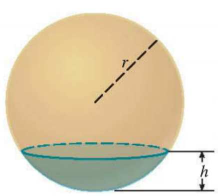
Math 232 Final Exam Review Questions

Sections covered: 6.1, 6.2, 6.4, 6.5, 7.1, 7.2 (powers of sine and cosine only), 7.3, 7.4, 7.5, 7.8, 10.1, 10.2, 11.1, 11.2, 11.3, 11.4, 11.5, 11.6, 11.7, 11.8, 11.9, 11.10, 11.11

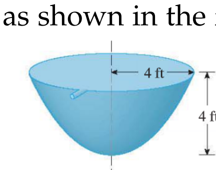
Note: There will be some true false and multiple choice concept questions on the exam. I recommend working true false questions and concept checks in the chapter review sections for practice.

Note: The following problems are mostly from the review problems in the textbook.

1. Find the area of the region bounded by the curves:
   (a) $y = 1 - 2x^2, y = |x|$
   (b) $x + y = 0, x = y^2 + 3y$
   (c) $y = a\sqrt{x}, y = x^2$
2. Set up the integral to find the volume of the solid obtained by rotating the region bounded by the given curves about the specified axis.
   (a) $x = 0, x = 9 - y^2$, about $x = -1$
3. Each integral represents the volume of a solid. Describe the solid.
   (a) $\int_0^{\pi/2} 2\pi \cos^2 x \, dx$
   (b) $\int_0^{\pi} \pi(4 - \sin^2 x) \, dx$
   (c) $\int_0^{\pi} \pi(2 - \sin x)^2 \, dx$
4. The base of a solid is a square with vertices located at $(1, 0), (0, 1), (-1, 0), (0, -1)$. Each cross-section perpendicular to the x-axis is a semicircle. Find the volume of the solid.
5. A monument in the shape of a square pyramid has height 20 meters. Its base is a square of side length 5 meters. Find the volume of the monument.
6. (p. 459 # 5a) Show that the volume of a segment of height h of a sphere of radius r is $V = \frac{1}{3}\pi h^2(3r - h)$.



7. A force of 30 N is required to maintain a spring stretched from its natural length of 12 cm to a length of 15 cm. How much work is done in stretching the spring from 12 cm to 20 cm?
8. A 1600 lb elevator is suspended by a 200 ft cable that weighs 10 lb/ft. How much work is required to raise the elevator from the basement to the third floor, a distance of 30 ft?
9. A tank full of water has the shape of a paraboloid of revolution as shown in the figure. That is, its shape is obtained by rotating a parabola about a vertical axis.



   (a) If its height is 4 ft and the radius at the top is 4 ft, find the work required to pump the water out of the tank.

(b) After 4000 ft-lb of work has been done, what is the depth of the water remaining in the tank?

10. A steel tank has the shape of a circular cylinder oriented vertically with diameter 4 m and height 5 m. The tank is currently filled to a level of 3 m with cooking oil that has a density of 920 kg/m^3. Compute the work required to pump the oil out through a 1-m spout at the top of the tank.

11. Find the average value of the function $f(t) = t\sin(t^2)$ on the interval $[0, 10]$.

12. Integrate by hand:

   (a) $\int \frac{dt}{2t^2+3t+1}$
   (b) $\int_0^{\pi/2} \sin^3\theta \cos^2\theta \, d\theta$
   (c) $\int_1^2 \frac{\sqrt{x^2-1}}{x} \, dx$
   (d) $\int_0^{\pi/6} t\sin 2t \, dt$
   (e) $\int_1^2 x^5 \ln x \, dx$
   (f) $\int \frac{e^{2x}}{1+e^{4x}} \, dx$
   (g) $\int \frac{x^2+2}{x+2} \, dx$
   (h) $\int e^x \cos x \, dx$

13. Evaluate the integral or prove that it is divergent.

   (a) $\int_0^4 \frac{\ln x}{\sqrt{x}} \, dx$
   (b) $\int_0^\infty \frac{\ln x}{x^4} \, dx$
   (c) $\int_0^1 \frac{1}{2-3x} \, dx$
   (d) $\int_1^\infty \frac{2+\sin x}{\sqrt{x}} \, dx$

14. Determine if the integral converges or diverges and prove your answer.

   (a) $\int_1^\infty \frac{1}{\sqrt{1+x^4}} \, dx$
   (b) $\int_1^\infty \frac{x+1}{\sqrt{x^4-x}} \, dx$
   (c) $\int_0^1 \frac{\sec^2 x}{x\sqrt{x}} \, dx$

15. Find the length of the curve $y = \frac{x^4}{16} + \frac{1}{2x^2}$, $1 \le x \le 2$

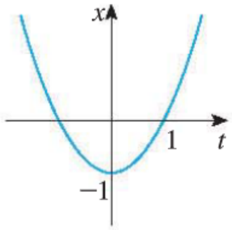
16. Find parametric equations for the following curves:

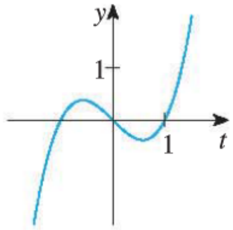
   (a) The curve $y = \sqrt{x}$
   (b) The curve $(x-3)^2 + (y-5)^2 = 36$
   (c) The line segment between the points $(-2, 5)$ and $(3, 7)$.

