(x) < 0$

* $f''(3) = f''(9) = 0$;
for x in the interval $(3, 9)$, $f''(x) < 0$;
for x in the interval $(-\infty, 3) \cup (9, \infty)$, $f''(x) > 0$



2. (6 pts) The velocity, in meters per second, of an object traveling horizontally is given by $v(t) = s'(t) = t\sqrt{3t^2 + 6}$ for $t \geq 0$. At time 1 second, the object is at position 5 meters. Note that velocity is non-negative.

Find s(t). How far does the object travel during the interval from 1 second to 5 seconds?

s(t) =	distance traveled is meters

3. (6 points) Let $H(x) = x^4 - 72x^2$. Determine the open interval/s on which H is "simultaneously" both increasing and concave up.

4. (6 pts) Each page of a publicity flyer is to have 1-inch margins at the top and bottom and $\frac{1}{2}$ –inch margins on the sides. The total area of each page is to be 72 square inches. Use calculus to determine the dimensions (length and width; see the figure) that the page should have in order to result in the maximal area for the printed matter in the center. Use <u>either</u> the first <u>or</u> the second derivative test to verify that you have a maximum for area.

Length, L, is ________________ in.; width, W, is __________________ in.

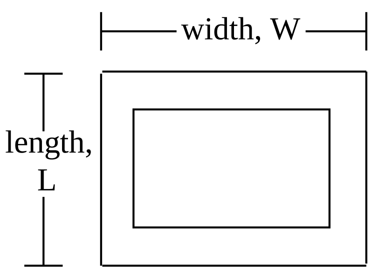


FIGURE IS NOT TO SCALE

5. Use an algebraic method to find, if it exists, $\lim_{x \to 2} \left(\frac{x^2 - x - 2}{x^2 + x - 6} \right)$. No credit for other methods. Your answer may be one of the following: a finite real number, ∞, − ∞, or the abbreviation "DNE" for "The limit does not exist."

6. If $f(x) = e^{7x}\left(3x + e^{-5x}\right)$, find $f''(x)$.

$f''(x) =$

7. If x denotes the number of widgets supplied and p denotes the unit price of widgets in dollars, the supply function is given by $p = 10 + 0.01x$. Use calculus to find the producers' surplus (in dollars) if the unit price is set at 50 dollars.

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8. If $Q(x) = \left(\dfrac{15 + x}{3 + x^2}\right)^{3/2}$, find $Q'(1)$.

$Q'(1) =$

9. Let $f(x) = 3x - 2x^2$. Use the limit definition of the derivative to find $f'(x)$. No credit for use of any other method.

$f'(x) =$

10. Find the average value of $f(x) = \frac{1}{x}$ over the interval $[-5, -1]$.

11. On the coordinate grid below, sketch and **<u>shade</u>** the region D in Quadrant I enclosed by the graphs of $g(x) = 4x$ and $h(x) = x^3$. **Label** the first-quadrant point/s of intersection of the two curves with (x, y)-coordinates. Which one of the following integrals gives the area of region D? DO NOT EVALUATE.

a. $\int_0^8 (x^3 - 4x)\,dx$ b. $\int_0^8 (4x - x^3)\,dx$

c. $\int_0^2 (x^3 - 4x)\,dx$ d. $\int_0^2 (4x - x^3)\,dx$

e. $\int_2^8 (4x + x^3)\,dx$

12. Given that $H(x) = 65 + 3(x-4)^{2/3}$ find the absolute minimum and absolute maximum values of H on the interval [3, 12].

minimum value of H is	; this occurs at x =
maximum value of H is	; this occurs at x =

13. A spherical balloon has radius r and volume $V = \dfrac{4\pi r^3}{3}$. Use differentials to approximate the change in the balloon's volume if its radius decreases from 10 cm to 8 cm. Express your answer as a multiple of π.

Change in volume $\approx$ (π) cm^3

4. Let $f(x) = \begin{cases} ax + 4, & \text{if } x < 3 \\ b, & \text{if } x = 3 \\ x^2, & \text{if } x > 3 \end{cases}$.

