1. Find the slope of the line tangent to the curve $x + \frac{xy^2}{\pi^2} + \tan y = 2$ at the point $(1, \pi)$

A. $\frac{2\pi^2}{\pi + 2}$

B. $-\frac{2\pi}{\pi + 2}$

C. $-\pi$

D. $\pi$

E. $\pi^2 - 2$

F. $-\frac{2\pi^2}{\pi^2 + 2}$

G. $\frac{2 + \pi}{\pi}$

2. Find the exact area bounded by the curve $y = x^2 - 1$, the $x$-axis, $x$ = -1, and $x$ = 3.

A. $\frac{16}{3}$

B. $\frac{20}{3}$

C. 6

D. $\frac{8}{3}$

E. 8

F. $\frac{26}{3}$

G. 4

3. Use L'Hospital's Rule to determine the limit. $A$ and $C$ are constants. $C, A > 0$

$$\lim_{x \to \infty} \frac{\ln(A + e^{Cx})}{x}$$

A. $e^C$

B. 1

C. $C$

D. $\infty$

E. $A$

F. 0

G. $C^2$

4. Use logarithmic differentiation to find the derivative of $y = (1 + \frac{1}{x})^{2x}$

A. $\frac{dy}{dx} = 2(1 + \frac{1}{x})^{2x}\left[1 + \frac{1}{x} - \frac{1}{x+1}\right]$

B. $\frac{dy}{dx} = 4x(1 + \frac{1}{x})^{2x}\left[\ln(1 + \frac{1}{x})\right]$

C. $\frac{dy}{dx} = 2(1 + \frac{1}{x})^{2x}\left[\ln(1 + \frac{1}{x}) - \frac{x^2}{x+1}\right]$

D. $\frac{dy}{dx} = 2(1 + \frac{1}{x})^{2x}\left[\ln(1 + \frac{1}{x}) - \frac{1}{x+1}\right]$

E. $\frac{dy}{dx} = 2\left[\ln(1 + \frac{1}{x})\right]^x$

5. If $g(x)=\int_0^{x^2} f(t)\ dt = x\ln x$ , find $f(1)$.

A. $\frac{1}{2}$ F. 1

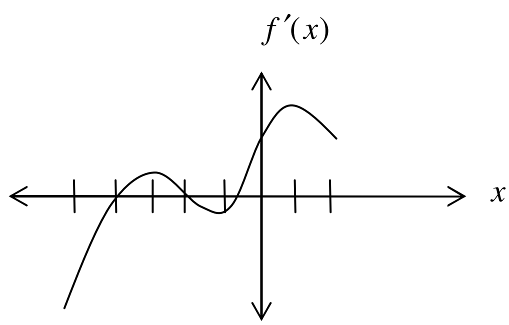
B. $-\frac{1}{2}$ G. 0

C. $\sqrt{2}$

D. $-\sqrt{2}$

E. $-1$

6. Given the graph of $f'(x)$ on [-5,2], answer the following questions about the graph of $f(x)$. Answers to the nearest integer.




Where is the graph of $f(x)$ simultaneously increasing and concave down? ___________________

7. Suppose a function $f(x)$ is differentiable everywhere and $f(0)=2$ , $f'(x)\le 4$ . Apply the Mean Value Theorem to the interval [0,6] to determine how large $f(6)$ can possibly be.

A. 4 F. 24

B. 26 G. none of these

C. 2

D. 22

E. 6

8. Find $y'$ if $y=\sin^{-1}(x^{3/2})$ .

A. $\dfrac{x^{3/2}}{\sqrt{1-x^2}}$ F. $\dfrac{3x^{1/2}}{2\sqrt{1-x^3}}$

B. $\dfrac{3x^{1/2}}{2\sqrt{1-x^2}}$ G. none of these

C. $\dfrac{-3x^{1/2}}{2\sqrt{1-x^3}}$

D. $\dfrac{-x^{3/2}}{\sqrt{1-x^3}}$

E. $\dfrac{-3x^{1/2}}{2\sqrt{1-x^2}}$

9. Find $B$ such that $\lim_{x\to -3^-} \dfrac{x^2-4}{x^2+7x+B} = -\infty$ .

10. Consider a function $g(x)$ with derivative $g'(x) = x^3(x-2)^2(x+8)^9$.
For what value(s) of $x$ does $g(x)$ have a local maximum?