17. Sketch the parametric curve and eliminate the parameter to find the Cartesian equation of the curve.

   (a) $x = 2\cos\theta, y = 1 + \sin\theta$
   (b) $x = t^2 + 4t, y = 2 - t, -4 \le t \le 1$

18. Use the graphs of $x = f(t)$ and $y = g(t)$ to sketch the parametric curve $x = f(t)$, $y = g(t)$. Indicate with arrows the direction in which the curve is traced as t increases.





19. Find the length of the curve $x = 3t^2, y = 2t^3$ between the origin and the point $(12, 16)$ on the x-y plane.

20. Determine whether the sequence is convergent or divergent. If it is convergent, find its limit.

(a) $a_n = \frac{2+n^3}{1+2n^3}$
(b) $a_n = \cos(n\pi/2)$
(c) $a_n = \frac{n \sin n}{n^2+1}$
(d) $a_n = \frac{\ln n}{\sqrt{n}}$
(e) $\left\{(1 + 3/n)^{4n}\right\}$
(f) $\left\{\frac{(-10)^n}{n!}\right\}$
(g) $a_n = \frac{(-1)^n 3^n}{2^{2n}}$

21. A series $\sum_{n=1}^{\infty} a_n$ has partial sums $s_n = 2 - (\frac{1}{3})^n$. Decide whether the series converges or diverges. Justify your answer. If it converges, find the sum.

22. Determine whether the series converges or diverges. Justify your answer. State the convergence test and check that any necessary conditions apply.

(a) $\sum_{n=1}^{\infty} \frac{1}{n+3^n}$
(b) $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n+2}$
(c) $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2+2}$
(d) $\sum_{n=1}^{\infty} \frac{n^2 2^{n-1}}{(-5)^n}$
(e) $\sum_{n=2}^{\infty} \frac{2n^2-3n+6}{n^3-1}$
(f) $\sum_{n=1}^{\infty} \left(\frac{1}{n^3} + \frac{1}{3^n}\right)$
(g) $\sum_{k=1}^{\infty} \frac{1}{k\sqrt{k^2+1}}$
(h) $\sum_{n=1}^{\infty} \frac{3^n n^2}{n!}$
(i) $\sum_{n=1}^{\infty} \frac{n!}{n^n}$
(j) $\sum_{n=1}^{\infty} \frac{\sin 2n}{1+2^n}$
(k) $\sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2n-1)}{2 \cdot 5 \cdot 8 \cdot \cdots \cdot (3n-1)}$
(l) $\sum_{n=1}^{\infty} \frac{1}{n \ln n}$

23. Determine whether the series is conditionally convergent, absolutely convergent, or divergent. Justify your answer.

(a) $\sum_{n=2}^{\infty} \frac{(-1)^n \sqrt{n}}{\ln n}$
(b) $\sum_{n=2}^{\infty} \frac{(-1)^n \ln n}{\sqrt{n}}$

24. Find the sum of the series.

(a) $\sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{2^{3n}}$

(b) $\sum_{n=4}^{\infty} \frac{3}{n^2-4}$

(c) $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^n}{3^{2n}(2n)!}$

(d) $1 - e + \frac{e^2}{2!} - \frac{e^3}{3!} + \frac{e^4}{4!} - \cdots$

(e) $\dfrac{1}{1 \cdot 2} - \dfrac{1}{3 \cdot 2^3} + \dfrac{1}{5 \cdot 2^5} - \dfrac{1}{7 \cdot 2^7} + \cdots$

25. Express $10.1\overline{35} = 10.135353535353535$ as a ratio of integers.

26. Find the partial sum s_3 for the series and estimate the error in using it as an approximation for the sum of the series.

(a) $\sum_{n=1}^{\infty} 1/n^6$

(b) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{3n^2+n}$

27. For each of the above two sequences, how many terms are needed to approximate the sum to within 0.001?

28. Find the radius of convergence and the interval of convergence of the series.

(a) $\sum_{n=1}^{\infty} (-1)^n \frac{x^n}{n^2 5^n}$

(b) $\sum_{n=1}^{\infty} \frac{2^n (x-2)^n}{(n+2)!}$

(c) $\sum_{n=0}^{\infty} \frac{2^n (x-3)^n}{\sqrt{n+3}}$

29. Find the Taylor series of $f(x) = \sin x$ at $a = \pi/6$

30. Find the Maclaurin series for f and its radius of convergence.

(a) $f(x) = \frac{x^2}{1+x}$

(b) $f(x) = \ln(4-x)$

(c) $f(x) = xe^{2x}$

(d) $f(x) = 10^x$

(e) $f(x) = \sin(x^4)$

(f) $f(x) = 6x^3 - 4x^2 + 2x + 7$

31. Use series to approximate $\int_0^1 x \arctan(x^4)\, dx$ correct to two decimal places.

32. Use series to evaluate the following limit: $\lim_{x\to 0} \frac{\sin x - x}{x^3}$

33. Use a degree 3 Taylor polynomial, centered at $a = 1$, to approximate $f(x) = \ln(1 + 2x)$.
Use Taylor's inequality to estimate the accuracy of the approximation when $0.5 \le x \le 1.5$.