Use the three conditions for continuity of a function at a point to find the values of a and b that make f continuous at 3. Use complete sentences in describing your results.

a =	b =

15. Let $f(x) = 4 + \left(\ln(x^2)\right)^3$ for $x > 0$.

a. Find $f'(x)$.

$f'(x) =$

b. Find (in point-slope form) the equation of the line tangent to the graph of f at the point with x-coordinate e.

$y -$ _______________ = ____________ ($x -$ ______________)

16. Let $f(x) = \dfrac{72x^5 + 180x^4}{3x - 2x^4}$.

a. Use an algebraic method to find, if it exists, $\lim_{x \to \infty} f(x)$. No credit for other methods.

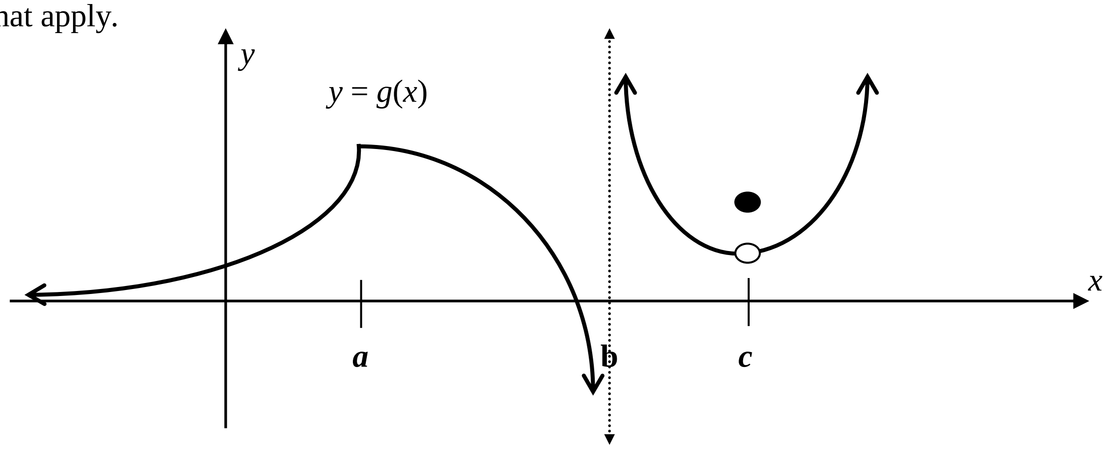
Your answer may be one of the following: a finite real number, ∞, $-\infty$, or the abbreviation "DNE" for "The limit does not exist."

$\lim_{x \to \infty} f(x) =$

b. **Work not required for this part.** Give the equation of the horizontal asymptote, if any, corresponding to the limit in part a. If there is no such asymptote, write "None."

equation of asymptote:

17. **Work not required.** Which of the following describe the graph of g shown below? Circle all that apply.



(i) g is differentiable at c

(ii) g is differentiable at a

(iii) the line $y = 0$ is an asymptote for g

(iv) $\lim_{x \to b^-} g(x) = \infty$

(v) the line $x = b$ is an asymptote for g

(vi) g is continuous at a

18. Students at Light Blue U held a Dance Marathon to benefit the children's hospital there. The number of students who had registered to volunteer t days after the start of the sign-up campaign was modeled by $Q(t) = \dfrac{400}{1 + 49e^{-0.4t}}$.

(a) At the very beginning of the campaign, only the Dance Marathon organizers were registered. How many organizers were there?

(b) Some readers of the campus newspaper complained that the volunteer registration campaign was lasting forever. If registration could have continued indefinitely, what value would the number of volunteer registrations have approached?

HAVE YOU FOLLOWED THE INSTRUCTIONS ON THE FIRST PAGE? HAVE YOU ABIDED BY THE HONOR CODE? IF SO, SIGN THE PLEDGE ON PAGE 1 BEFORE YOU TURN IN YOUR EXAM.