A. 0
B. 0, 2
C. 0, 2, -8
D. 2
E. 2, -8
F. -8
G. 0, -8

11. Given $g(1)=6$ , $g'(1)=-1$, find $\dfrac{d}{dx}\left(\dfrac{2\cdot g(x)}{x^2+1}\right)$ when $x=1$.

A. $-24$
B. $-15$
C. $-7$
D. $-1$
E. 20
F. 1
G. 7

12. The following limit represents the derivative of some function $f(x)$ at some number $a$. State such an $f(x)$ and $a$.

$$\lim_{h\to 0}\frac{(1+h)^8-1}{h}$$

$f(x)=$ ☐

a = ☐

13. Consider the graph of $f(x)=\dfrac{x-B}{x^2-B^2}$ , $B$ a constant.
At what value(s) of $x$ does a hole (removable discontinuity) in the graph occur?

A. $-B^2$ F. $B$

B. $B^2$ G. $-B$

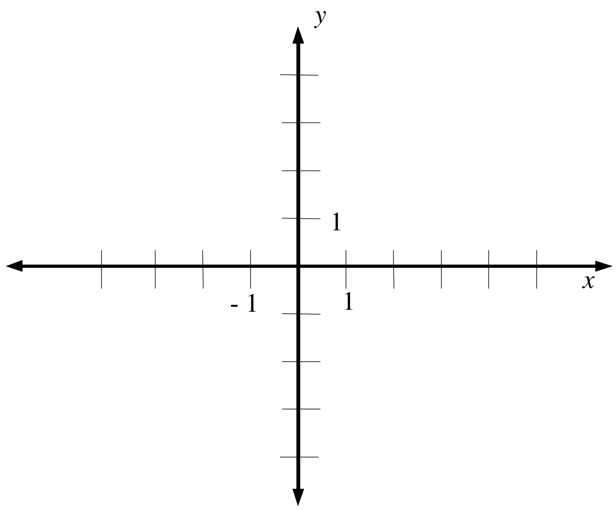
C. $\pm B^2$

D. $\pm B$

E. no holes

14. Sketch the graph of a function $f(x)$ that has ***exactly one discontinuity*** and satisfies the following conditions.

- $f(2) = 4$
- $f'(2)$ does not exist
- $f'(x) > 0$ on $(-\infty,-1) \cup (-1,2)$, $f'(x) < 0$ on $(2,\infty)$
- $f''(x) > 0$ on $(-\infty,-1) \cup (-1,2) \cup (2,\infty)$
- $\lim_{x \to \pm\infty} f(x) = 1$, $\lim_{x \to -1^+} f(x) = 3$, $\lim_{x \to -1^-} f(x) = \infty$




15. Given the function $f(x) = x^{2/3}$,

a) Find the linear approximation, L($x$), to the function at $a = 8$.

L($x$) =

b) Use your result from part (a) to approximate the value of the function at $x = 7$ .
Leave answer in fraction form. Do not use your calculator.

$7^{2/3} \approx$

16. A particle moves along a path described by $y = 4 - x^2$. At what point along the curve are $x$ and $y$ changing at the same rate?

A. $(-\frac{1}{2}, \frac{15}{4})$ F. $(2,0)$

B. $(-2,0)$ G. $(1,3)$

C. $(\frac{1}{2}, \frac{15}{4})$

D. always at the same rate

E. never at the same rate

17. A rectangle with base on the $x$-axis has its upper vertices on the curve $y = 12 - x^2$. Find the maximum area of such a rectangle.

A. 144 F. 32

B. 2 G. 16

C. 8

D. 12

E. 64

18. a) Express the limit of a Riemann sum, $\lim_{n\to\infty}\sum_{i=1}^{n}\left(5+\left(x_i^*\right)^2\right)\Delta x$ on [2,5], as a definite integral.

b) Find the horizontal asymptote for the graph of $f(x)=\dfrac{Ax^4-Bx^2}{C-Dx^4}$ . $A,B,C,D$, constant.

$y=$

19. A side of a square field is measured to be 10 feet with a possible error of $\frac{1}{12}$ $ft$. Use differentials to approximate the maximum error in the calculated area of the field.

A. $\frac{25}{3}$ $ft^2$

B. 20 $ft^2$

C. $\frac{5}{3}$ $ft^2$

D. $\frac{5}{6}$ $ft^2$

E. 121 $ft^2$

F. 100 $ft^2$

G. none of these

20. If Newton's method is used to solve $2x^3 + 2x + 1 = 0$ with first approximation $x_1$ = -1, what is the second approximation, $x_2$ ?

A. $-\frac{1}{2}$ F. $-\frac{3}{4}$

B. $-\frac{5}{8}$ G. $-\frac{7}{8}$

C. $-\frac{3}{8}$

D. $-\frac{3}{5}$

E. $-\frac{1}{4}$

21. On a certain planet, a ball is thrown upward with a speed of 33 feet per second from the edge of a cliff 44 feet above the ground. When does it hit the ground? (use $a(t) = -22$ )

A. $1s$ F. $11s$

B. $4\,s$ G. none of these

C. $\frac{3}{2}s$

D $55\,s$

E. $\frac{3+\sqrt{41}}{4}\,s$

22. Determine the absolute minimum value of the function $f(x) = e^{\cos x}$ .

**Show all work. Exact value only. No credit for calculator approximations.**

absolute min value =

23. Evaluate the definite integral.
**Exact answer only - no calculator approximations**. **Show work.**

$$\int_1^3 \frac{x \;\; - 4x^2}{x^3}\, dx$$

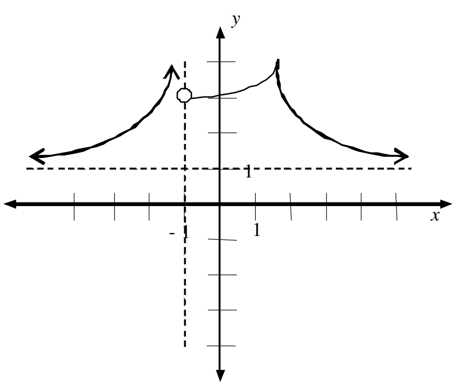
24. Evaluate the integral.

$$\int \left(e^{-5x} - 3\sec^2 x\right) dx$$

25. Evaluate the definite integral $\int_0^{\frac{\pi}{2}} \frac{\cos x}{\sqrt{2+\sin x}}\, dx$. **Show all work for credit. Exact answer only - no calculator approximations.**

Answers

1. B
2. E
3. C
4. D
5. A
6. $(-3,-2)\cup(1,2)$
7. B
8. F
9. 12
10. F
11. C
12. $f(x)=x^8$, $a=1$
13. F
14.




15. a) $L(x)=\frac{x}{3}+\frac{4}{3}$,

    b) $7^{2/3}\approx\frac{11}{3}$
16. A
17. F
18. a) $\int_2^5 \left(5+x^2\right)dx$

    b) $y=-\frac{A}{D}$
19. C
20. B
21. B
22. $e^{-1}$
23. $\frac{2}{3}-4\ln 3$
24. $-\frac{1}{5}e^{-5x}-3\tan x+c$
25. $2\left(\sqrt{3}-\sqrt{2}\right)$